First Semester	**Final Exam**	**King Saud University**
(without calculators)	**Time allowed: 3 hours**	**College of Science**
Sunday 5-5-1442	**240 Math**	**Math. Department**

Q1: Suppose (1,2,3) is a solution of the following linear system:

$$x_1 + 2x_2 - x_3 = b_1$$
$$2x_1 + 3x_2 - 3x_3 = b_2$$

Find the **values** of b_1, b_2. (2 marks)

Q2: Show that the matrix A is invertible, where $A^2 + 3A = B$ and det(B)=2. (2 marks)

Q3: Let V be the subspace of $\mathbb{R}^3$ **spanned** by the set S={ v_1=(1, 2,3), v_2=(2, 4,6), v_3=(4, 6, 6)}. Find a **subset** of S that forms a basis of V. (4 marks)

Q4: Show that $A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$ is diagonalizable and find a matrix P that diagonalizes A. (6 marks)

Q5: Assume that the vector space $\mathbb{R}^3$ has the Euclidean inner product. Apply the Gram-Schmidt process to transform the following basis vectors (1,-2,0), (2,1,-1), (0,1,1) into an **orthonormal basis**. (8 marks)

Q6: Let V be an inner product space, let v_o be any fixed vector in V, and let $T: V \to \mathbb{R}$ be the map defined by $T(v) = \langle v, v_o \rangle$ for all v in V. Show that:

(a) T is a linear transformation. (4 marks)

(b) If $v_o \in \ker(T)$, then $v_o = 0$ and $\ker(T) = V$. (2 marks)

Q7: Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by:
$T(x_1, x_2) = (3x_1 - x_2, -2x_1, x_1 + x_2)$.

(a) Find $[T]_{S,B}$ where S is the standard basis of $\mathbb{R}^3$ and B={v_1=(1,1),v_2=(1,0)}. (4 marks)

(b) Show that T is one-to-one. (2 marks)

Q8: Show that:

(a) If $T: V \to W$ is a linear transformation, then the kernel of T is a subspace of V. (2 marks)

(b) If 1 and -1 are the eigenvalues of a square matrix A of order 2, then we have that $A^{100} = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. (2 marks)

(c) If u and v are orthogonal vectors in an inner product space, then: $\|u+v\|^2 = \|u\|^2 + \|v\|^2$. (2 marks)