

Q1: If $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$, $B^T = \begin{bmatrix} 1 & 2 \\ 0 & 2 \\ -1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ and $P(x) = \frac{1}{4}x^2 - x + 2$, then

find the following:

(a) $P(A)$ (3 marks)

(b) $\text{adj}(A)$ in details (2 marks)

(c) the inverse of C (3 marks)

(d) the solution set of $Bx=0$ by Gauss-Jordan Elimination. (3 marks)

(e) $T_B(1,2,3)$. (1 mark)

Q2: Find the determinant of the following matrix, then find the cofactor C_{12} :

(4 marks)

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 5 & 4 & 4 \\ 3 & 6 & 6 & 7 \\ 4 & 8 & 10 & 8 \end{bmatrix}$$

Q3: (a) Prove that if A is an invertible matrix, then $\det(A^{-1}) = (\det(A))^{-1}$. (2 marks)

(b) Prove that if A is an invertible symmetric matrix, then A^{-1} is symmetric.

(2 marks)

(c) If $B = \begin{bmatrix} 1 & 5 \\ 1 & 2 \end{bmatrix}$, then find $\text{tr}(B)$. (1 mark)

(d) If A is a square matrix of order 2 such that $\det(A) = 3$, then find $\det(2(A^T)^{-1})$. (2 marks)

(e) If the solution set of the system $Ax=b$ is $\{(2r+1, s-1): r, s \in \mathbb{R}\}$, then find the solution set of the system $Ax=0$. (2 marks)