## Exercise

Note: +S: slack variable, -S: surplus variable, $URS: unresticted,$

**Q1: Put the following LP in the standard form:**

1- **Max $Z = X_1 - X_2 - 3X_3$**

Subject to

$2X_1 + X_2 - X_3 \leq 2$

$X_1 - 3X_2 + 2X_3 \leq 3$

$X_1 + X_2 - X_3 \geq -2$ *(-1)

$X_1 \geq 0, X_2 \leq 0, X_3 \, URS$

The standard form

Replace $X_2$ with $(-X_2^-)$ , and Replace $X_3$ with $({X_3}^+ - {X_3}^-)$

Max $Z = X_1 + {X_2}^- - 3{X_3}^+ + 3{X_3}^-$

Subject to:

$2X_1 - {X_2}^- - X_3^+ + X_3^- + \boldsymbol{S_1} = 2$

$X_1 + 3{X_2}^- + 2{X_3}^+ - 2{X_3}^- + \boldsymbol{S_2} = 3$

$-X_1 + {X_2}^- + {X_3}^+ - {X_3}^- + \boldsymbol{S_3} = 2$

$X_1, {X_2}^-, {X_3}^+, {X_3}^-, S_1, S_2, S_3 \geq 0$

2- **Min $Z = 80X_1 + X_2$**

Subject to
$0.2X_1 + 0.32X_2 \leq 0.25$
$X_1 + X_2 = 1$
$X_1 \geq 0, X_2 \geq 0$

Min $\boldsymbol{Z} = 80\boldsymbol{X_1} + \boldsymbol{X_2}$
Subject to
$0.2X_1 + 0.32X_2 + S_1 = 0.25$
$X_1 + X_2 = 1$

$X_1, X_2, S_1 \geq 0$

**3- Min $Z = 3X_1 + 8X_2 + 4X_3$**

Subject to

$X_1 + X_2 \geq 8$

$2X_1 - 3X_2 \leq 0$

$X_2 \geq 9$

$X_1 \geq 0, X_2 \geq 0, X_3\ URS$

Replace $X_3$ with $({X_3}^+ - {X_3}^-)$

The standard form

Min $Z = 3X_1 + 8X_2 + 4({X_3}^+ - {X_3}^-)$

Subject to

$X_1 + X_2 - S_1 = 8$

$2X_1 - 3X_2 + S_2 = 0$

$X_2 - S_3 = 9$

$X_1, X_2, {X_3}^+, {X_3}^-, S_1, S_2, S_3 \geq 0$

**Q2: For the LP, answer the following questions?**

**Max $Z = 5X_1 + 4X_2$**

Subject to

$6X_1 + 4X_2 \leq 24$

$X_1 + 2X_2 \leq 6$

$X_1 \geq 0, X_2 \geq 0$

a) Express the problem in equation form (standard form).
b) Determine all basic solutions and classify them as feasible and infeasible.
c) Use direct substitution in the objective function to determine the optimum basic feasible solution.
d) Verify graphically that the solution obtained in (c) is the optimum LP solution.

The standard form

Max $Z = 5X_1 + 4X_2$

Subject to

$6X_1 + 4X_2 + \boldsymbol{S_1} = 24$

$X_1 + 2X_2 + \boldsymbol{S_2} = 6$

$X_1 \geq 0, X_2 \geq 0, S_1 \geq 0, S_2 \geq 0$

Note:

(m) linear equations or basic variables; (n−m) non-basic variables

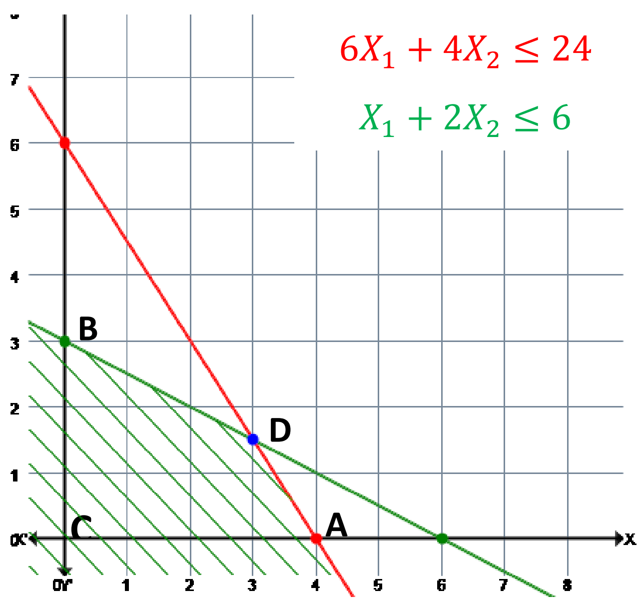
To find total number of basic solutions by use $\binom{n}{m} = nC_m$ "combinations"

All basic solutions are not necessarily feasible.

We have **m=2** constraints and **n=4** variables, thus **n-m=2** Non-basic variables (zero variables).

Total number of Basic solutions are $\binom{4}{2} = 6$

| Non-basic Variables | Basic Variables & Basic Solution | Feasibility Status | Extreme point | Objective Value |
|---|---|---|---|---|
| $S_1, S_2$ | $X_1 = 3, X_2 = 1.5$ | Feasible | D | 21 |
| $S_2, X_2$ | $X_1 = 6, S_1 = -12$ | Infeasible | | |
| $S_1, X_2$ | $X_1 = 4, S_2 = 2$ | Feasible | A | 20 |
| $S_2, X_1$ | $X_2 = 3, S_1 = 12$ | Feasible | B | 12 |
| $S_1, X_1$ | $X_2 = 6, S_2 = -6$ | Infeasible | | |
| $X_1, X_2$ | $S_1 = 24, S_2 = 6$ | Feasible | C | 0 |



**1** $S_1 = S_2 = 0$

$6X_1 + 4X_2 = 24$

$-6 * \quad X_1 + 2X_2 = 6$

$(4-12)X_2 = 24 - 36$

$-8X_2 = -12$

$X_2 = 1.5$

$X_1 + 2(1.5) = 6$

$X_1 = 6 - 3 = 3$

$X_1 = 3$

**2** $S_2 = X_2 = 0$

$X_1 + 2X_2 + S_2 = 6$

$X_1 = 6$

$6X_1 + 4X_2 + S_1 = 24$

$6(6) + 4(0) + S_1 = 24$

$S_1 = 24 - 36$

$S_1 = -12$

**3** $S_1 = X_2 = 0$

$6X_1 + 4X_2 + S_1 = 24$

$6X_1 = 24$

$X_1 = 4$

$X_1 + 2X_2 + S_2 = 6$

$4 + 2(0) + S_2 = 6$

$S_2 = 2$

**4** $S_2 = X_1 = 0$

$X_1 + 2X_2 + S_2 = 6$

$2X_2 = 6$

$X_2 = 3$

$6X_1 + 4X_2 + S_1 = 24$

$4(3) + S_1 = 24$

$S_1 = 24 - 12$

$S_1 = 12$

**5** $S_1 = X_1 = 0$

$6X_1 + 4X_2 + S_1 = 24$

$4X_2 = 24$

$X_2 = 6$

$X_1 + 2X_2 + S_2 = 6$

$2(6) + S_2 = 6$

$S_2 = 6 - 12$

$S_2 = -6$

**6** $X_1 = X_2 = 0$

$6X_1 + 4X_2 + S_1 = 24$

$S_1 = 24$

$X_1 + 2X_2 + S_2 = 6$

$S_2 = 6$

1- **Max $Z = 3X_1 + 2X_2$** **H.W**

Subject to

$2X_1 + 4X_2 \leq 8$

$X_1 + X_2 \leq 2$

$X_1 \geq 0, X_2 \geq 0$

a) Express the problem in equation form.

b) Determine the all basic solutions and classify them as feasible and infeasible.

c) Use direct substitution in the objective function to determin the optimum basic feasible solution.

d) Verify graphically that the solution obtained in (c) is the optimum LP solution.