<u>DRAFT</u>

KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

Mid-term Exam / Math-244 (Linear Algebra) / Semester 441

Max. Marks: 30 **Max.Time: 2 hrs**

Question 1: [Marks: 3+4+3]**:**

a) **Let** $\boldsymbol{X} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & -\mathbf{1} \\ \mathbf{1} & \mathbf{0} & -\mathbf{2} \\ \mathbf{1} & \mathbf{1} & -\mathbf{3} \end{bmatrix}$ and $Y = \begin{bmatrix} \mathbf{2} & -\mathbf{1} & \mathbf{0} \\ \mathbf{1} & -\mathbf{2} & \mathbf{1} \\ \mathbf{1} & -\mathbf{1} & \mathbf{0} \end{bmatrix}$. **Show** that *RREF* of the matrix $\boldsymbol{X}$ is same as the *RREF* of $\boldsymbol{Y}$.

b) **Let** the real matrix $\begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \boldsymbol{a} & \boldsymbol{b} & \boldsymbol{c} \\ \boldsymbol{a}^2 & \boldsymbol{b}^2 & c^2 \end{bmatrix}$ be non-invertible. **Show** that $a = b$ or $b = c$ or $b = c$.

c) **Let** $\boldsymbol{A}$ be a matrix with $|\boldsymbol{A}| = 1$ and $adj(\boldsymbol{A}) = \begin{bmatrix} \mathbf{1} & \mathbf{0} & -\mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$. Find $\boldsymbol{A}$.

Question 2: [Marks: 3+4+3]**:**

a) **Explain**! **Why** the following system of linear equations has no non-trivial solution?

$$\begin{aligned} x - \ y + \ z &= 0 \\ x + \ y + \ z &= 0 \\ 4x + 2y + z &= 0. \end{aligned}$$

b) **Find** the values of λ for which the following system of equations has a unique solution:

$$\begin{aligned} 2x_1 + 3x_2 + \quad\quad\quad\quad x_3 &= -1 \\ x_1 + 2x_2 + \quad\quad\quad\quad x_3 &= 0 \\ 3x_1 + \ x_2 + (\lambda^2 - 6)\, x_3 &= \lambda - 3. \end{aligned}$$

c) **Solve** the following system of linear equations by using the Cramer's rule:

$$\begin{aligned} 4x + 2\,y + \ z &= 3 \\ x + \ y + \ z &= 2 \\ x - \ y + \ z &= 1. \end{aligned}$$

Question 3: [Marks: 3+3+4]

a) **Show** that $\boldsymbol{F} := \{\boldsymbol{A} \in \boldsymbol{M}_n(\mathbb{R}) : \boldsymbol{A}^T = \boldsymbol{A}\}$ is a subspace of the vector space $\boldsymbol{M}_n(\mathbb{R})$.

b) **Show** that the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$ are linearly independent in $\boldsymbol{M}_2(\mathbb{R})$

c) **Let** $\boldsymbol{W}$ be subspace generated by $\{(1, 0, 1, 1), (1, -1, 2, 0), (5, 1, 4, 0)\}$ in the Euclidean space $\mathbb{R}^4$. Find a basis and dimension of $\boldsymbol{W}$.

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