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Lecture ⑬ Markov chains ch 3

تابع

* Transition probs

$$P_{ij} = pr\{\underset{\text{المستقبل}}{X_{n+1}=j} \mid \underset{\text{الحاضر}}{X_n = i}\}$$ Defn of Markov

* $pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_n = i_n\}$ Joint prob. theorem

$$= P_{i_0} P_{i_0 i_1} P_{i_1 i_2} \cdots P_{i_{n-1} i_n}$$

we get $P_{i_0} = pr\{X_0 = i_0\}$ from initial dist n

pb 3.1.1 p. 81 Textbook

Given

$$P = \overbrace{\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0.1 & 0.2 & 0.7 \\ 1 & 0.9 & 0.1 & 0 \\ 2 & 0.1 & 0.8 & 0.1 \end{array}}^{\text{states}} \Rightarrow \text{Transition prob. Mx}$$

states

$P_0 = pr\{X_0 = 0\} = 0.3$, $P_1 = pr\{X_0 = 1\} = 0.4$ and $P_2 = pr\{X_0 = 2\} = 0.3$

⇒ initial distribution

Determine $pr\{X_0 = 0, X_1 = 1, X_2 = 2\}$?

Ans: $pr\{X_0 = \overset{i_0}{0}, X_1 = \overset{i_1}{1}, X_2 = \overset{i_2}{2}\}$

$$= P_{i_0} P_{i_0 i_1} P_{i_1 i_2} = P_0 P_{01} P_{12}, \quad P_0 = pr\{X_0 = 0\} = 0.3$$

$$= 0.3(0.2)(0) = 0$$

هنا العملية بدأت من state 0

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ملاحظة

لو كان المطلوب هو

$pr\{X_0 = 1, X_1 = 0, X_2 = 2\}$

[illegible]

$pr\{X_0 = 1, X_1 = 0, X_2 = 2\}$

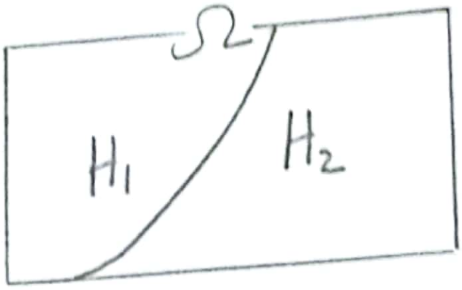
$$= P_1 P_{10} P_{02}, \quad P_1 = pr\{X_0 = 1\} = 0.4$$

$$= 0.4(0.9)(0.7)$$

$$= 0.252$$

هنا العملية بدأت من state 1

prop for Conditional prob.



- For any two events A_1 and A_2

$pr(A_1, A_2 | H_i)$

$= pr(A_1 | A_2, H_i)\, pr(A_2 | H_i) \quad , \; i = 1, 2$

where $H_1 \cup H_2 = \Omega$ (Sample space)

pb 3.1.2 p. 81
Textbook

$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0.7 & \boxed{0.2} & 0.1 \\ 1 & 0 & \boxed{0.6} & 0.4 \\ 2 & 0.5 & 0 & 0.5 \end{array}$$

states

states

الماضي

① $pr\{X_2 = 1, X_3 = 1 | X_1 = 0\}$

$= pr\{X_3 = 1 | X_2 = 1, \cancel{X_1 = 0}\} \cdot pr\{X_2 = 1 | X_1 = 0\}$

المستقبل الحاضر

prop. of Cond. prob.

$= pr\{X_3 = 1 | X_2 = 1\} \cdot pr\{X_2 = 1 | X_1 = 0\}$

Markov prop.

$= P_{11} \, P_{01} = 0.6\,(0.2) = 0.12$

ii) $pr\{X_1=1, X_2=1 \mid X_0=0\}$

$$= pr\{\underset{\text{المستقبل}}{X_2=1} \mid \underset{\text{الحاضر}}{X_1=1}, \cancel{X_0=0}\} \cdot pr\{X_1=1 \mid X_0=0\}$$

prop. of Cond. prob.

$$= pr\{X_2=1 \mid X_1=1\} \cdot pr\{X_1=1 \mid X_0=0\}$$

Markov prop.

$$= P_{11}\, P_{01} = 0.6(0.2) = 0.12$$

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Pb 3.1.3 p. 83 Textbook

let G → for good product item

D → " bad " "

Given
$$\begin{cases} P_{GG} = \alpha \quad , \quad P_{GD} = 1-\alpha \\ P_{DD} = \beta \quad , \quad P_{DG} = 1-\beta \end{cases}$$

حل: If the 1st item is good, What is the prob. that the first defective item to appear is the fifth item ?

Ans:

$$pr\{X_2=G, X_3=G, X_4=G, X_5=D \mid X_1=G\}$$

$$pr\{X_2=G, X_3=G, X_4=G, X_5=D \mid X_1=G\}$$

$$= pr\{X_5=D \mid X_4=G, \cancel{X_3=G}, \cancel{X_2=G}, \cancel{X_1=G}\}$$

$$\cdot\, pr\{X_4=G \mid X_3=G, \cancel{X_2=G}, \cancel{X_1=G}\}\; pr\{X_2=G \mid X_1=G\}$$

$$\cdot\, pr\{X_3=G \mid X_2=G, \cancel{X_1=G}\}$$

By using Cond. prob. prop.

By using Markov def$\underline{n}$

$$= P_{GD}\, P_{GG}^3 = (1-\alpha)\,\alpha^3 = \alpha^3(1-\alpha)$$

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