EXERCISES CH3

PART A

Q1) *Let A = {a, b, c, d, e, f}*

B = {4, 9, 2, 7}

C = {Larry, Moe, Curly}

D = {cat, dog}

E = {2, 4, 6, 8, 10, ...}

N = {1, 2, 3, 4, 5, 6, 7, ...}

F = {x|x is a person in this room}

G = {b, c, f} ,

determine whether each statement is true or false.

1. $7 \in B$
2. $n(A) = a$
3. $B \subseteq N$
4. $dog \subseteq D$
5. $\{7\} \in B$
6. $\{6, 8, 10\} \subseteq E$
7. $\{6, 8, 10\} \subseteq \{6, 8, 10\}$

Q2) *Let*

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

X = {2, 4, 6, 8, 10}

W = {x|x is an odd number}

Y = {3},

determine whether each statement is true or false.

1. $X' = W$

2. $Y \subseteq W$

3. $Y \subset W$

4. $X \subseteq U$

5. $\{\ \} \subseteq X$

Q3) *Let A = {a, b, c, d, e, f} How many subsets does A have?*

Q4)*Let*

U = {1, 2, 3, 4, 5, 6, 7,...}

S = {x|x is less than 10}

How many subsets does S have?

Q5) *Let*

U = { 1, 2, 3, 4, 5, 6, 7 } *S = {1, 2, 4, 7}*

T = {1, 2, 4, 5, 6, 7} *V = {4, 5, 6}*

Decide whether each statement is true or false.

1. $S \subset T$ ***2.*** *n(S) = 7* ***3.*** $4 \notin V$ ***4.*** $T \subseteq V$ ***5.*** $S \subseteq T$

6. *{2, 4, 6} = {x|x is an even number}* ***7.*** $T \subseteq \{\ \}$ ***8.*** $7 \in T$

9. *T has 36 subsets* ***10.*** *V has 7 proper subsets* ***11.*** $\{\ \} \subseteq V$

12. *V = {x|x > 3}* ***13.*** *V = {1, 2,3}* ***14.*** *{7, 4, 2, 1} = S*

15. $\{1\} \subset T$ ***16.*** $\{1\} \subseteq T$ ***17.*** $V \subseteq \{5, 6\}$

Q6) *Let*

U = {a, b, c, d, e, f} *A = {a, c, e, f}* *B = {c, d, e}* *C = {e, f}*

Find each of the following:

1. A'

2. B'

3. C'

4. $B \cup C$

5. $A \cap C$

6. $B \cap C$

7. $(A \cup B)'$

8. $A' \cup B'$

9. $B' \cap C$

10. $A \cup (B' \cap C)$

Q7) *Let*

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

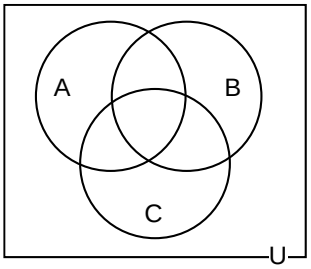
$S = \{2, 5, 7, 9\}$

$V = \{3, 4, 5, 6, 7\}$

$T = \{1, 3, 4, 5, 8, 9\}$

Find $(S' \cap T)'$

Q8) *On a standard three-circle Venn diagram like the one shown below, shade the region(s) corresponding to the given set expression.*



1. $(A' \cap B) \cap C'$

2. $A \cap (B \cap C')$

3. $(A \cup B) \cap C$

4. $(A \cap C') \cup B'$

5. $(A' \cup B)' \cap C$

6. $A' \cup (B' \cap C)$

Q9) *Let U = { b, c, d, e, f, g, h, i, j } V = {e} W = {c, f, g, j} Find* $(V' \cup W')'$

Q10) *Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 } T = {2, 3, 9} V = {8, 9, 10}* $Find\ (T' \cup V)'$

Q11) *Let U = { 1, 2, 3, 4, 5, 6, 7, 8 } T = {2, 5} V = {1, 2, 3, 7, 8}* $Find\ (V \cap T')'$

Q12) $U = \{1, 2, 3, 4, 5, 6, 7\}$ $S = \{2, 4, 5\}$ $T = \{3, 5, 7\}$
$V = \{2, 3, 4, 5, 7\}$ $W = \{1, 2, 3, 4, 6\}$ $Find\ (S \cap V)' \cup (W' \cup T)$

Q13) *Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 } S = {1, 2, 5, 6, 7} T = {5, 6}*
$V = \{1, 2, 3, 5, 6, 9\}$ $Find\ S' \cup (T \cap V)$

PART B

1. Let R be a relation defined on the set $A = \{-2,-1,0,1,2,3,4\}$

$$m, n \in A \quad , \quad m\,R\,n \Leftrightarrow n = m^2$$

(i) What are the ordered pairs in the relation R ?
(ii) Find the domain and the image of R ?

2. Let R be a relation defined on the set $A = \{1,2,3,4,5\}$

$$x, y \in A\,, \qquad x\,R\,y \Leftrightarrow xy \leq 9$$

(i) What are the ordered pairs in the relation R ?
(ii) Find the domain and the image of R ?
(iii) Draw the directed graph (diagraph) that represents R ?

3. Let R be a relation defined on the set $A = \{-4,-3,-2,-1,0,1,2,3,4\}$

$$x, y \in A\,, \qquad x\,R\,y \Leftrightarrow y = 2x$$

(i) What are the ordered pairs in the relation R ?
(ii) Find the domain and the image of R ?

4 . Suppose R is a relation defined on the set $A = \{-2,-1,0,1,2\}$, as

$$x, y \in A, \qquad x\,R\,y \Leftrightarrow |x - y| < 2$$

(i) What are the ordered pairs in the relation R ?
(ii) Draw the directed graph (diagraph) that represents R
(iii) Determine whether the relation R is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

5 . Let R be a relation defined on the set $A = \{-2, -1, 0, 1, 2, 3, 4\}$

$$a, b \in A, \qquad a \, R \, b \Leftrightarrow a^2 = b^2$$

(i) What are the ordered pairs in the relation R ?
(ii) Find the domain and the image of R ?
(iii) Draw the directed graph (diagraph) that represents R ?
(iv) Determine whether the relation R is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

6 . Let R be a relation defined on the set $A = \{-2, -1, 0, 1, 2\}$

$$a, b \in A, \qquad a \, R \, b \Leftrightarrow a.b < 0$$

(i) What are the ordered pairs in the relation R ?
(ii) Find the domain and the image of R ?
(iii) Draw the directed graph (diagraph) that represents R ?
(iv) Find R^2 .
(v) Determine whether the relation R is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

7 . Let R be a relation defined on the set $A = \{-2, -1, 0, 1, 2\}$

$$a, b \in A, \qquad a \, R \, b \Leftrightarrow a.b \geq 2$$

(i) What are the ordered pairs in the relation R ?
(ii) Find the domain and the image of R ?
(iii) Draw the directed graph (diagraph) that represents R ?
(iv) Find R^2 .
(v) Determine whether the relation R is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

8. Let $S = \{(1,1),(1,2),(1,3),(2,2),(3,1),(3,3)\}$ be a relation on the set $B = \{1, 2, 3\}$

(i) Draw the directed graph (diagraph) that represents S ?
(ii) Find S^2 , $\bar{S}$, $So\,\bar{S}$
(iii) Find $\mathbf{M}_S$
(iv) Determine whether the relation S is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*

9. Let $S = \{(1,v),(1,w),(2,u),(2,v),(3,w)\}$ and $T = \{(1,u),(1,w),(2,v),(2,w),(3,u),(3,v)\}$ are relations from the set $A = \{1,2,3\}$ to the set $B = \{u,v,w\}$.

(i) Find $\bar{S}$, $\bar{S} \cap T$, $T - \bar{S}$
(ii) Find $T^{-1} o\, S$
(iii) Find $S^{-1} \circ T$

10. Suppose R is a relation defined on the integers set $\mathbb{Z}^+ = \{1,2,3,\ldots\}$

$m, n \in \mathbb{Z}^+$, $m\,R\,n \Leftrightarrow m + n = 20$

Determine whether the relation T is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

11. Suppose T is a relation defined on the integers set $\mathbb{Z}$

$m, n \in \mathbb{Z}$, $m\,T\,n \Leftrightarrow m + n \geq 2$

Determine whether the relation T is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

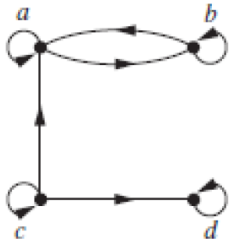
12. Let T be a relation defined on the integers set $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$m, n \in \mathbb{Z}$, $m\,T\,n \Leftrightarrow m + n$ *is odd*

Determine whether the relation T is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

13. Determine whether the relation for the directed graphs shown in the Figure is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

Solution:



14. Let T be a relation defined on $\mathbb{Z}$ such that:

$$a, b \in \mathbb{Z}, \quad a\,T\,b \Leftrightarrow |a| = |b|$$

(i) Show that T is an equivalence relation .