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Lecture (11) M107

Determinants

• **Evaluating Determinant by using row operations**

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & 4 & 8 \end{bmatrix}$, find each of the following:

(i) $\det(A)$

(ii) If $A_1 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 8 \\ 0 & 1 & 2 \end{bmatrix}$, find $\det(A_1)$

(iii) If $A_2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, find $\det(A_2)$

(iv) If $A_3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$, find $\det(A_3)$

Ans: (i) $\det(A) = \cancel{1(8-8)} - \cancel{0} + 2(4-3) = \boxed{2}$ by using 1st column

or $\det(A) = \cancel{1(8-8)} - 2(0-4) + 3(0-2)$ by using 1st row

$= 8 - 6 = 2$

(ii) $\det(A_1) = \cancel{1(8-8)} - 2(4-3) + 0 = \boxed{-2}$ by using 1st column

Note that: $A \Rightarrow A_1$ by interchange of two rows
then $\det(A_1) = -\det(A)$

(iii) $\det(A_2) = \cancel{1(4-4)} - 0 + 1(4-3) = 1$ by using 1st column

i.e $\det(A_2) = \frac{1}{2}\det(A)$

Note that: $A \Rightarrow A_2$ by multiplication of one row of A by a constant k then $\det(A_2) = k\det(A)$, $k \neq 0$

$\frac{\cdot}{2}$

In our Example, We multiply 3rd row of Matrix A by $\frac{1}{2}$ to get Matrix A_2

$\therefore \det(A_2) = \frac{1}{2}\det(A)$

$= \frac{1}{2}(2) = 1$

(iv) $\det(A_3) = 1(2-0) = 2$ by using 1st column

or $\det(A_3) = 2(1-0) = 2$ by using 3rd row

or $\det(A_3) = 1(2-0) - \cancel{2(0-0) + 3(0-0)} = 2$ by using 1st row

Note that: $A \Rightarrow A_3$ by addition of a multiple of one row to another row then $\det(A_3) = \det(A)$

In our Example, We multiply the 1st row of A by -2, then add to 3rd row (i.e $-2R_1 + R_3$) to get Matrix A_3

, So $\det(A_3) = \det(A) = 2$

* Some prop.s for Determinants

(1) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 0 & 0 \\ 4 & 5 & 6 \end{bmatrix} \Rightarrow \det(A) = |A| = 0$ (row of Zeros)

(2) If $A = \begin{bmatrix} 0 & 1 & -4 \\ 0 & 2 & 5 \\ 0 & 3 & 7 \end{bmatrix} \Rightarrow \det(A) = 0$ (column of Zeros)

(3) If $A = \begin{bmatrix} 1 & -7 & 5 \\ 0 & 2 & 4 \\ 0 & 0 & -3 \end{bmatrix} \Rightarrow \det(A) = 1(2)(-3) = -6$

product of the diagonal elements

upper triangular Mx

(4) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ -5 & 3 & 3 \end{bmatrix} \Rightarrow \det(A) = 1(2)(3) = 6$

Lower Triangular MX

(5) If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \Rightarrow \det(A) = -1(2)(-3) = 6$

diagonal MX

(6) If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -4 & 6 \\ 5 & 7 & 0 \end{bmatrix} \Rightarrow \det(A) = 0$

If we have two proportional rows or (Columns) in a Matrix then its determinant equals Zero (Vanishes) ينعدم (ويتلاشى)

* **Check**

$\det(A) = 1(0-42) + 2(0-30) + 3(14+20)$ by using 1st row of A

$\det(A) = -42 - 60 + 102 = 0$

EX By using Row Reduction, evaluate $\det(A)$ where $A = \begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}$

Ans

$R_1 \leftrightarrow R_2 \Rightarrow \det(A) = - \begin{vmatrix} 3 & -6 & 9 \\ 0 & 1 & 5 \\ 2 & 6 & 1 \end{vmatrix} = -3 \begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 2 & 6 & 1 \end{vmatrix}$

$-2R_1 + R_3 \Rightarrow \det(A) = -3 \begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 10 & -5 \end{vmatrix}$

$-10R_2 + R_3 \Rightarrow \det A = -3 \begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & -55 \end{vmatrix} = -3(-55) \begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{vmatrix}$

$\det A = -3(-55)(1) = 165$