Exercise of Transportation problem

Example 1: A Company has 2 production facilities S1 and S2 with production capacity of 100 and 110 units per week of a product, respectively. These units are to be shipped to 3 warehouses D1, D2 and D3 with requirement of 80,70 and 60 units per week, respectively. The transportation costs (in $) per unit between factories to warehouses are given in the table below.

A)

Destination / Sources	D_1	D_2	D_3	Supply
S_1	1	2	3	100
S_2	4	1	5	110
Demand	80	70	60	

Find initial basic feasible solution (IBFS) to the following transportation problem using NWCM, then optimize the solution using MODI method (Modified Distribution Method -UV method) .

Answer:

$$\text{Min } Z = x_{11} + 2x_{12} + 3x_{13} + 4x_{21} + x_{22} + 5x_{23}$$

$$x_{11} + x_{12} + x_{13} \le 100$$

$$x_{21} + x_{22} + x_{23} \le 110$$

$$x_{11} + x_{21} \ge 80$$

$$x_{12} + x_{22} \ge 70$$

$$x_{13} + x_{23} \ge 60$$

Min $Z = \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij}$

s.t

$\sum_{j=1}^{m} x_{ij} \le s_i$

$\sum_{i=1}^{n} x_{ij} \ge d_j$

$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j = 210$, so we don't need dummy demand or dummy supply.

starting point is the north-west corner of the table.

$min(S_1 = 100, D_1 = 80) = \mathbf{80}$, This satisfies the total demand of D_1 and leaves 100 - 80 = 20 units with S_1.

$min(S_1 = 20, D_1 = 70) = \mathbf{20}$, This exhausts the capacity of S_1 and remain 70 - 20 = 50 units for D_2.

$min(S_2 = 110, D_2 = 50) = \mathbf{50}$, This satisfies the total demand of D_2 and leaves 110 - 50 = 60 units with S_2.

$min(S_2 = 60, D_3 = 60) = \mathbf{60}$, This satisfies S_2 and D_3.

Destination / Sources	D_1	D_2	D_3	Supply		
S_1	1 80	2 20	3	100	20	0
S_2	4	1 50	5 60	110	60	0
Demand	80	70	60			
	0	50 0	0			

Initial feasible solution (IBFS) is:

$$X_{11} = 80,\ X_{12} = 20,\ X_{22} = 50,\ X_{23} = 60\ , X_{13} = 0, X_{21} = 0$$

The total transportation cost:

$$TTC = Z = 80 * 1 + 20 * 2 + 50 * 1 + 60 * 5 = \mathbf{470}\$$$

The number of allocated cells = 4 is equal to m + n - 1 = 3 + 2 - 1 = 4, so the solution could be improved.

Optimality test using MODI method...

$$\boldsymbol{\delta_{kj} = v_j + u_i - C_{kj}},$$

1. Find u_i and v_j for all occupied cells (i, j), where $v_j + u_i = C_{ij}$
   - Let u_1=0
   - $c_{11} = u_1 + v_1 \Rightarrow v_1 = c_{11} - u_1 \Rightarrow v_1 = 1 - 0 \Rightarrow v_1 = 1$
   - $c_{12} = u1 + v2 \Rightarrow v2 = c12 - u1 \Rightarrow v2 = 2 - 0 \Rightarrow v2 = 2$
   - $c_{22} = u2 + v2 \Rightarrow u2 = c22 - v2 \Rightarrow u2 = 1 - 2 \Rightarrow u2 = -1$
   - $c_{23} = u_2 + v_3 \Rightarrow v_3 = c_{23} - u_2 \Rightarrow v_3 = 5 + 1 \Rightarrow v_3 = 6$
2. Find $\boldsymbol{\delta_{kl} = v_l + u_k - C_{kl}}$ for all ***unoccupied*** cells (k, l). IF all $\boldsymbol{\delta_{kl}} \leq 0$, the solution is optimal solution.
3. Now choose the maximum positive value from all $\boldsymbol{\delta_{kj}}$ **(opportunity cost)** = $\boldsymbol{\delta_{13} = 3}$ and draw a closed path $S1D3 \rightarrow S2D3 \rightarrow S2D2 \rightarrow S1D2$ with plus/minus sign allocation.
4. Minimum allocated value among all negative position (-) on closed path $\theta = 20$ Subtract 20 from all (-) and Add it to all (+).

		V₁=1	V₂=2	V₃=6	
	Destination / Sources	D₁	D₂	D₃	Supply
$U_1=0$	S_1	1 80	- 2 20	3 + δ_{13}= 3	100
$U_2=-1$	S_2	4 δ_{21}= - 4	+ 1 50	- 5 60	110
	Demand	80	70	60	

5. Repeat the step 1 to 4, until an optimal solution is obtained.

		V_1= 1	V_2= -1	V_3= 3	
	Destination / Sources	D₁	D₂	D₃	Supply
U_1= 0	S_1	1 80	2 δ_{12}= -3	3 20	100
U_2= 2	S_2	4 δ_{21}= -1	1 70	5 40	110
	Demand	80	70	60	

The new solution (*):

$$X_{11} = 80,\ X_{13} = 20,\ X_{22} = 70,\ X_{23} = 40, X_{12} = X_{21} = 0$$

The minimum total transportation cost: $Z^* = 80 * 1 + 20 * 3 + 70 * 1 + 40 * 5 = 410\$$

The number of allocated cells = 4 is equal to m + n - 1 = 3 + 2 - 1 = 4.

All $\boldsymbol{\delta_{kj}} \leq 0$, so solution (*) is an optimal solution.

B) same previous example (A) but change S2 to 130 rather than 110.

Answer:

Destination / Sources	D_1	D_2	D_3	Supply
S_1	1	2	3	100
S_2	4	1	5	130
Demand	80	70	60	230 / 210

Here Total Demand = 210 is less than Total Supply = 230. So, we add a **dummy demand** constraint with 0 unit cost and with allocation 20. ($\boldsymbol{x_{14} + x_{24} \geq 20}$)

Destination / Sources	D_1	D_2	D_3	D_4(Dummy)	Supply
S_1	1	2	3	0	100
S_2	4	1	5	0	130
Demand	80	70	60	20	230=230

		$V_1=1$	$V_2=2$	$V_3=6$	$V_4=1$		
	Destination / Sources	D_1	D_2	D_3	D_4(Dummy)	Supply	
$U_1=0$	S_1	1 80	- 2 20	+ 3 $\delta_{13}=3$	0 $\delta_{14}=1$	100	20 0
$U_2=-1$	S_2	4 $\delta_{21}=-4$	+ 1 50	- 5 60	0 20	130	80 20 0
	Demand	80	70	60	20		
		0	50 0	0	0		

Initial feasible solution (IBFS) is:

$$X_{11} = 80,\ X_{12} = 20,\ X_{22} = 50,\ X_{23} = 60,\ X_{24} = 20, X_{13} = X_{14} = X_{21} = 0$$

The minimum total transportation cost:

$$TTC = Z = 80 * 1 + 20 * 2 + 50 * 1 + 60 * 5 + 20 * 0 = 470$$

Here, the number of allocated cells = 5 is equal to m + n - 1 = 2 + 4 - 1 = 5

Not all $\boldsymbol{\delta_{kj}} \leq 0$, so IBFS is **not** an optimal solution.

		$V_1 = 1$	$V_2 = -1$	$V_3 = 3$	$V_4 = -2$	
	Destination / Sources	D_1	D_2	D_3	D_4(Dummy)	Supply
$U_1 = 0$	S_1	1 80	2 $\delta_{12} = -3$	3 20	0 $\delta_{14} = -2$	100
$U_2 = 2$	S_2	4 $\delta_{21} = -1$	1 70	5 40	0 20	110
	Demand	80	70	60	20	

The new solution (*):

$$X_{11} = 80,\ X_{13} = 20,\ X_{22} = 70,\ X_{23} = 40,\ X_{24} = 20,\ X_{12} = X_{21} = 0$$

$$Z^* = 80 * 1 + 20 * 3 + 70 * 1 + 40 * 5 = 410\$$$

The number of allocated cells = 5 is equal to m + n - 1 = 4+ 2 - 1 = 5.

All $\delta_{kj} \leq 0$, so solution (*) is an optimal solution.

C) same previous example in part (B) but change D1, D2 and D3 to 90,80 and 100 units per week, respectively.

Answer:

Destination / Sources	D_1	D_2	D_3	Supply
S_1	1	2	3	100
S_2	4	1	5	130
Demand	90	80	100	230 / 270

Here Total Demand = 270 is greater than Total Supply = 230. So, we add a dummy supply constraint with 0 unit cost and with allocation 40. ($x_{31} + x_{32} + x_{33} \leq 40$)

		$V_1 = 1$	$V_2 = 2$	$V_3 = 6$			
	Destination / Sources	D_1	D_2	D_3	Supply		
$U_1 = 0$	S_1	1 90	- 2 10	+ 3 $\delta_{13} = 3$	100	10	0
$U_2 = -1$	S_2	4 $\delta_{21} = -4$	+ 1 70	- 5 60	130	60	0
$U_3 = -6$	S_3 (Dummy)	0 $\delta_{12} = -5$	0 $\delta_{12} = -4$	0 40	40	0	
	Demand	90	80	100	270 / 270		
		0	70	40			
			0	0			

Initial feasible solution (IBFS) is:

$X_{11} = 90,\ X_{12} = 10,\ X_{22} = 70,\ \ X_{23} = 60,\ \ X_{33} = 40, X_{13} = X_{21} = X_{31} = X_{32} = 0$

The total transportation cost:

$TTC = Z = 90 * 1 + 10 * 2 + 70 * 1 + 60 * 5 + 40 * 0 = 480\$$

Here, the number of allocated cells = 5 is equal to m + n - 1 = 3 + 3 - 1 = 5

Not all $\boldsymbol{\delta}_{\mathbf{kj}} \leq 0$, so IBFS is **not** an optimal solution.

		V₁= -2	V₂= -4	V₃= 0	
	Destination / Sources	D₁	D₂	D₃	Supply
U₁= 3	S₁	1 90	2 δ₁₂= -3	3 10	100
U₂= 5	S₂	4 δ₂₁= -1	1 80	5 50	130
U₃= 0	S₃ (Dummy)	0 δ₁₂= -2	0 δ₁₂= -4	0 40	40
	Demand	90	80	100	

All $\boldsymbol{\delta}_{\mathbf{kj}} \leq 0$, so the optimal solution is :

$X_{11} = 90,\ \ X_{13} = 10,\ \ X_{22} = 80,\ X_{23} = 50, X_{33} = 40, X_{12} = X_{21} = X_{31} = X_{32} = 0$

The minimum total transportation cost: $Z^* = 90 * 1 + 10 * 3 + 80 * 1 + 50 * 5 = 450\$$

The number of allocated cells = 5 is equal to m + n - 1 = 3 + 3 - 1 = 5.

Degenerate case

Example 2: A company has factories at S1, S2 and S3 which supply to warehouses at D1, D2, D3 and D4. Weekly factory capacities are 18, 3 and 30 units, respectively. Weekly warehouse requirement are 21, 15, 9 and 6 units, respectively. Unit shipping costs (in Dollar) are as follows:

Destination / Sources	D₁	D₂	D₃	D₄	Supply
S₁	8	21	44	28	18
S₂	4	0	24	4	3
S₃	20	32	60	36	30
Demand	21	15	9	6	

Solution:

Destination / Sources	D_1	D_2	D_3	D_4	Supply	
S_1	8 18	21	44	28	18	0
S_2	4 3	0	24	4	3	0
S_3	20	32 15	60 9	36 6	30	15 6 0
Demand	21	15	9	6	51 51	
	3 0	0	0	0		

Initial feasible solution (IBFS) is:

$$X_{11} = 18,\ X_{21} = 3,\ X_{32} = 15,\ \ X_{33} = 9, X_{34} = 6$$

The total transportation cost:

$TTC = Z = 8 * 18 + 4 * 3 + 32 * 15 + 60 * 6 = \mathbf{1392}\$$

The number of allocated cells =5≠ **m + n - 1 = 3 + 4 - 1 = 6,** then **degeneracy** does exist**.**

Note: this solution is degenerate.

To resolve degeneracy, we proceed by allocating a small quantity $(\boldsymbol{\varepsilon})$ to one or more (if needed) unoccupied cells that have lowest transportation costs, so as to allocate $m + n - 1$ cells.

The quantity $\boldsymbol{\varepsilon}$ is assigned to *cell (2,2)* , which has the minimum transportation cost = **0**.

	Iteration-1	V_1= 36	V_2= 32	V_3= 60	V_4= 36	
	Destination / Sources	D_1	D_2	D_3	D_4	Supply
U_1= -28	S_1	8 18	21 δ_{12} = -17	44 δ_{13} = -12	28 $\delta_{14} = -20$	18
U_2= -32	S_2	- 4 3	+ 0 ε	24 $\delta_{23} = 4$	4 $\delta_{24} = 0$	3
U_3= 0	S_3	+ 20 $\delta_{31} = 16$	- 32 15	60 9	36 6	30
	Demand	21	15	9	6	51 51

To Find u_i and v_j for all occupied cells (i, j), where $v_j + u_i = C_{ij}$

- Let u_3 =0
- $c_{32} = u_3 + v_2 \Rightarrow v_2 = c_{32} - u_3 \Rightarrow v_2 = 32 - 0 = 32$
- $c_{33} = u_3 + v_3 \Rightarrow v_3 = c_{33} - u_3 \Rightarrow v_3 = 60 - 0 \Rightarrow v_3 = 60$

- $c_{34} = u_3 + v_4 \Rightarrow v_4 = c_{34} - u_3 \Rightarrow v_4 = 36 - 0 = 36$
- $c_{22} = u_2 + v_2 \Rightarrow u_2 = c_{22} - v_2 \Rightarrow u_2 = 0 - 32 = -32$
- $c_{21} = u_2 + v_1 \Rightarrow v_1 = c_{21} - u_2 \Rightarrow v_1 = 4 - (-32) = 36$
- $c_{11} = u_1 + v_1 \Rightarrow u_1 = c_{11} - v_1 \Rightarrow u_1 = 8 - 36 = -28$

It is clear that not all $\boldsymbol{\delta_{kj}} \leq 0$, so IBFS is **not** an optimal solution.

	Iteration-2	V_1= 20	V_2= 32	V_3= 60	V_4= 36	
	Destination / Sources	**D_1**	**D_2**	**D_3**	**D_4**	**Supply**
U_1= -12	**S_1**	- 8 18	21 $\delta_{12} =$ -1	+ 44 $\delta_{13} = 4$	28 $\delta_{14} =$ - 4	**18**
U_2= -32	**S_2**	4 $\delta_{21} = -16$	0 $\varepsilon + 3$	24 $\delta_{23} = 4$	4 $\delta_{24} = 0$	**3**
U_3= 0	**S_3**	+ 20 3	32 12	- 60 9	36 6	**30**
	Demand	**21**	**15**	**9**	**6**	51 / 51

The new solution (*) is:

$$X_{11} = 18,\ X_{22} = \boldsymbol{\varepsilon + 3},\ X_{31} = 3, X_{32} = 15,\ \ X_{33} = 9, X_{34} = 6$$

$$X_{12} = X_{13} = X_{13} = X_{14} = X_{21} = X_{23} = X_{24} = 0$$

The total transportation cost:

$$TTC = Z = 8 * 18 + 0 * (\boldsymbol{\varepsilon + 3}) + 20 * 3 + 32 * 12 + 60 * 9 + 36 * 6 = \mathbf{1344}\$$$

The number of allocated (occupied) cells = 6= **m + n - 1 = 3 + 4 - 1 = 6,** so the solution could be improved.

$$find\ u_i\ and\ v_j\ \Rightarrow \cdots$$

It is clear that **not** all $\boldsymbol{\delta_{kj}} \leq 0$, so solution (*) is **not** an optimal solution.

	Iteration-3	V_1= 20	V_2= 32	V_3= 56	V_4= 36	
	Destination / Sources	**D_1**	**D_2**	**D_3**	**D_4**	**Supply**
U_1= -12	**S_1**	8 9	21 $\delta_{12} =$ -1	44 9	28 $\delta_{14} =$ -4	**18**
U_2= -32	**S_2**	4 $\delta_{21} = -16$	0 $\varepsilon + 3$	24 $\delta_{23} = 0$	4 $\delta_{24} = 0$	**3**
U_3= 0	**S_3**	20 12	32 12	60 $\delta_{33} = -4$	36 6	**30**
	Demand	**21**	**15**	**9**	**6**	51 / 51

The new solution (**) is:

$$X_{11} = 9, X_{12} = 9,\ X_{22} = \boldsymbol{\varepsilon + 3},\ X_{31} = 12, X_{32} = 12, X_{34} = 6$$
$$X_{12} = X_{13} = X_{21} = X_{23} = X_{24} = X_{33} = 0$$

The minimum total transportation cost:

$$TTC = Z = 8 * 9 + 44 * 9 + 0 * (\boldsymbol{\varepsilon + 3}) + 20 * 12 + 32 * 12 + 36 * 6 = \mathbf{1308}\$$$

The number of allocated cells = 6= **m + n - 1 = 3 + 4 - 1 = 6,** so the solution could be improved.

$$find\ u_i\ and\ v_j\ \Rightarrow \cdots$$

All $\boldsymbol{\delta_{kj}} \leq 0$, so solution (**) is an optimal solution.

Note: alternate solution is available with unoccupied cell (2,4) , but with the same optimal value.

Example 3: Find the optimal solution and minimum total cost to the following transportation problem:

Destination / Sources	D_1	D_2	D_3	Supply
S_1	5	4	2	70
S_2	6	3	2	50
S_3	1	5	1	10
Demand	50	50	30	130 / 130

Solution:

		V_1= 5	V_2= 4	V_3= 3		
	Destination / Sources	D_1	D_2	D_3	Supply	
U_1= 0	S_1	- 5 50	+ 4 20	2 δ= 1	70	20 0
U_2= -1	S_2	6 δ= -2	- 3 30	+ 2 20	50	20 0
U_3= -2	S_3	+ 1 δ= 2	5 δ= -3	- 1 10	10	
	Demand	50	50	30		
		0	30 0	10		

Initial feasible solution (IBFS) is:

$$X_{11} = 50,\ X_{21} = 20,\ X_{22} = 30,\ \ X_{23} = 20, X_{33} = 10$$

The total transportation cost:

$$TTC = Z = 5 * 50 + 4 * 20 + 3 * 30 + 2 * 20 + 1 * 10 = \mathbf{470}$$

Here, the number of allocated cells = 5= **m + n - 1 = 3 + 4 - 1 = 5,** so the solution could be improved.

Not all $\boldsymbol{\delta_{kj}} \leq 0$, so IBFS is **not** an optimal solution.

		V_1= 5	V_2= 4	V_3= 3	
	Destination / Sources	D_1	D_2	D_3	Supply
U_1= 0	S_1	5 40	- 4 30	+ 2 $\delta_{13} = 1$	70
U_2= -1	S_2	6 $\delta_{21} = -2$	+ 3 20	- 2 30	50
U_3= -4	S_3	1 10	5 $\delta_{32} = -5$	1 $\delta_{33} = -2$	10
	Demand	50	50	30	

The new solution (*):

$$X_{11} = 40,\ X_{12} = 30,\ X_{22} = 20,\ \ X_{23} = 30, X_{31} = 10, X_{13} = X_{21} = X_{32} = X_{33} = 0$$

The total transportation cost:

$$TTC = Z = 5 * 40 + 4 * 30 + 3 * 20 + 2 * 30 + 1 * 10 = \mathbf{450}$$

Here, the number of allocated (occupied) cells = 5= **m + n - 1 = 3 + 4 - 1 = 5,** so the solution could be improved.

Not all $\boldsymbol{\delta_{kj}} \leq 0$, so solution (*) is **not** an optimal solution.

		V_1= 5	V_2=?	V_3= 2	
	Destination / Sources	D_1	D_2	D_3	Supply
U_1= 0	S_1	5 40	4	2 30	70
U_2= ?	S_2	6	3 50	2	50
U_3= - 4	S_3	1 10	5	1	10
	Demand	50	50	30	

The new solution (**):

$$X_{11} = 40,\ X_{13} = 30,\ X_{22} = 50,\ \ X_{31} = 10, X_{12} = X_{21} = X_{23} = X_{32} = X_{33} = 0$$

The total transportation cost:

$TTC = Z = 5 * 40 + 2 * 30 + 3 * 50 + 1 * 10 = \mathbf{420}$

The number of allocated (occupied) cells =4 ≠ **m + n - 1 = 3 + 4 - 1 = 5, then degeneracy does exist (the solution cannot be improved)**

Note: Is the solution ** optimal?

To Find u_i and v_j for all occupied cells (i, j), where $v_j + u_i = C_{ij}$

- Let u_1=0, we get
- $u_1 + v_1 = 5 \Rightarrow v_1 = 5$
- $u_1 + v_3 = 2 \Rightarrow v_3 = 2$
- $u_2 + v_2 = 3 \Rightarrow u_2 = ?$, $v_2 = ?$ The u_2 *and* v_2 cannot be assigned because the occupied cells condition is not met.
- $u_3 + v_1 = 1 \Rightarrow u_3 + 5 = 1 \Rightarrow u_3 = -4$

To resolve degeneracy, we proceed by allocating a small quantity (ε) to one or more (if needed) unoccupied cells that have lowest transportation costs, so as to allocate $m + n - 1$ cells.

		V_1= 5	V_2=?	V_3= 2	
	Destination / Sources	D_1	D_2	D_3	Supply
U_1= 0	S_1	5 40	4	2 30	70
U_2= ?	S_2	6	3 50	2	50
U_3= - 4	S_3	1 10	5	1 ε	10
	Demand	50	50	30	

If the quantity ε is assigned to cell (3,3), which has the least transportation cost = 1. Obviously, assigning ε to cell *(3,3)* does not help in finding the values of u_2 *and* v_2.

To Find u_i and v_j for all occupied cells (i, j), where $v_j + u_i = C_{ij}$

- let u_1=0
- $u_1 + v_1 = 5 \Rightarrow v_1 = 5$
- $u_1 + v_3 = 2 \Rightarrow v_3 = 2$
- $u_2 + v_2 = 3 \Rightarrow u_2 = ?$, $v_2 = ?$ The u_2 *and* v_2 cannot be assigned because the occupied cells condition is not met.
- $u_3 + v_1 = 1 \Rightarrow u_3 + 5 = 1 \Rightarrow u_3 = -4$
- $u_3 + v_3 = 1 \Rightarrow -4 + 2 \neq 1$

Therefore, assigning ε to cell (2,3), which has the second least transportation cost=2.

		$V_1= 5$	$V_2= 3$	$V_3= 2$	
	Destination / Sources	D_1	D_2	D_3	Supply
$U_1= 0$	S_1	5 40	4 $\delta_{12} = -1$	2 30	70
$U_2= 0$	S_2	6 $\delta_{21} = -1$	3 50	2 ε	50
$U_3= - 4$	S_3	1 10	5 $\delta_{32} = -6$	1 $\delta_{33} = -3$	10
	Demand	50	50	30	

To Find u_i and v_j for all occupied cells (i, j), where $v_j + u_i = C_{ij}$

- Substituting, u_1=0, we get
- $u_1 + v_1 = 5 \Rightarrow v_1 = 5$
- $u_1 + v_3 = 2 \Rightarrow v_3 = 2$
- $u_2 + v_3 = 2 \Rightarrow u_2 + 2 = 2 \Rightarrow u_2 = 0$
- $u_2 + v_2 = 3 \Rightarrow 0 + v_2 = 3 \Rightarrow , v_2 = 3$
- $u_3 + v_1 = 1 \Rightarrow u_3 + 5 = 1 \Rightarrow u_3 = -4$

The new solution (***):

$X_{11} = 40,\ X_{13} = 30,\ X_{22} = 50, X_{23} = \boldsymbol{\varepsilon},\ X_{31} = 10, X_{12} = X_{21} = X_{32} = X_{33} = 0$

The minimum total transportation cost:

$TTC = Z = 5 * 40 + 2 * 30 + 3 * 50 + 2\,\boldsymbol{\varepsilon} + 1 * 10 = \mathbf{420} + \mathbf{2}\boldsymbol{\varepsilon}$

ε is small quantity close to zero, $\varepsilon \approx 0$

$$TTC = Z = 420$$

It is obvious that all $\boldsymbol{\delta_{kj}} \leq 0$, then solution (***) is optimal solution.

H.W Example 4: The ICARE Company has three factors located throughout a state with production capacity 40, 15 and 40 gallons. Each day the firm must furnish its four retail shops D1, D2, D3 with at least 25, 55 , and 20 gallons respectively. The transportation costs (in $.) are given below.

Destination / Sources	D_1	D_2	D_3	Supply
S_1	10	7	8	40
S_2	15	12	9	15
S_3	7	8	12	40
Demand	25	55	20	95 / 100

Q: Find the **optimum** transportation schedule and minimum total cost of transportation.

Answer:

The minimum total transportation cost =7×40+9×15+7×25+8×15+0×5=710

		$V_1= 3$	$V_2= 0$	$V_3=4$		
	Destination / Sources	D_1	D_2	D_3	Supply	
$U_1= 7$	S_1	10 25	7 15	8 $\delta_{13}= 3$	40	~~15~~ 0
$U_2= 12$	S_2	15 $\delta_{21}= 0$	- 12 15	+ 9 $\delta_{23}= 7$	15	0
$U_3=8$	S_3	7 $\delta_{31}= 4$	+ 8 25	- 12 15	40	~~15~~ 0
$U_4= -4$	S_4 (Dummy)	0 $\delta_{41}= -1$	0 $\delta_{42}= -4$	0 5	5	0
	Demand	25	55	20	100 / 100	
		0	~~40~~ ~~25~~ 0	~~5~~ 0		

$\theta = 15$ Subtract 15 from all (-) and Add it to all (+).

		$V_1= 10$	$V_2= 7$	$V_3=4$	
	Destination / Sources	D_1	D_2	D_3	Supply
$U_1= 0$	S_1	- 10 25	+ 7 15	8 $\delta_{13}= -4$	40
$U_2= 5$	S_2	15 $\delta_{21}= 0$	- 12 0	+ 9 15	15
$U_3=1$	S_3	7 $\delta_{31}= 4$	8 40	12 $\delta_{31}= -7$	40
$U_4= -4$	S_4 (Dummy)	+ 0 $\delta_{41}= 6$	0 $\delta_{42}= 3$	- 0 5	5
	Demand	25	55	20	100 / 100

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6

$\theta = 0$ Subtract from all (-) and Add it to all (+).

		$V_1= 0$	$V_2= -3$	$V_3=0$	
	Destination / Sources	D_1	D_2	D_3	Supply
$U_1= 10$	S_1	- 10 25	+ 7 15	8 $\delta_{13}= 2$	40
$U_2= 9$	S_2	15 $\delta_{21}= -6$	12 $\delta_{22}=-6$	9 15	15
$U_3=11$	S_3	+ 7 $\delta_{31}= 4$	- 8 40	12 $\delta_{31}= -1$	40
$U_4= 0$	S_4 (Dummy)	0 0	0 $\delta_{42}= -3$	0 5	5
	Demand	25	55	20	100 / 100

$\theta = 25$ Subtract 15 from all (-) and Add it to all (+).

		$V_1= 0$	$V_2= 1$	$V_3=0$	
	Destination / Sources	D_1	D_2	D_3	Supply
$U_1= 6$	S_1	10 $\delta_{11}=-4$	7 40	8 $\delta_{13}= -2$	40
$U_2= 9$	S_2	15 $\delta_{21}= -6$	12 $\delta_{22}=-2$	9 15	15
$U_3=7$	S_3	7 25	8 15	12 $\delta_{31}= -5$	40
$U_4= 0$	S_4 (Dummy)	0 0	0 $\delta_{42}= 1$	0 5	5
	Demand	25	55	20	100 / 100