King Saud University
College of Science
Department of Mathematics
1^{st} Semester 1443 H

Math-225
Final Exam
Duration: 3 hours

Question	Grade	Question	Grade
Question I (a)		Question III (b)	
Question I (b)		Question IV (a)	
Question I (c)		Question IV (b)	
Question II (a)		Question V (a)	
Question II (b)		Question V (b)	
Question III (a)		Total out of 40	

(1)	(2)	(3)	(4)	(5)

Question I:[5+3+2=10]

(a) Choose the correct answer (Write it down on the above table) :

(1) If the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$ has a solution $y_1 = 1$, the second solution is

(i) $y = e^{-x}$.
(ii) $y = e^{x}$.
(iii) $y = x$.
(iv) None of the previous.

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(2) The singular points of the differential equation $(9 - x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2y = 0$ are

(i) $x = 0$.
(ii) $x = -3,\ x = 3$.
(iii) $x = 0,\ x = -3,\ x = 3$.
(iv) None of the previous.

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(3) If the auxiliary equation of a homogeneous differential equation is $(m-3)(m+2)^2(m^2+4m+5)=0$ then

(i) $y = c_1e^{3x} + c_2e^{-2x} + c_3xe^{-2x} + e^{-2x}(c_4\cos(\frac{1}{2}x) + c_5\sin(\frac{1}{2}x))$.
(ii) $y = c_1e^{3x} + c_2e^{-2x} + c_3xe^{-2x} + c_4\cos(\frac{1}{2}x) + c_5\sin(\frac{1}{2}x)$.
(iii) $y = c_1e^{3x} + c_2e^{-2x} + c_3xe^{-2x} + e^{-2x}(c_4\cos x + c_5\sin x)$.
(iv) None of the previous.

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(4) $\mathcal{L}^{-1}\{\frac{1}{(s-a)^2+b^2}\}$ equals

(i) $e^{at}\sin(bt)$
(ii) $\frac{1}{b}e^{at}\sin(bt)$
(iii) $\frac{1}{b}e^{at}\cos(bt)$
(iv) None of the previous.

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(5) The solution $x^2+y^2=c^2$ of the differential equation $ydy=-xdx$ is a

(i) explicit solution.
(ii) implicit solution.
(iii) None of the previous.

(b) **Without solving**. Classify the differential equations below as separable, linear, exact, homogeneous and/or Bernoulli:

(i) $y' = \frac{3x^2+4x-4}{2y-4}$.
(ii) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0$.
(iii) $y' = 5y + e^{-2x}y^{-2}$.

(c) Determine the largest region of the xy-plane for which the differential equation has a unique solution

$$\frac{dy}{dx} = x - \sqrt{y-2}.$$

Question II:[3+4=7]

(a) Find the orthogonal trajectories of the family

$$cx^2 - y^2 = 1.$$

(b) Find the solution of the non-exact differential equation $(x^2-y^2+x)dx+2xydy = 0;\ x>0,\ y>0$ by using integration factor.

Question III:[(2+4)+3=9]

(a) Solve the following differential equations

(i) $y'' - 6y' - 2y = 0$
(ii) $x^2y'' - xy' + y = 2x;\quad x>0.$
$y(1)=0,\quad y'(1)=0.$

(b) Using the **superposition approach**, to find the form of the particular solution of the nonhomogenous differential equation:

$$y''' - 4y'' + 4y' = 5x^2 - 6x + 4x^2e^{2x} + 3e^{-2x}.$$

Question IV:[3+4=7]

(a) Solve the system of differential equations:

$$x'(t) = -3x + 4y$$

$$y'(t) = -2x + 3y$$

$$x(0) = -1,\ y(0) = 3$$

(b) Find the power series solutions about the ordinary point $x = 1$ for the following differential equation:

$$y'' - 2(x-1)y' + 2y = 0.$$

$$y(1) = 0, \quad y'(1) = 1.$$

Question V:[2+5=7]

(a) Find $\mathcal{L}^{-1}\left(\frac{s-3}{s^2-3s+2}\right)$.

(b) Use the Laplace transform to solve the initial value problem

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 0, \quad y(0) = 1,\ \frac{dy}{dt}(0) = 0.$$

[Hint: Use Part (a)]

Good Luck.