PHYS 201 : Problems in Determinants and Gauss Elimination

1. Use the method of augmented matrix to solve the following system:

$$\begin{aligned} 5x + 11y - 21z &= -22 \\ x + 2y - 4z &= -4 \\ 3x - 2y + 3z &= 11. \end{aligned}$$

2. Use the method of augmented matrix to solve the following system:

$$\begin{aligned} x + y - 2z + 3w &= 4 \\ 2x + 3y + 3z - w &= 3 \\ 5x + 7y + 4z + w &= 5 \end{aligned}$$

3. Use the method of augmented matrix to solve the following system:

$$\begin{aligned} x + y - 2z + 4t &= 5 \\ 2x + 2y - 3z + t &= 3 \\ 3x + 3y - 4z - 2t &= 1 \end{aligned}$$

4. Use the method of augmented matrix to solve the following system:

$$\begin{aligned} x + 2y + z &= 5 \\ 2x + 5y - z &= -4 \\ 3x - 2y - z &= 5 \\ 4x + y - 3z &= -2 \end{aligned}$$

5. For the following matrices find their inverse ones $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 4 & 1 & 5 \end{pmatrix}$$

6. The following matrices are called *triagonal.* Show that the detrminant of a triagonal matrix is just the product of the its diagonal elements

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}, \quad \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

7. Show that:

$$\begin{vmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & e & f \\ 0 & 0 & g & h \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \cdot \begin{vmatrix} e & f \\ g & h \end{vmatrix}.$$

8. If **A**, **B** are diagonal matrices show that

$$\det(\mathbf{A}\cdot\mathbf{B}) = \det(\mathbf{A})\cdot\det(\mathbf{B}).$$

9. Show that:

$$\begin{vmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

10. Show that:

$$\begin{vmatrix} a_{11}+ka_{21} & a_{12}+ka_{22} & a_{13}+ka_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

11. Show that:

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{vmatrix} = (x-y)(y-z)(z-x)$$

12. Show that:

$$\begin{vmatrix} 1+a^2 & a & 1 \\ 1+b^2 & b & 1 \\ 1+c^2 & c & 1 \end{vmatrix} = (a-b)(b-c)(a-c)$$

13. Show that the numbers a, b are roots of the equation

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix} = 0.$$

14. If $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 8$ what is

$$\begin{vmatrix} 2a_{11}-3a_{21} & 2a_{12}-3a_{22} & 2a_{13}-3a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \end{vmatrix} = ?$$

15. Use determinants to solve the following system:

$$\begin{aligned} 2x+3y-w&=1 \\ 4x+y+2w&=5 \\ x-y+w&=2 \end{aligned}$$

16. Solve the following system

$$\begin{aligned} 3x+2y&=7 \\ -4x+5y&=-40 \end{aligned}$$

17. Solve the following system

$$\begin{aligned} 2x+3y+4z&=0 \\ x-y+z&=0 \\ 7x+y+z&=0 \end{aligned}$$

18. If the following system has a unique solution then calculate *a*.

$$\begin{aligned} x+y+az&=6 \\ 2x+3y+4z&=0 \\ 3x+4y+5z&=1 \end{aligned}$$

19. Use determinants to solve the following system:

$$\begin{aligned} x+y-w&=1 \\ 2x+3y+\lambda w&=5 \\ x+\lambda y+3w&=2 \end{aligned}$$