PHYS 201 : Problems in Matrices

1. Find out those *x, y,* for which

$$\begin{bmatrix} 2x+6 & 8 & -3 \\ 1 & 18 & -8 \\ 7 & -2 & 11 \end{bmatrix} = \begin{bmatrix} x+2 & 8 & -3 \\ 1 & 2y & 2x \\ 7 & -2 & y+2 \end{bmatrix}$$

2. For which value of *x*, $\mathbf{A} = \mathbf{A}^{\mathrm{T}}$,

$$\mathbf{A} = \begin{bmatrix} 1 & x & 0 \\ 2-x & 2 & 1 \\ 0 & x^2 & 3 \end{bmatrix}$$

3. You are given the following matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}.$$

Calculate the matrix $\mathbf{A} - \mathbf{B} - \mathbf{C}$.

4. What is the matrix **X** for which

$$\mathbf{X} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

5. Find the matrix **X** if

$$4 \cdot \begin{bmatrix} 2 & 1 & 3 \\ -1 & 0 & 1 \end{bmatrix} + 2\mathbf{X} = 6 \cdot \begin{bmatrix} -1 & 3 & 4 \\ 5 & 1 & 0 \end{bmatrix}.$$

6. Solve the following matrix equation, $2(\mathbf{X} + \mathbf{B}) = 3(\mathbf{X} - \mathbf{A}) - 4\mathbf{B}$, if

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & -3 \\ 2 & 0 & 6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -3 & -1 & 0 \\ 7 & 0 & 3 \end{bmatrix}$$

7. Calculate $\mathbf{A} \cdot \mathbf{B}$ and $\mathbf{B} \cdot \mathbf{A}$ if,

$$\mathbf{A} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}.$$

8. If **A**, **B** are $n \times n$ square matrices, show:

a) $(\mathbf{A} \pm \mathbf{B})^2 = \mathbf{A}^2 \pm \mathbf{A} \cdot \mathbf{B} \pm \mathbf{B} \cdot \mathbf{A} + \mathbf{B}^2$

b) When $(\mathbf{A} \pm \mathbf{B})^2 = \mathbf{A}^2 \pm 2\mathbf{A} \cdot \mathbf{B} + \mathbf{B}^2$?

9. Show that $\mathbf{A}\cdot\mathbf{B}-\mathbf{C}=0$ if,

$$\mathbf{A}=\begin{bmatrix}1 & -1\\ 0 & 1\\ 2 & 0\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}-1 & 1 & 0\\ 1 & 0 & 1\\ 0 & 2 & 2\end{bmatrix}.$$

10. Find the product $\mathbf{A}\cdot\mathbf{B}$, $\mathbf{B}\cdot\mathbf{A}$ if,

$$\mathbf{A}=\begin{bmatrix}3 & 2 & 1\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}2\\ 3\\ 0\end{bmatrix}.$$

11. Find the product $\mathbf{A}\cdot\mathbf{B}$, $\mathbf{B}\cdot\mathbf{A}$ if,

$$\mathbf{A}==\begin{bmatrix}3 & 0 & -1\\ 0 & 4 & 2\\ 5 & -3 & 1\end{bmatrix},\quad \mathbf{B}==\begin{bmatrix}1 & -5 & 0\\ 4 & 1 & -2\\ 0 & -1 & 3\end{bmatrix}$$

12. Show that $\mathbf{A}^2=-3\mathbf{A}-4\mathbf{I}$ if,

$$\mathbf{A}==\begin{bmatrix}-1 & -1\\ 2 & -2\end{bmatrix}.$$

13. Show that: a) $\mathbf{A}^2(x)+\mathbf{B}^2(x)=\mathbf{0}$, b) $\mathbf{B}^2(x)-\mathbf{A}^2(x)=2\mathbf{I}$.

14. Solve the equation

$$5\cdot\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}-3\mathbf{X}=4\cdot\begin{bmatrix}-4 & 7\\ 3 & 8\end{bmatrix}$$

15. Show that $\mathbf{A}\cdot\mathbf{B}=\mathbf{A}\cdot\mathbf{C}$, if

$$\mathbf{A}=\begin{bmatrix}0 & 3\\ 0 & 0\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}2 & 1\\ 3 & 0\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}7 & 5\\ 3 & 0\end{bmatrix}.$$

16. If for two matrices we can define $\mathbf{A}+\mathbf{B}$, $\mathbf{A}\cdot\mathbf{B}$ show that these matrices are square matrices.

17. Find the matrices $\mathbf{A}$, $\mathbf{B}$ for which,

$$\mathbf{A}+\mathbf{B}=\begin{bmatrix}3 & 6\\ 3 & 9\end{bmatrix},\quad 2\mathbf{A}+\mathbf{B}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}.$$

18. Show that $(\mathbf{A}\cdot\mathbf{B})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\cdot\mathbf{A}^{\mathrm{T}}$. If,

$$\mathbf{A} == \begin{bmatrix} 2 & 1 & 4 \\ 1 & 3 & -1 \\ 0 & -2 & 5 \end{bmatrix}, \quad \mathbf{B} == \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & 4 \\ -1 & 1 & -3 \end{bmatrix}.$$

19. Find the matrix **X** such that $\mathbf{A}\cdot\mathbf{X} = \mathbf{B}$, if

$$\mathbf{A} = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} a \\ b \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

20. If,

$$\mathbf{A} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

show that

$$\mathbf{A}^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, \quad (n \in \mathrm{N}).$$

21. If,

$$\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$$

show that

$$\mathbf{A}^n = \begin{bmatrix} n+1 & -n \\ n & -n+1 \end{bmatrix}, \quad (n \in \mathrm{N}).$$

22. If,

$$\mathbf{A}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

show that $\mathbf{A}(a)\cdot\mathbf{A}(b) = \mathbf{A}(a+b)$.

23. If,

$$\mathbf{A} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

show that

$$\mathbf{A}^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}, \quad (n \in \mathrm{N}).$$

24. Verify the Jacobi identity: $\left[\mathbf{A},\left[\mathbf{B},\ \mathbf{C}\right]\right] = \left[\mathbf{B},\left[\mathbf{A},\mathbf{C}\right]\right] - \left[\mathbf{C},\left[\mathbf{A},\mathbf{B}\right]\right]$.

25. Show that $(\mathbf{A}+\mathbf{B})\cdot(\mathbf{A}-\mathbf{B}) = \mathbf{A}^2 - \mathbf{B}^2$ if **A** and **B** commute, i.e. $\left[\mathbf{A},\ \mathbf{B}\right] = 0$.

26. Show that $\mathbf{K}^n = \mathbf{1}$, given that

$$\mathbf{K}^n = \begin{bmatrix} 0 & 0 & i \\ -i & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \quad (n \in \mathrm{N})$$

27. Show that $\left[\mathbf{A},\mathbf{B}\right] = \mathbf{C},\ \left[\mathbf{A},\mathbf{C}\right] = \mathbf{0},\ \left[\mathbf{B},\ \mathbf{C}\right] = \mathbf{0}$ if,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

28. The following three matrices (i, j, and k) plus the unit matrix **1** forma a basis for the so called *quaternions.* Show that:

(a) $\mathrm{i}^2 = \mathrm{j}^2 = \mathrm{k}^2 = -\mathbf{1}$

(b) $\mathrm{i}\cdot\mathrm{j} = -\mathrm{j}\cdot\mathrm{i} = \mathrm{k}$

(c) $\mathrm{j}\cdot\mathrm{k} = -\mathrm{k}\cdot\mathrm{j} = \mathrm{i}$

(d) $\mathrm{k}\cdot\mathrm{i} = -\mathrm{i}\cdot\mathrm{k} = \mathrm{j}$

$$\mathrm{i} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad \mathrm{j} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \mathrm{k} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

29. The three Pauli spin matrices are

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Show that:

(a) $\sigma_i^2 = \mathbf{1}$

(b) $\sigma_i \sigma_j = i\sigma_k, \quad (i,j,k) = (1,2,3),(2,3,1),(3,1,2)$ (cyclic permutation)

These matrices were used by Pauli in the nonrelativistic theory of electron spin.