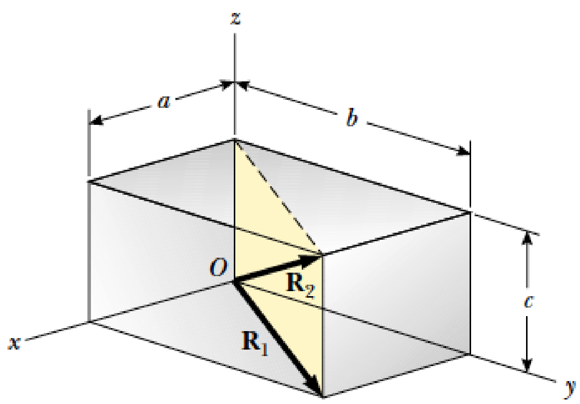
PHYS 201
Dr. Vasileios Lempesis
Handout – 3: Problems in vectors

1. Prove that two vectors must have equal magnitudes if their sum is perpendicular to their difference.

2. Two vectors **A**, **B** have precisely the same magnitudes. For the magnitude of **A+B** to be hundred times larger than the magnitude of **A-B** what must be the angle between them?

3. If the component of a vector **A** along the direction of a vector **B** is zero what can you conclude about these two vectors?

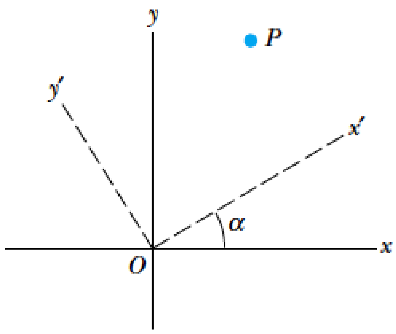
4. In the figure below obtain a vector expression for diagonals $\mathbf{R}_1$ and $\mathbf{R}_2$. Find the angle between these two vectors.



5. A point *P* has coordinates (*x*, *y*) as shown in the figure below. We define a new set of axes x' and y'. Show that the coordinates of the point *P* in the two systems of axes are related as follows:

$$x' = x\cos a + y\sin a$$
$$y' = -x\sin a + y\cos a$$

Express this result in a matric form



6. Simplify $(\mathbf{u}+\mathbf{v})\times(\mathbf{u}-\mathbf{v})$.

7. Verify Cauchy-Schwartz inequality in the following cases:
a) **u** = (-3, 1, 0), **v** = (2, -1, 3) b) **u** = (0, 2, 2, 1), **v** = (1, 1, 1, 1)

8. Find a unit vector in the opposite direction of the vector **v** = (-12, -5).

9. Prove that for two vectors $\mathbf{v}=(v_1,\ v_2,\ ...,v_N)$ and $\mathbf{w}=(w_1,\ w_2,\ ...,w_N)$ we have: **v** + **w** = **w** + **v**.

10. If **u** = (3, 2, 2, 1) and **v** = (1, 1, -1, 1). Find the components of (-2 **v** + 3**u**).

11. Which of the following vectors of R^6 is parallel to vector **v** = (-2, 1, 0, 3, 5, 1):

a) (0, 0, 0, 0, 0, 0) b) (0, 1, 2, 3, 10, 1) c) (-4, 2, 0, 6, 10, 2)

12. Given two points *A*(3, 2, 2, 1) and *B*(1, 1, -1, 1), find their distance.

13. Given two vectors **u** = (3, 2, 2, 1) and **v** = (1, 1, -1, 1), find their dot product.

14. Show that the vectors **u** = (-2, 3, 1, 4) and **v** = (1, 2, 0, -1) qre orthogonal.

15. Verify the Pythagoras Theorem for the vectors: **u** = (-2, 3, 1, 4) and **v** = (1, 2, 0, -1).

16. Prove Pythagoras Theorem in a n-dimensional space.

17. Prove that $\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v})=0$.

18. Calculate the product $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})$ for the vectors:

$$\mathbf{v}=3\mathbf{i}-2\mathbf{j}-5\mathbf{k},\ \ \mathbf{v}=\mathbf{i}+4\mathbf{j}-4\mathbf{k},\ \ \mathbf{v}=3\mathbf{j}+2\mathbf{k}$$