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Math 107: Lecture 1

Textbook: ~~Linear Algebra~~ by H. Anton

- System of linear equations

$AX = b$, where $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$ Coefficient Matrix

Matrix form

$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$, $b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m\times 1}$ Matrix of Constant terms

Variables (unknown) Matrix

* Solving system of linear Eqns by using Gaussian elimination method

Consider the following system of linear Eqns

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Coefficients Mx Variables Mx Constants Mx

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Step ① Form Augmented Matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix}$$

Step ② By using elementary row operations

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & A_{12} & A_{13} & B_1 \\ 0 & 1 & A_{23} & B_2 \\ 0 & 0 & 1 & B_3 \end{bmatrix}$$

row echelon form

المصفوفة تكون على صيغة

درجية صفية.

Step ③

Find the solution by back-substitution.

EX Solve the following system of linear Equations by using Gaussian-elimination method

$$x_1 + x_2 + 2x_3 = 8$$

$$-x_1 - 2x_2 + 3x_3 = 1$$

$$3x_1 - 7x_2 + 4x_3 = 10$$

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Ans: the augmented Mx is

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix}$$

R → Row

$R_1 + R_2$, $-3R_1 + R_3$

$$\Longrightarrow \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix}$$

$-10R_2 + R_3$

$$\Longrightarrow \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -104 \end{bmatrix}$$

$-R_2$, $-\frac{1}{52}R_3$

$$\Longrightarrow \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

row echelon form (ref)

$$\Longrightarrow \quad x_3 = 2 \quad (1)$$

$$x_2 - 5x_3 = -9 \quad (2)$$

$$x_1 + x_2 + 2x_3 = 8 \quad (3)$$

Subs. (1) in (2) we get $x_2 = 1$

$\therefore$ (3) $\Rightarrow$ $x_1 = 3$

$\therefore$ The soln is $x_1 = 3$, $x_2 = 1$, $x_3 = 2$