PHYS 201
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Handout –4: Problems in vectors

1) Let $V = R^3 = \{(a,b,c) \mid a,b,c \in R\}$. We are going to check if the following subsets of R^3 are vector subspaces:

$W_1 = \{(a,b,0) \mid a,b \in R\}$
$W_2 = \{(a,b,1) \mid a,b \in R\}$
$W_3 = \{(a,b,1) \mid a,b \in R\} \cup \{(0,0,0)\}$
$W_4 = \{(a,a,a) \mid a \in R\}$
$W_5 = \{(a,b,a+2b) \mid a,b \in R\}$

2) Let $V = M_{3\times3}$, the vector space of 3x3 matrices in R. Check if the subset of all the *upper triangular* matrices

$$W = \{\begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} \mid a,b,c,d,e,f \in R\}$$

is a vector subspace of V:

3) Let $V = R^3 = \{(a,b,c) \mid a,b,c \in R\}$ and $u_1 = (1,0,0)$, $u_2 = (0,1,0)$. The space which is produced by the two vectors is:

4) We consider the vector space R^2. Check if the following vectors are linearly independent or not.

a) $u_1 = (1,2)$ and $u_2 = (2,4)$
b) $u_1 = (1,0)$ and $u_2 = (0,1)$
c) $u_1 = (1,0)$ and $u_2 = (1,1)$

5) We consider the vector space $V=M_{2x2}=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a,b,c,d \in R\right\}$ and the vectors:

$$u_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},\ u_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},\ u_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

a) Find the subspace $\langle u_1, u_2, u_3 \rangle$

b) Show that u_1, u_2, u_3 are linearly independent

c) If $u_4 = \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}$, show that u_1, u_2, u_3, u_4 are linearly dependent.

6) We consider the vector space R^3. The vectors

$$e_1 = (1,0,0),\ e_2 = (0,1,0),\ e_3 = (0,0,1)$$

and the vector $u = (7,8,9)$:

$$u = 7e_1 + 8e_2 + 9e_3 .$$

Consider the vector $e_4 = (1,1,1)$. Show that $u = (7,8,9)$ is not written in a unique way as a linear combination of e_1, e_2, e_3, e_4:

7) Show that the following vectors are a base of R^3

$$u_1 = (1,0,0),\ u_2 = (1,1,0),\ u_3 = (1,1,1)$$

8) Consider the vector space

$$M_{2x3} = \left\{ \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \mid a,b,c,d,e,f \in R \right\}.$$

Show that the vectors

$$e_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ e_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ e_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e_4 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},\ e_5 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},\ e_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

are a base of the space.

9) We consider the vector space R^3. The vectors

$$e_1 = (1,0,0),\ e_2 = (0,1,0),\ e_3 = (0,0,1)$$

form a base of R^3:

10) In the same space R^3 we are going to show that that

$$u_1 = (1,0,0),\ u_2 = (1,1,0),\ u_3 = (1,1,1)$$

form a base.

11) We consider the vector space

$$M_{2x3} = \left\{ \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \mid a,b,c,d,e,f \in R \right\}.$$

Show that the vectors

$$e_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ e_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ e_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e_4 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},\ e_5 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},\ e_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

form a base.