

Q1: If $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ and $P(x) = x^2 + x - 2$, then find

the following:

(a) $P(A)$ (3 marks)

(b) $\text{adj}(BB^T)$ in details (2 marks)

(c) the inverse of C (3 marks)

(d) Solution of $Bx=0$ by Gauss-Jordan Elimination. (4 marks)

(e) $T_B(1,2,3)$. (2 marks)

Q2: Find the determinant of the following matrix, then find the cofactor C_{12} :

(5 marks)

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 3 & 4 \\ 1 & 2 & 3 & 5 \\ 1 & 2 & 5 & 4 \end{bmatrix}$$

Q3: (a) If $E = \begin{bmatrix} a & b & a \\ e & -2a & e \\ a & a & a \end{bmatrix}$, then find $\det(E)$ and $\text{tr}(E)$. (2 marks)

(b) Prove that if A is an invertible symmetric matrix, then A^{-1} is symmetric.

(2 marks)

(c) If A is an invertible matrix of size 3×3 and $|A|=2$, then find $|2(A^T)^{-1}|$.

(2 marks)