Fluid Mechanics

Matter is normally classified as being in one of three states: Solid, Liquid, or Gas.

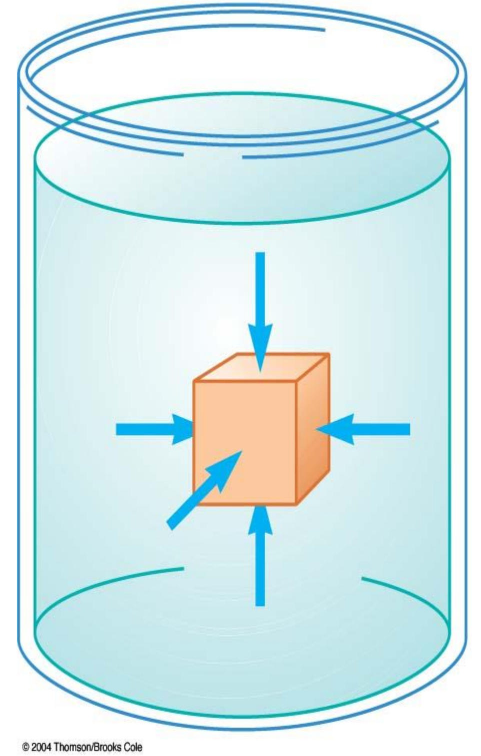
- Solid has a definite volume and shape.
- Liquid has a definite volume but no definite shape.
- Gas has neither a definite volume nor a definite shape.

- A fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container.

- Both liquids and gases are fluids.

Pressure

Fluids do not sustain shearing stresses or tensile stresses.Thus, the only stress that can be exerted on an object submerged in a static fluid is one that tends to compress the object from all sides. In other words, the force exerted by a static fluid on an object is always perpendicular to the surfaces of the object.



$P = F/A \ (N/m^2)$ $\qquad$ $1 \text{ Pa} = 1 \text{ N/m}^2$

Table 14.1

Densities of Some Common Substances at Standard Temperature (0°C) and Pressure (Atmospheric)

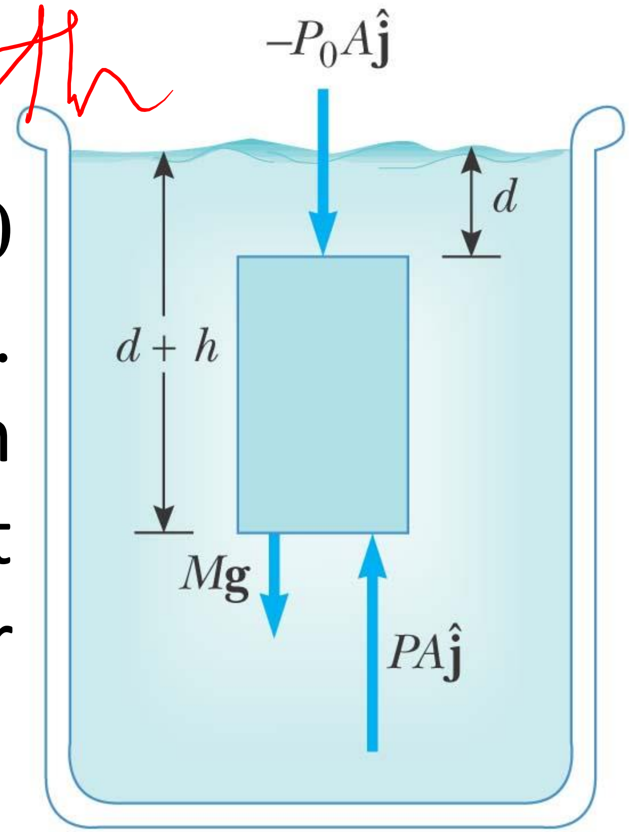
Substance	ρ (kg/m^3)	Substance	ρ (kg/m^3)
Air	1.29	Ice	0.917×10^3
Aluminum	2.70×10^3	Iron	7.86×10^3
Benzene	0.879×10^3	Lead	11.3×10^3
Copper	8.92×10^3	Mercury	13.6×10^3
Ethyl alcohol	0.806×10^3	Oak	0.710×10^3
Fresh water	1.00×10^3	Oxygen gas	1.43
Glycerin	1.26×10^3	Pine	0.373×10^3
Gold	19.3×10^3	Platinum	21.4×10^3
Helium gas	1.79×10^{-1}	Seawater	1.03×10^3
Hydrogen gas	8.99×10^{-2}	Silver	10.5×10^3

Variation of Pressure with Depth

As divers well know, water pressure increases with depth. Likewise, atmospheric pressure decreases with increasing altitude. for this reason, aircraft flying at high altitudes must have pressurized cabins.

$$P = P_o \pm \rho g h$$

d depth

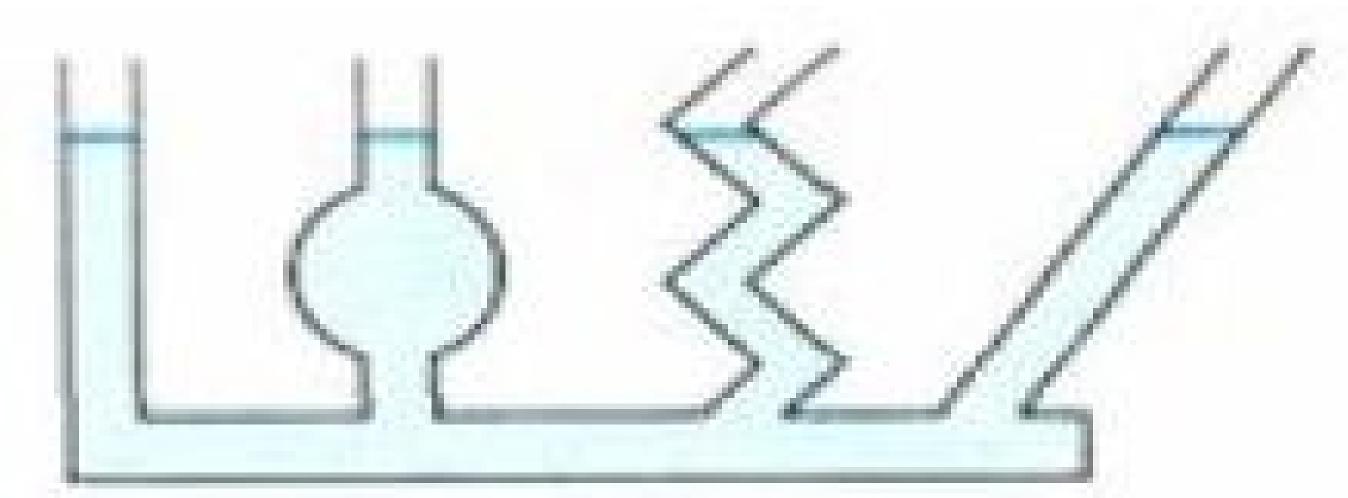


where P_o is the pressure at $h = 0$ and ρ is the density of the fluid. That is, the pressure P at a depth h below a point in the liquid (at which the pressure is P_o) is greater by an amount ρgh.

$$\Delta P = \rho g h$$

If the liquid is open to the atmosphere, P_o is the pressure at the surface of the liquid, atmospheric pressure, ($P_o = P_{atm} = 1.013 \text{x} 10^5 \ N/m^2$)

Equation (**$P = P_o + \rho g h$**) implies that If the liquid is open to the atmosphere, the pressure is the same at all points having the same depth h, independent of the shape of the container.



Pascal's law: a change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

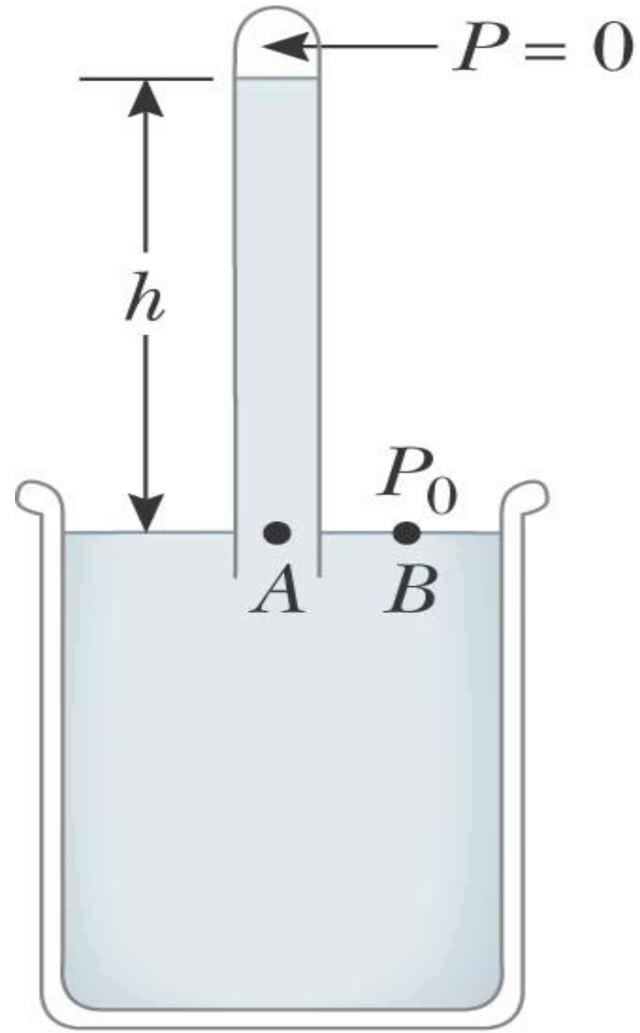
EX 1: calculate the force exerted on your eardrum ($A = 1x10^{-4}\ m^2$) due to the water above when you are swimming at the bottom of a pool that is 6 m deep ($\rho_w = 1000\ kg/\ m^{3)}$. The air inside the middle ear is normally at atmospheric pressure $P_o = 1x10^5\ N/\ m^2$.

Slide 9

Measuring the Pressure

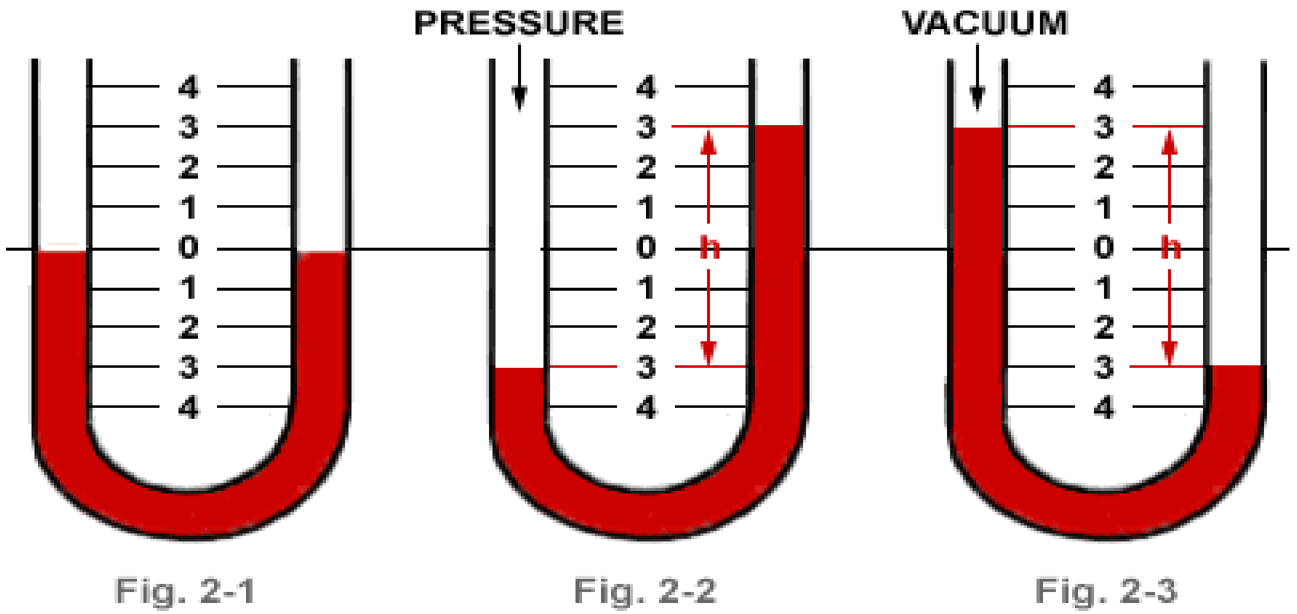
Based on a calculation,
one atmosphere of pressure
is defined to be the pressure
equivalent of a column of
mercury that is exactly 76 cm
in height at 0°C.

$$P_o = \rho_{Hg}\, g\, \mathbf{h}$$

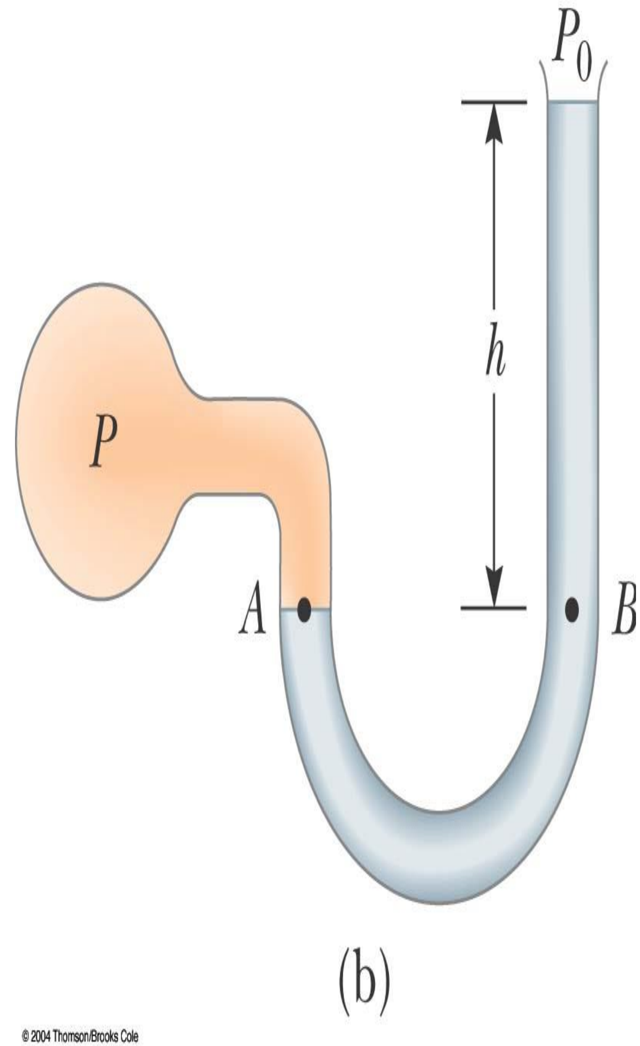


(a)

A device for measuring the pressure of a gas contained in a vessel is the **open-tube manometer** illustrated in Figure.



One end of a U-shaped tube containing a liquid is open to the atmosphere, and the other end is connected to a system of unknown pressure *P*.
The pressures at points *A* and *B* must be the same and the pressure at *A* is the unknown pressure of the gas.



Therefore, equating the unknown pressure *P* to the pressure at point *B*, we see that

$P = P_o + \rho gh.$

The difference in pressure $(P - P_o) = \rho g h$.

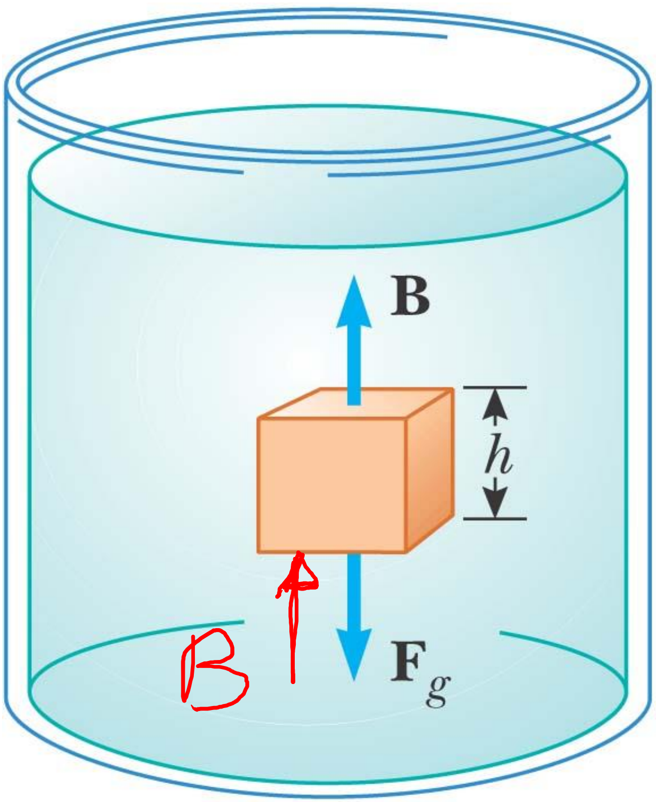
The pressure *P* is called the absolute pressure, while the difference $(P - P_o)$ is called the **gauge pressure.**

For example, the pressure you measure in your bicycle tire is gauge pressure.

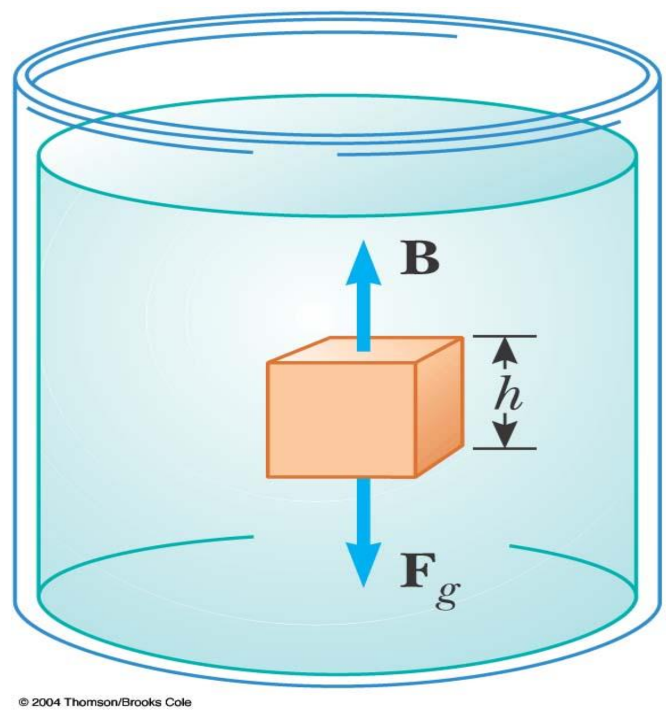
It is difficult tried to push a closed object under water because of the large upward force exerted by the water on the ball.
The upward force exerted by a fluid on any immersed object is called a **buoyant force**.

The magnitude of the buoyant force always equals the weight of the fluid displaced by the object.

This statement is known as Archimedes' principle.



To understand the origin of the buoyant force, consider a cube immersed in a liquid as in Figure. The pressure P_b at the bottom of the cube is greater than the pressure P_t at the top by an amount (ρ_{fl} gh), where h is the height of the cube and ρ_{fl} is the density of the fluid.



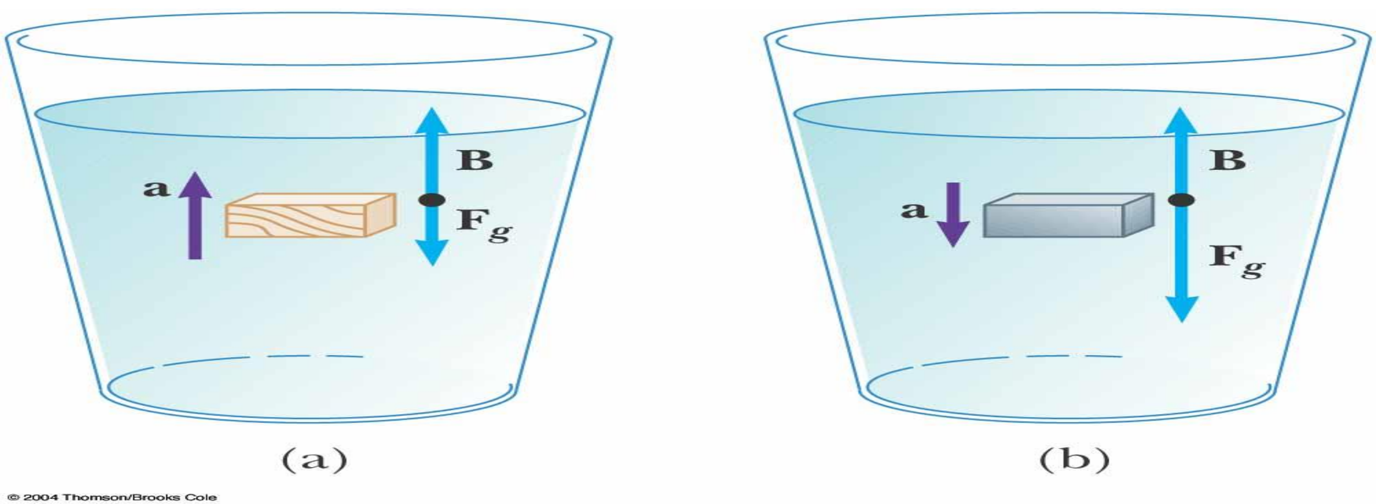
The pressure at the bottom of the cube causes an ***upward*** **force equal to *$P_b A$***, where *A* is the area of the bottom face. The pressure at the top of the cube causes a *downward* force equal to *$P_t A$*. The resultant of these two forces is the buoyant force B.

$$B = A\Delta P = A\,(P_b - P_t) = A\,\rho_{fl}\,gh$$
$$= \rho_{fl}\,gV = \rho_{fl}\,V\,g = Mg$$

where *Mg* is the weight of the fluid displaced by the cube.

Case 1: Totally Submerged Object

When an object is totally submerged in a fluid of density ρ_{fluid}, the magnitude of the upward buoyant force is $B = \rho_{fluid}\, g V_{obj}$, where V_{obj} is the volume of the object. If the object has a mass M and density $\boldsymbol{\rho_{obj}}$, its weight is equal to $F_g = Mg = \boldsymbol{\rho_{obj}} V_{obj}\, g$, and



the net force on it is:

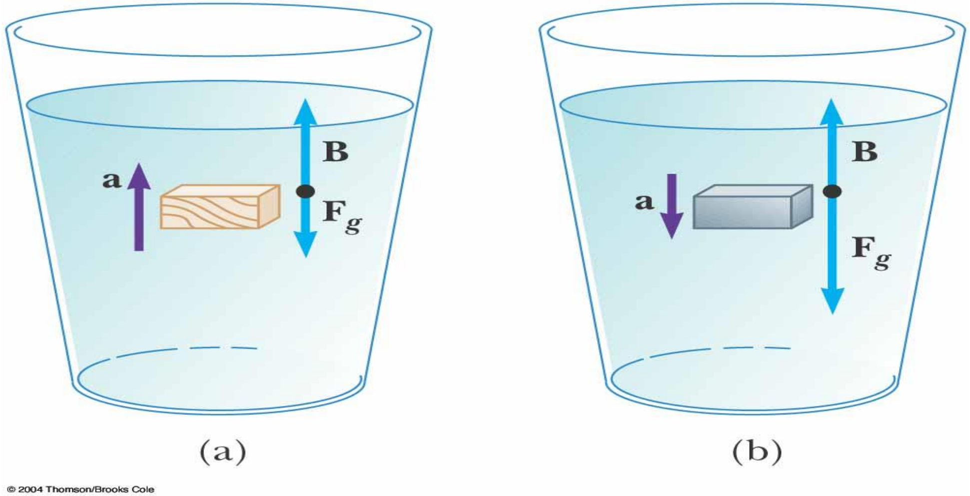
$$B - F_g = (\rho_{fluid} - \rho_{obj})\ V_{obj}.\ g$$ Hence,

- If the density of the object is less than the density of the fluid, then the downward gravitational force is less than the buoyant force, and the unsupported object accelerates upward (Fig.a).

- If the density of the object is greater than the density of the fluid, then the upward buoyant force is less than the downward gravitational force and the unsupported object sinks (Fig. b

- If the density of the submerged object equals the density of the fluid, the net force on the object is zero and it remains in equilibrium.

Thus, the direction of motion of an object submerged in a fluid is determined only by the densities of the object and the fluid and the net force equals $B - F_g = (\rho_{\text{fluid}} - \rho_{\text{obj}})\, V_{\text{obj}} \cdot g$



8- A Ping-Pong ball has a diameter of 3.8 cm and average density of 0.084 g/cm^3. What force is required (F_{appl}) to hold it completely submerged under water?

$\rho = \frac{m}{V}$

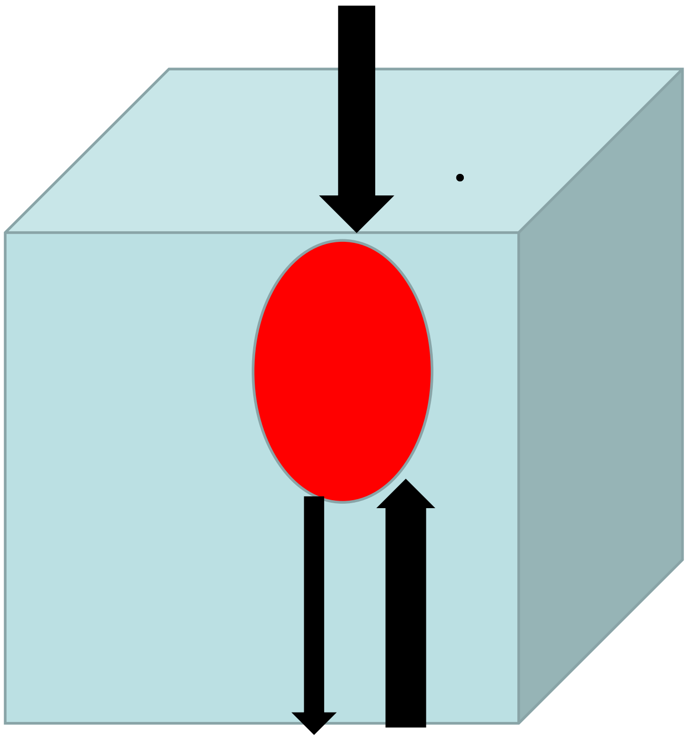
At balance $\sum F = 0$

$$F_{appl} + mg - B = 0$$

$$F_{appl} = B - mg =$$ الجسم مغمور كليا

$$= \rho_{Fl} V_{Fl}\, g - \rho_{ob} V_{obj}\, g$$

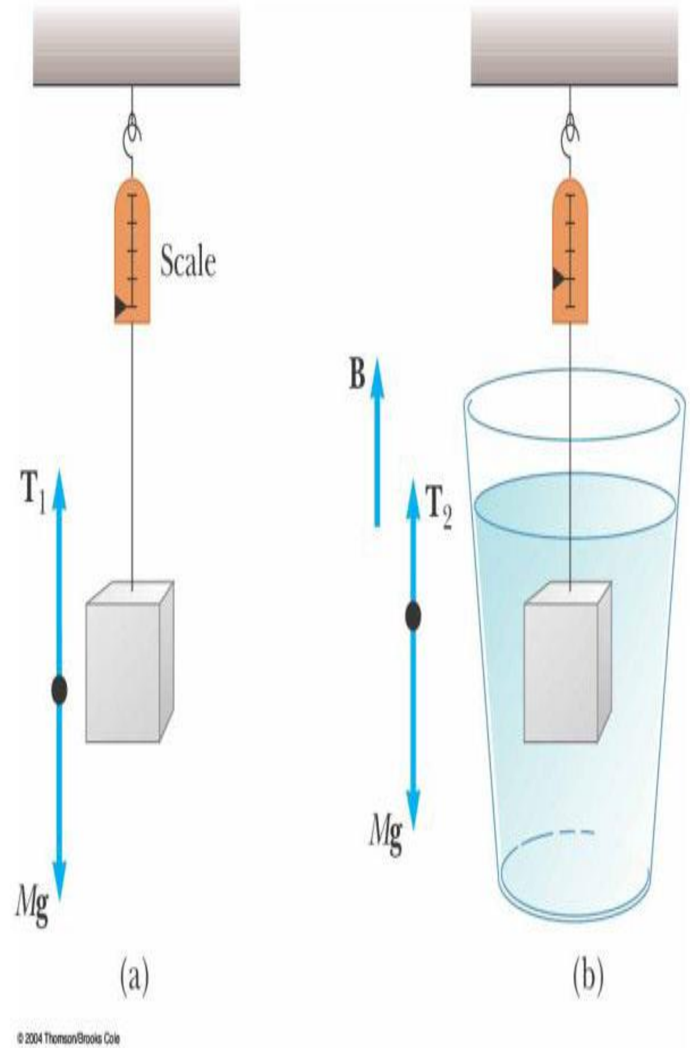
$$= (\rho_{Fl} - \rho_{obj}) V_{obj}\, g = 0.25 \text{ N}$$



12- A piece of aluminum with mass 1 kg and density 2 700 kg/m^3 is suspended from a string and then completely immersed in a container of water (Figure). Calculate the tension in the string

(a) before and

(b) after the metal is immersed.

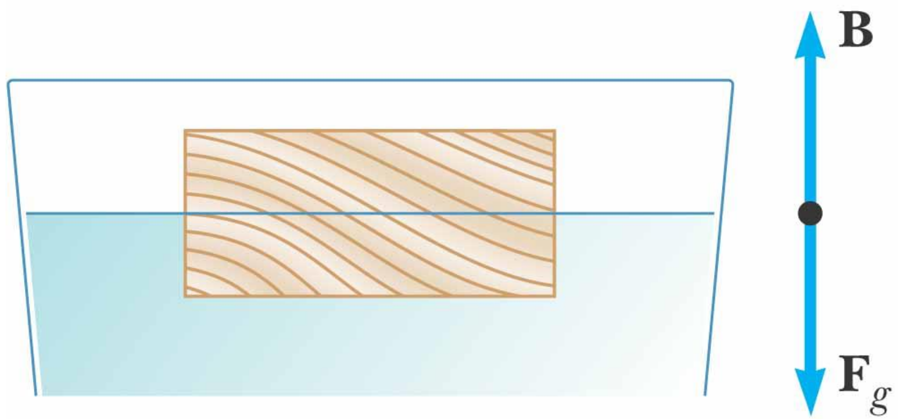


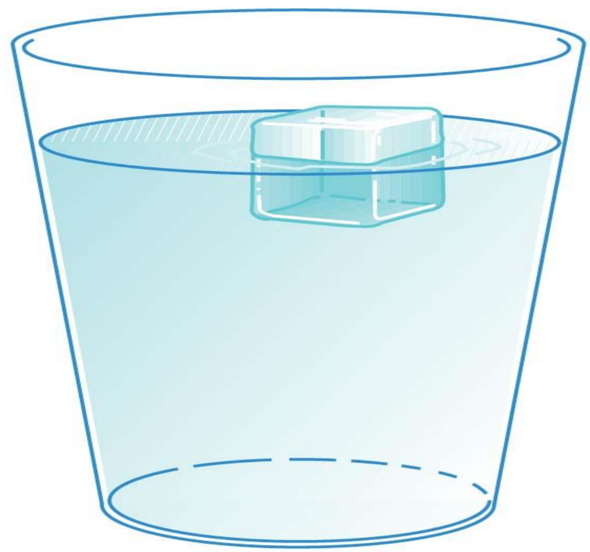
Case 2: Floating Object

Now consider an object (of volume V_{obj} and density $\rho_{obj} < \rho_{fluid}$) in static equilibrium floating on the surface of a fluid.

The object is only *partially* submerged (Fig).

In this case, the upward buoyant force is balanced by the downward gravitational force acting on the object.





If *Vfl* is the volume of the fluid displaced by the object (this volume is the same as the volume of that part of the object that is beneath, under, the surface of the fluid), the buoyant force has a magnitude:

$$B = \rho_{\text{fluid}} V_{\text{fluid}}\ g$$

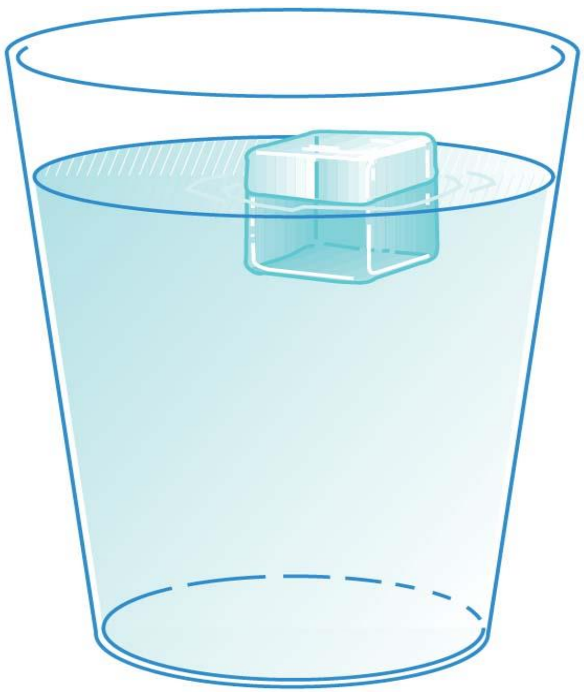
the weight of the object $F_g = Mg = \rho_{\text{obj}} V_{\text{obj}}$,g

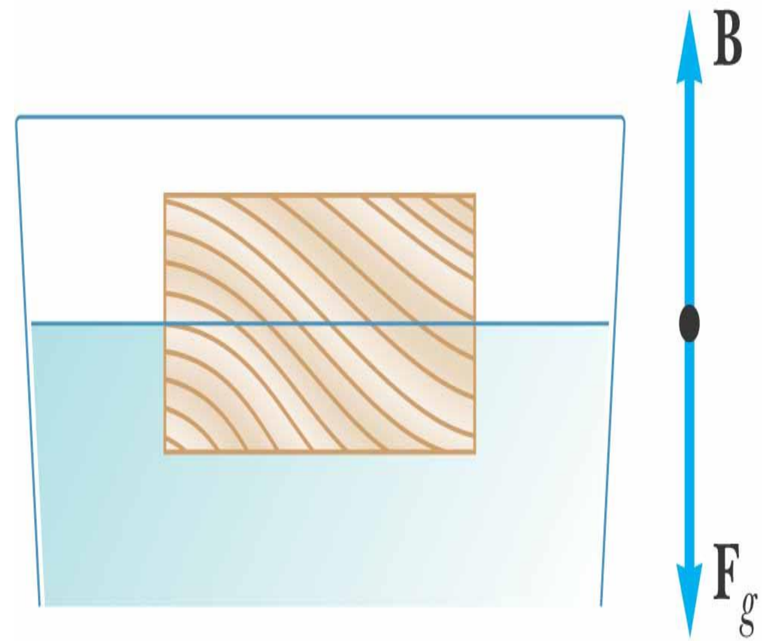
and because $Fg = B$, we see that

$$\rho_{fluid}\, V_{fluid}\, g = \rho_{obj}\, V_{obj}\, g$$

$$\left(V_{fluid} / V_{obj}\right) = \left(\rho_{obj} / \rho_{fluid}\right)$$

This equation tells us that the fraction of the volume of a floating object to that below the fluid surface is equal to the ratio of the density of the object to that of the fluid.





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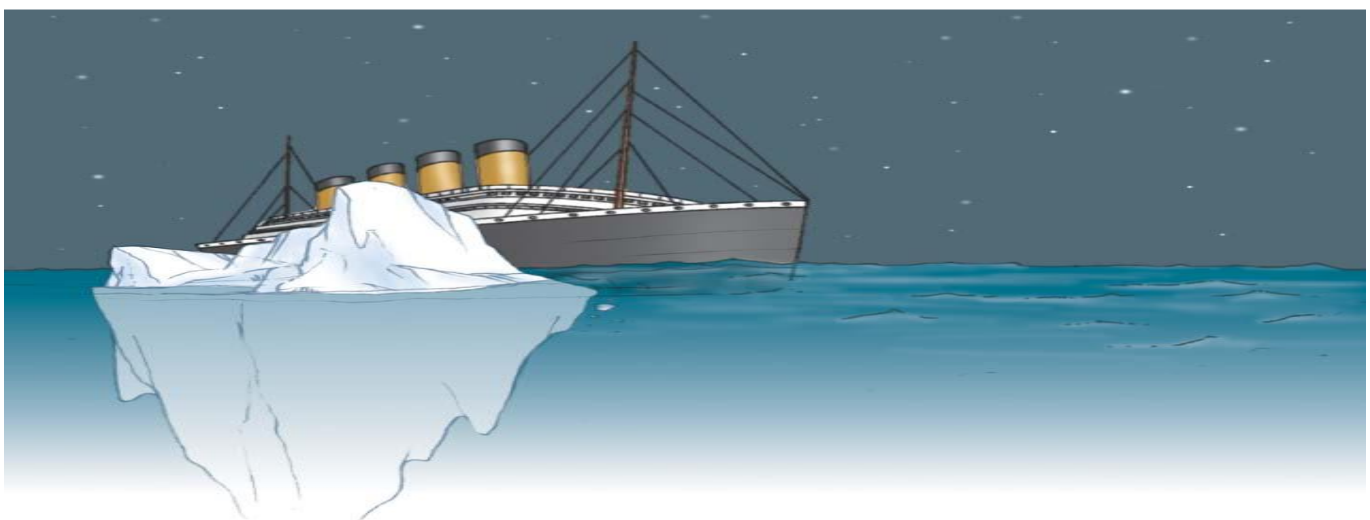
Ex3: An iceberg floating in seawater, as shown in Figure is extremely dangerous because most of the ice is below the surface. This hidden ice can damage a ship that is still a considerable distance from the visible ice.
What fraction of the iceberg lies below the water level?



(b)

Solution This problem corresponds to Case 2. The weight of the iceberg is $F_g = \rho_i V_i g$, where $\rho_i = 917 \text{ kg/m}^3$ and V_i is the volume of the whole iceberg.

The magnitude of the up- ward buoyant force equals the weight of the displaced water:

$$B = \rho_w V_w g$$

$$\frac{\rho_{ob}}{\rho_{FL}} = \frac{\rho_{ice}}{\rho_{W}} = \frac{V_W}{V_{ice}}$$

$$B = \rho_w V_w g,$$

V_w , the volume of the displaced water, and it is equal to the volume of the ice beneath the seawater. (the shaded region in Fig.)

Density of seawater:

$\rho w = 1030\ \mathrm{kg/m^3}$.

The fraction of ice beneath the water's surface is:

$$(V_{bth} / V_i) = (V_w / V_i) = (\rho_i / \rho_w) =$$

$$(917/1030) = 0.890 \times 100 = 89\ \%$$

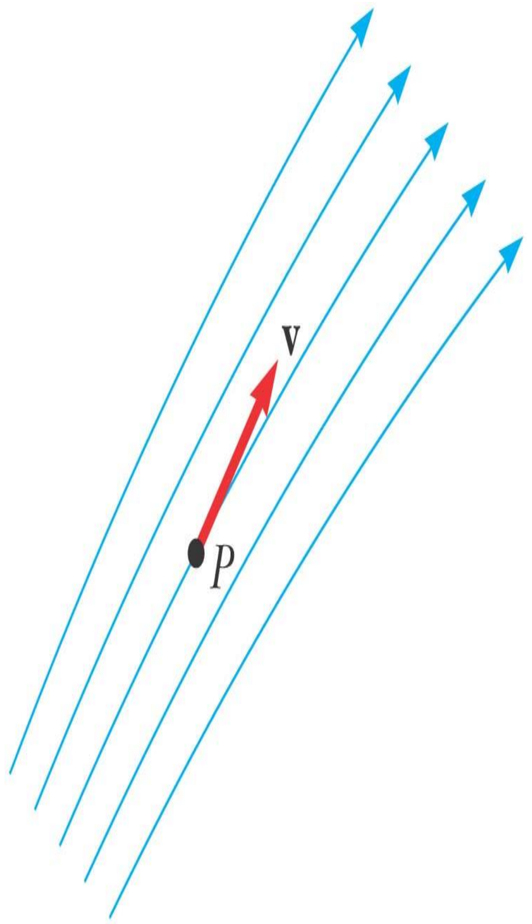
Fluid Dynamics

When fluid is in motion, its flow can be characterized as being one of two main types:

- The flow is said to be steady, or laminar, if each particle of the fluid follows a smooth path, such that the paths of different particles never cross each other. In steady flow, the velocity of fluid particles passing any point remains constant in time.
- Above a certain critical speed, fluid flow becomes turbulent; turbulent flow is irregular flow.

Ideal fluid flow has the following four assumptions:

1. **The fluid is nonviscous**.
   In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
2. **The flow is steady.** In steady (laminar) flow, the velocity of the fluid at each point remains constant.
3. **The fluid is incompressible**. The density of an incompressible fluid is constant.
4. **The flow is irrotational.** The fluid has no angular momentum about any point.



The path taken by a fluid particle under steady flow is called a **streamline** (figure).

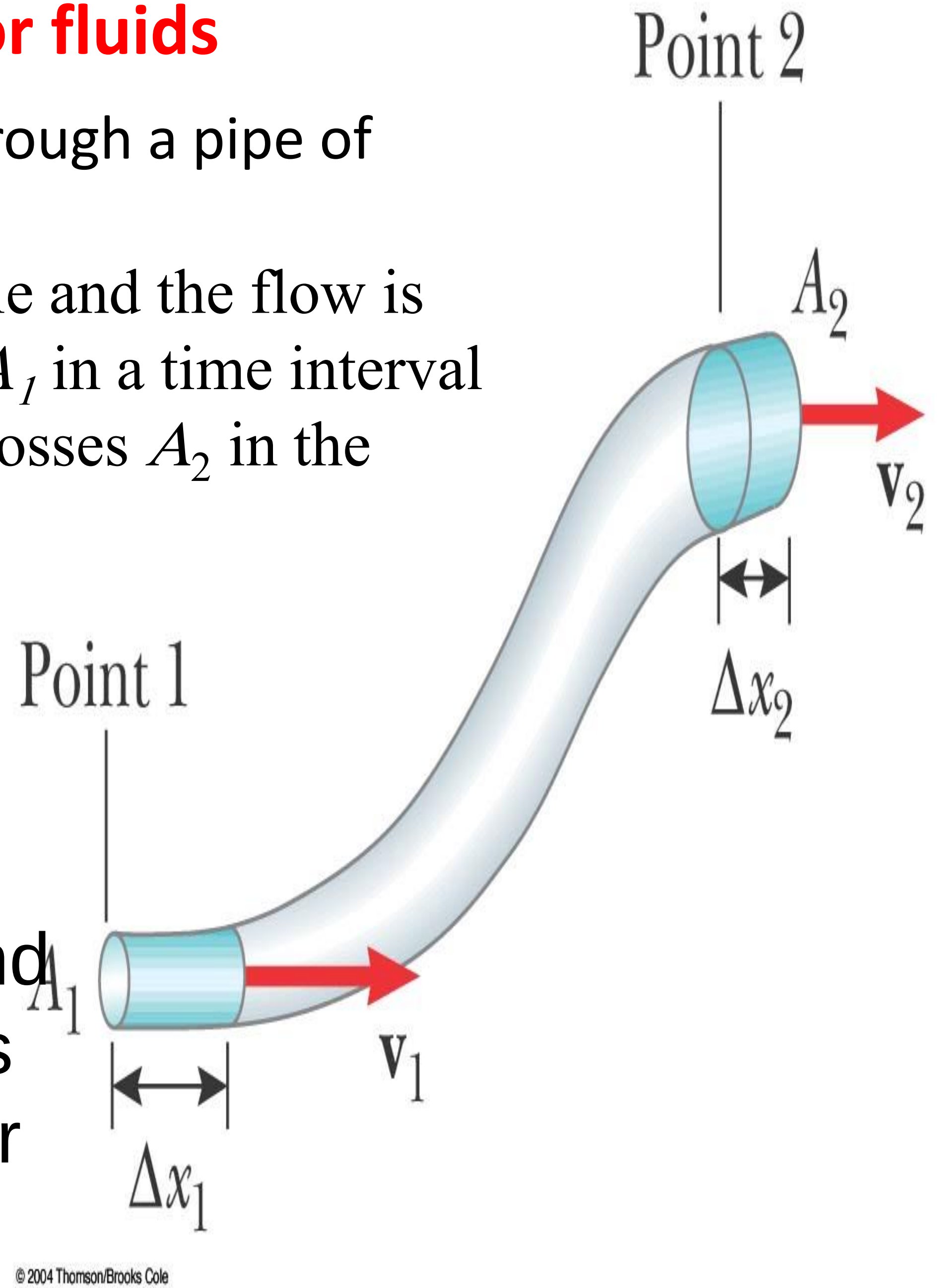
Fig. 14.13b, p.430

Equation of continuity for fluids

Consider an ideal fluid flowing through a pipe of nonuniform size (Figure).

Because the fluid is incompressible and the flow is steady, the mass $m1$ that crosses A_1 in a time interval Δt must equal the mass $m2$ that crosses A_2 in the same time interval Δt. That is:

Volume Flow Rate



$$A_1 v_1 = A_2 v_2 = \text{constant}$$

The product of the area and the fluid speed at all points along a pipe is constant for an incompressible fluid.

 Ex: $10 \times 10 = 5 \times 20 = 25 \times 4 = 100$

The condition $\boldsymbol{Av}$ = **constant**

$= L^2 (L/T) = L^3/T =$ Volume / Time = (m^3/s)

is equivalent to the statement that the volume of fluid that enters one end of a tube in a given time interval equals the volume leaving the other end of the tube in the same time interval if no leaks are present.

Ex4: Water flows through a fire hose of diameter 6.4 cm at a rate of 0.0120 m^3/s. The fire hose ends in a nozzle of inner diameter 2.2 cm. What is the speed with which the water exits the nozzle?

$R_1 = 6.4 \times 10^{-2} m$

$r_1 = 3.2 \times 10^{-2} m$

$A = \pi r^2$

$R_2 =$

$r_2 = 1.1 \times 10^{-2} m$

A_2

$A_1 v_1 = A_2 v_2 = 120 \times 10^{-4} \frac{m^3}{s}$

$$A_1 = A_h = \pi\, r_1^2$$
$$A_2 = A_n = \pi\, r_2^2$$

$$Q\ (m^3/s) = A_1 v_1 = A_2 v_2$$

$$V_2 = Q / A_2$$

$$v_2 = \frac{Q}{A_2} = \frac{0.0120\ m^3/s}{A_2\ m^2} = \qquad m/s$$

Bernoulli's Equation

The relationship between fluid speed, pressure, and elevation was first derived in 1738 by the Swiss physicist Daniel Bernoulli.

Consider the flow of a segment of an ideal fluid through a nonuniform pipe in a time Δt, as illustrated in Figure

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 =$$

$$= P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

This is Bernoulli's equation as applied to an ideal fluid.

It is often expressed as

$$P + \frac{1}{2}\rho v^2 + \rho g y = const.$$

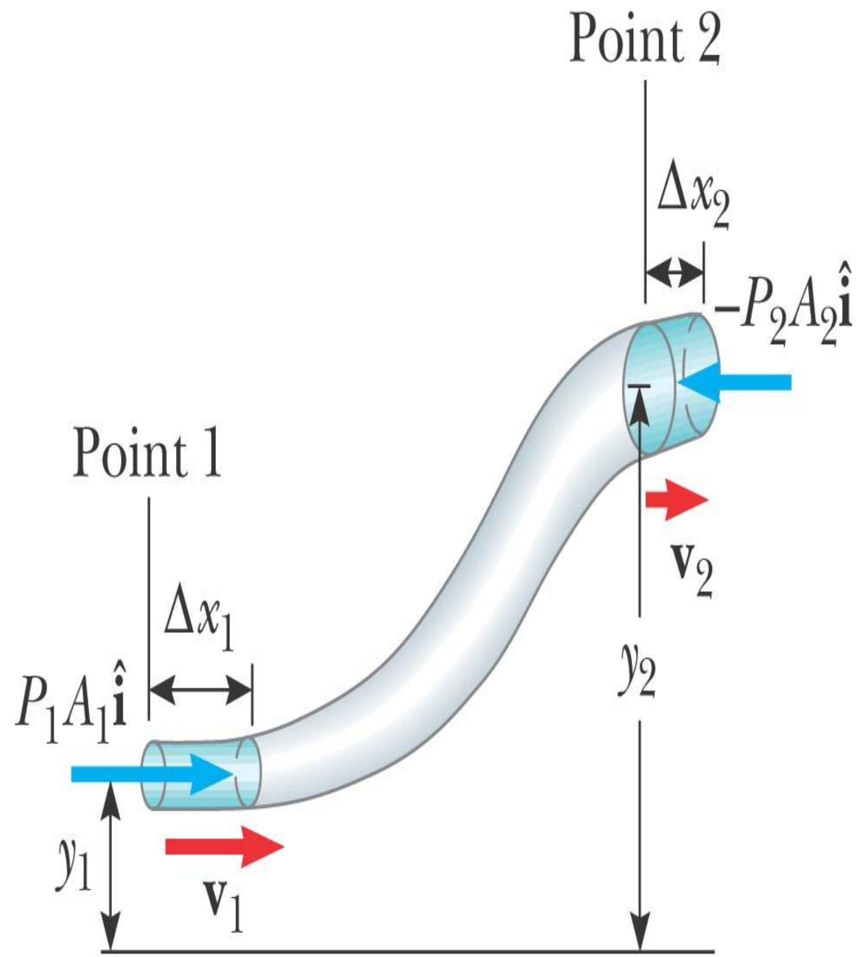


Fig. 14.19, p.433

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$$

Bernoulli's Equation means that:

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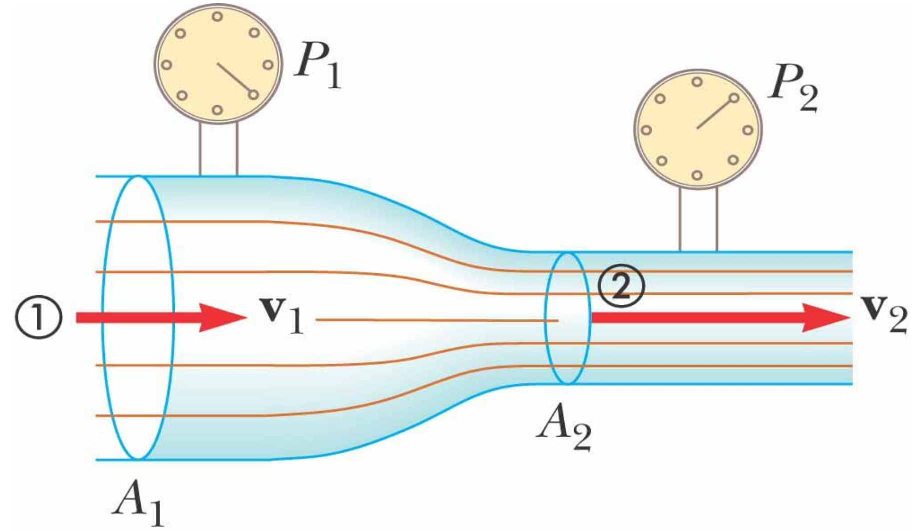
$$\mathbf{P} + \frac{\boldsymbol{Kinetic\ Energy}}{\boldsymbol{Volume}} + \frac{\boldsymbol{Potential\ Energy}}{\boldsymbol{Volume}} = \text{Constant}$$

Other Applications of Fluid Dynamics

The horizontal constricted pipe illustrated in Figure known as a ***Venturi tube***, can be used to measure the flow speed of an incompressible fluid.
Determine the flow speed at point 2 if the pressure difference P_1 & P_2 is known.



(a)

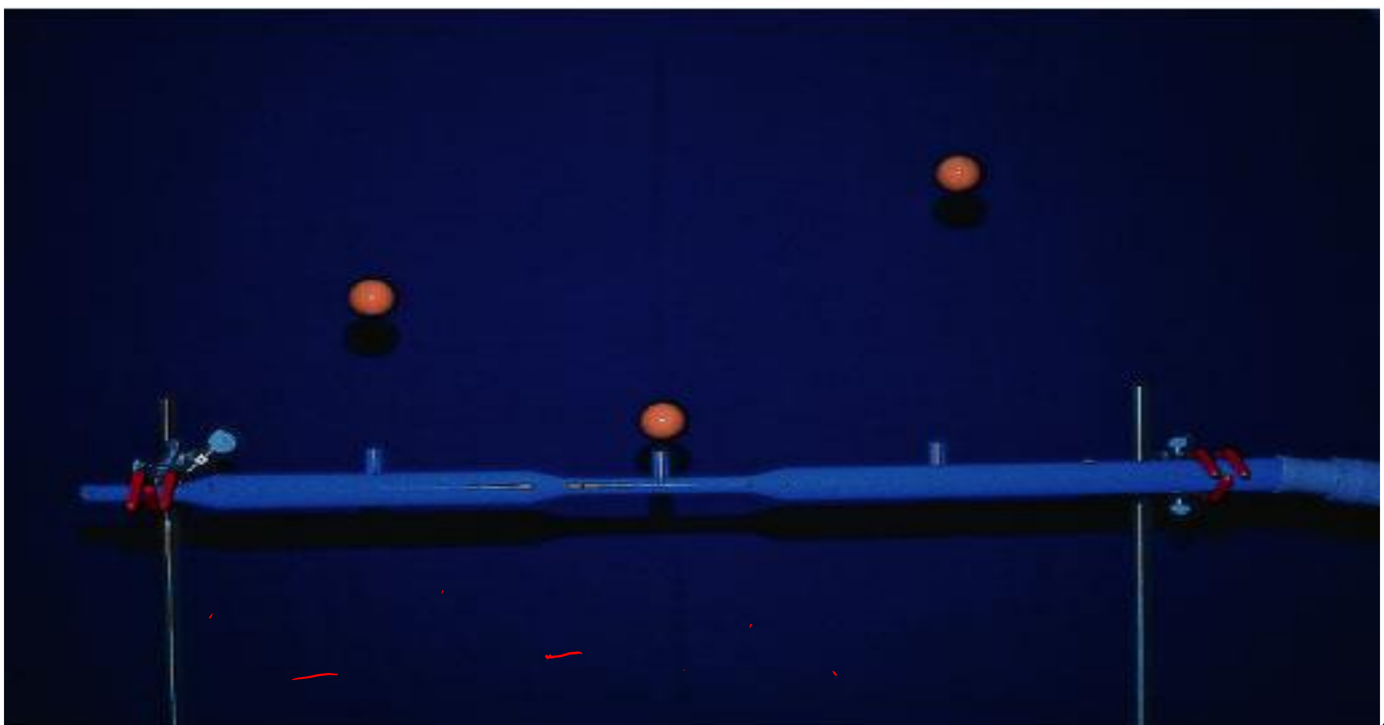
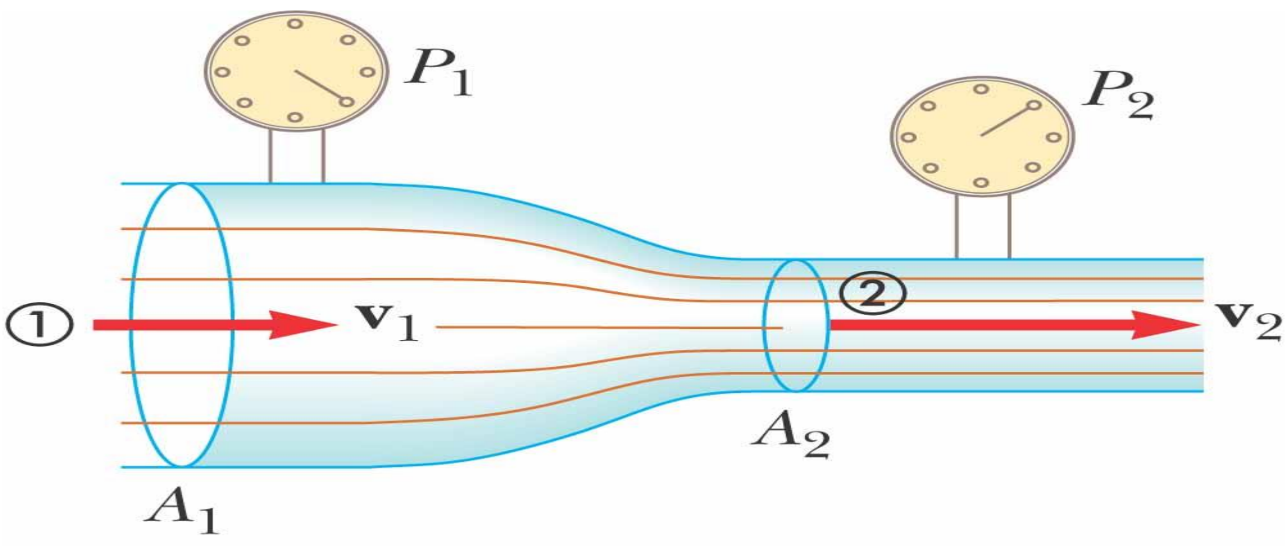


Fig. 14.20a, p.435

d) $P_1 + \frac{KE_1}{V} = P_2 + \frac{KE_2}{V} = P_3 + \frac{KE_3}{V} = \text{Const}$

$A_1 > A_2 \Rightarrow v_2 > v_1 \Rightarrow P_2 < P_1$

As area decreases, velocity increases and so pressure decreases



(a)

Fig. Q14.31, p.439

Consider the streamlines that flow around an airplane wing as shown in Figure. Let us assume that the airstream approaches the wing horizontally from the right with a velocity v_1. The tilt of the wing causes the airstream to be deflected downward with a velocity v_2. Because the airstream is deflected by the wing, the wing must exert a force on the airstream. According to Newton's third law, the airstream exerts a force F on the wing that is equal in magnitude and opposite in direction. This force has a vertical component called the lift (or aerodynamic lift) and a horizontal component called drag.

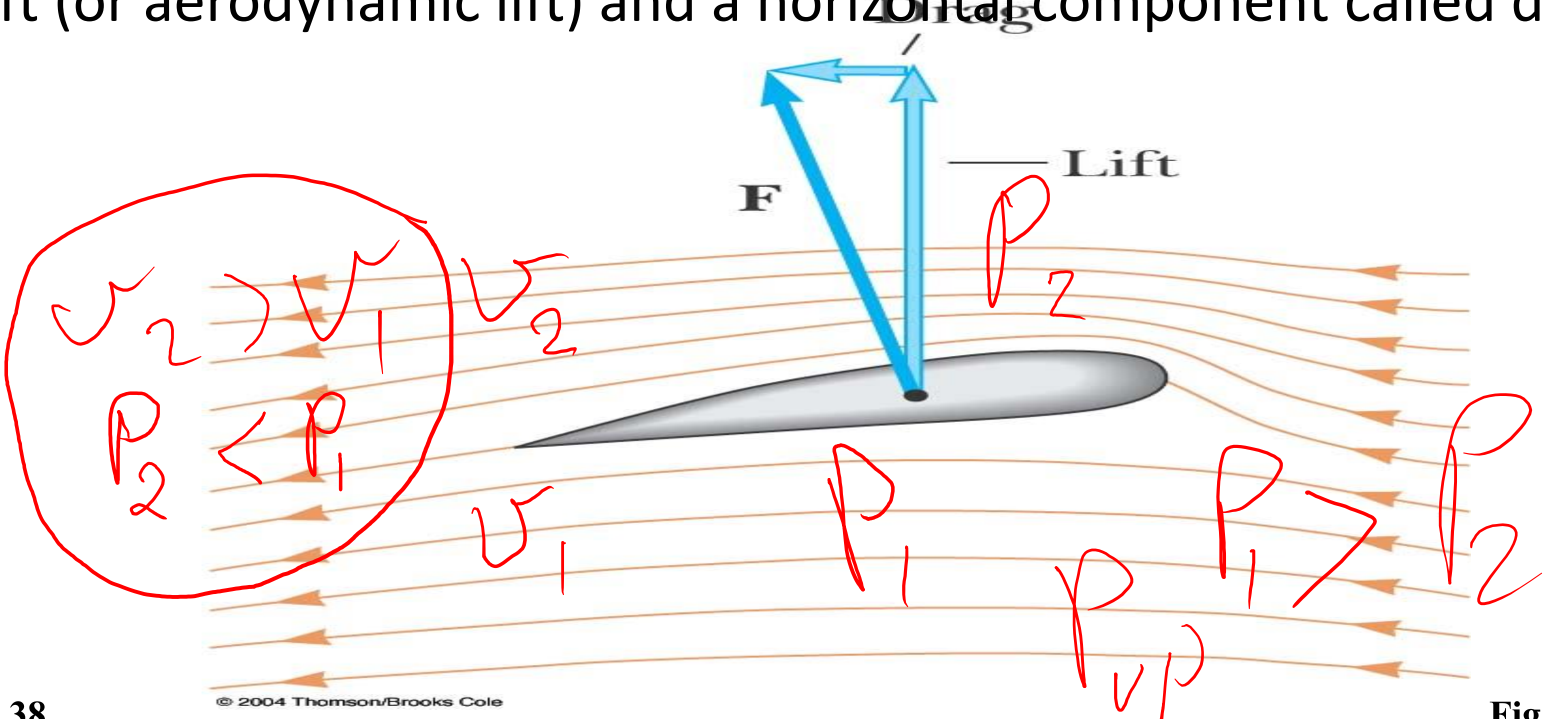
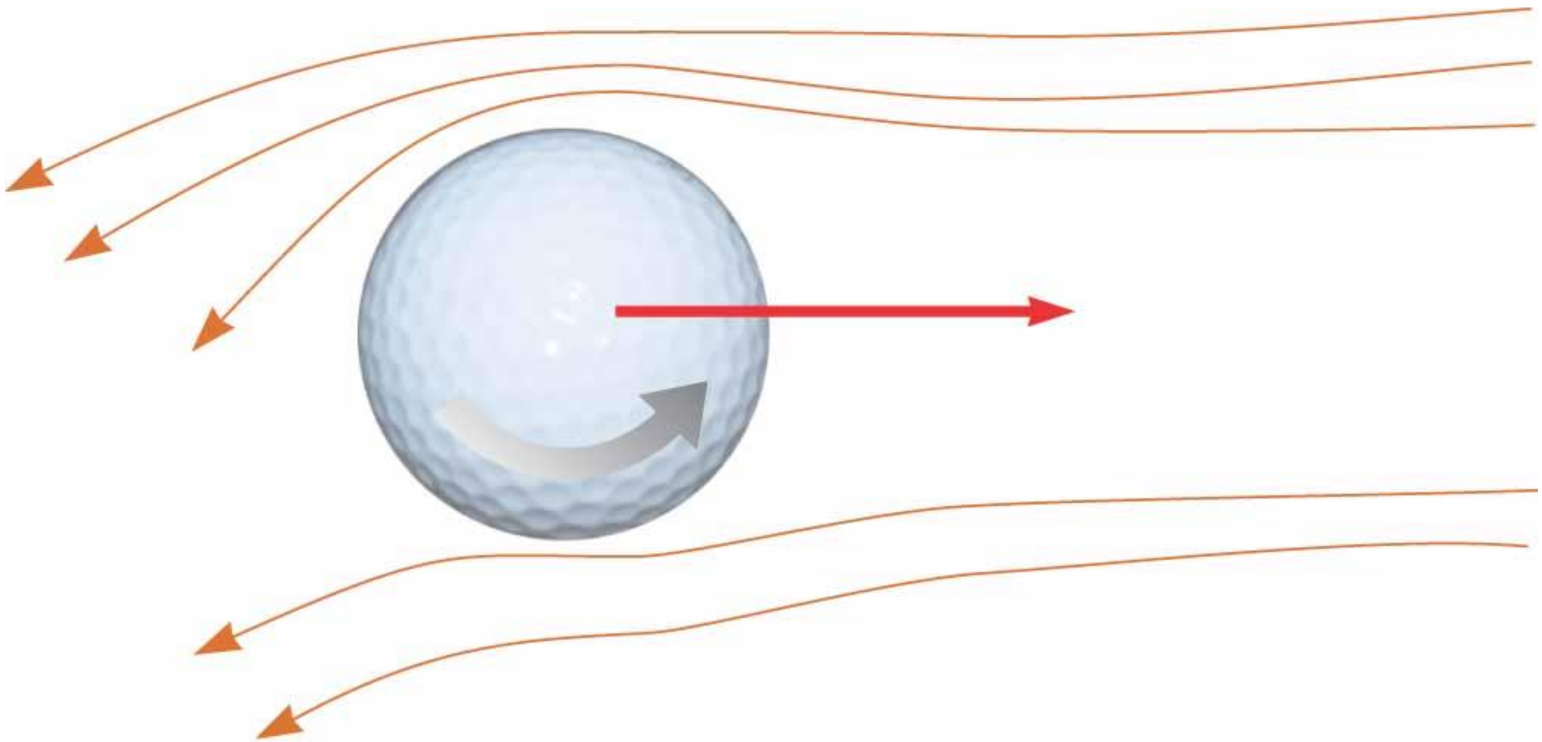


Fig. 14.22, p.436



Because of the deflection of air, a spinning golf ball experiences a lifting force that allows it to travel much farther than it would if it were not spinning.

Fig. 14.23, p.437

A number of devices operate by means of the pressure differentials that result from differences in a fluid's speed. For example, a stream of air passing over one end of an
open tube, the other end of which is immersed in a liquid, reduces the pressure above the tube, as illustrated in Figure. This reduction in pressure causes the liquid to rise into
the air stream. The liquid is then dispersed into a fine spray of droplets. You might recognize that this so-called atomizer is used in perfume bottles and paint sprayers.