Chapter 6

Circular Motion

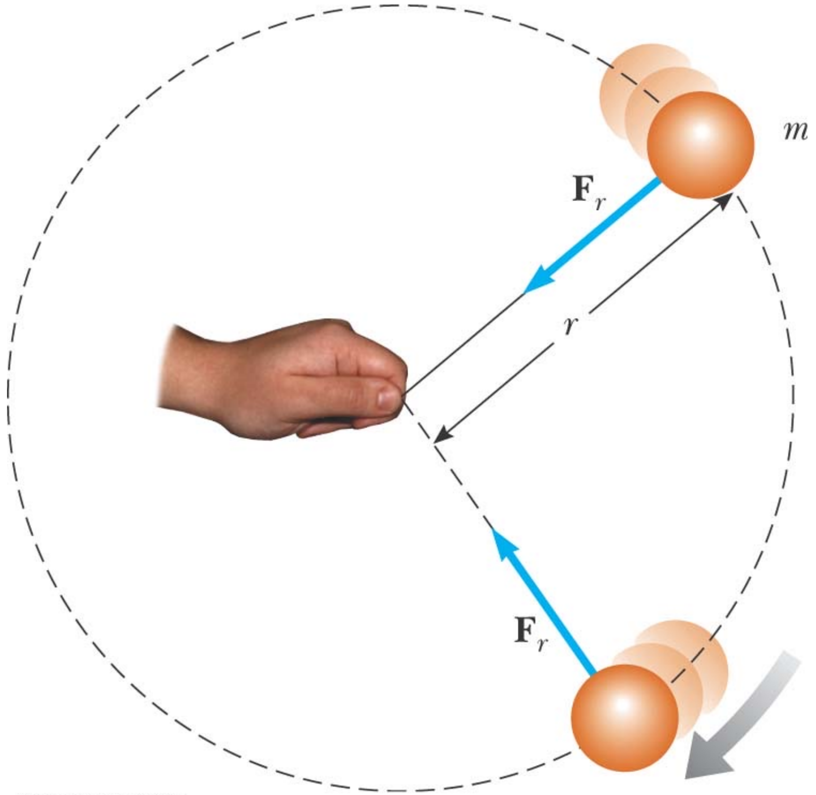
and

Other Applications of Newton's Laws

Uniform Circular Motion

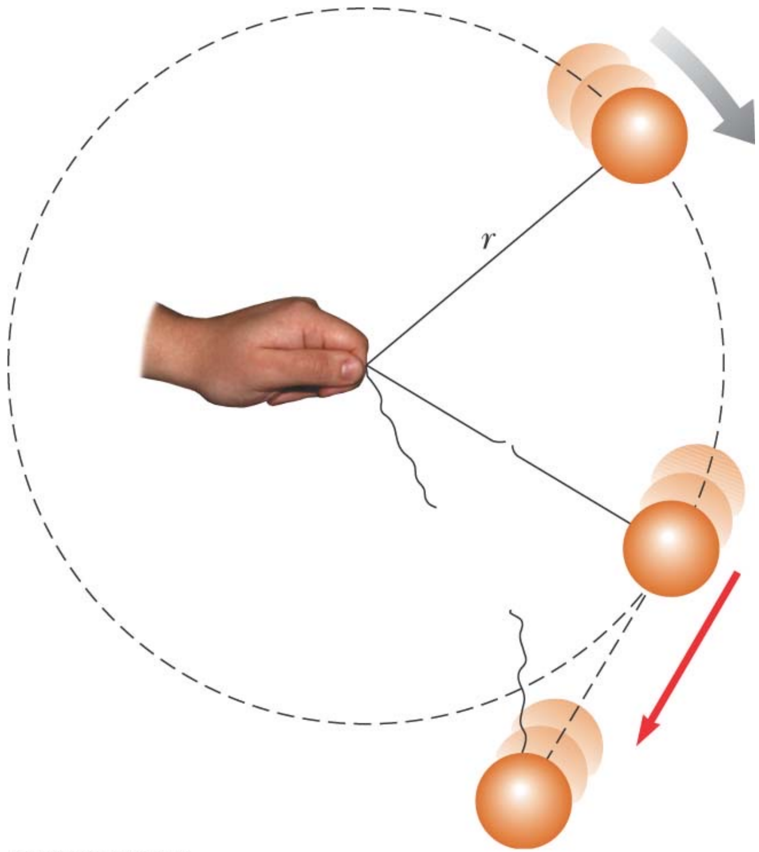
- A force, $\mathbf{F}_r$, is directed toward the center of the circle
- This force is associated with an acceleration, $\mathbf{a}_c$
- Applying Newton's Second Law along the radial direction gives

$$\sum F = ma_c = m\frac{v^2}{r}$$



Uniform Circular Motion, cont

- A force causing a centripetal acceleration acts toward the center of the circle
- It causes a change in the direction of the velocity vector
- If the force vanishes, the object would move in a straight-line path tangent to the circle



Centripetal Force

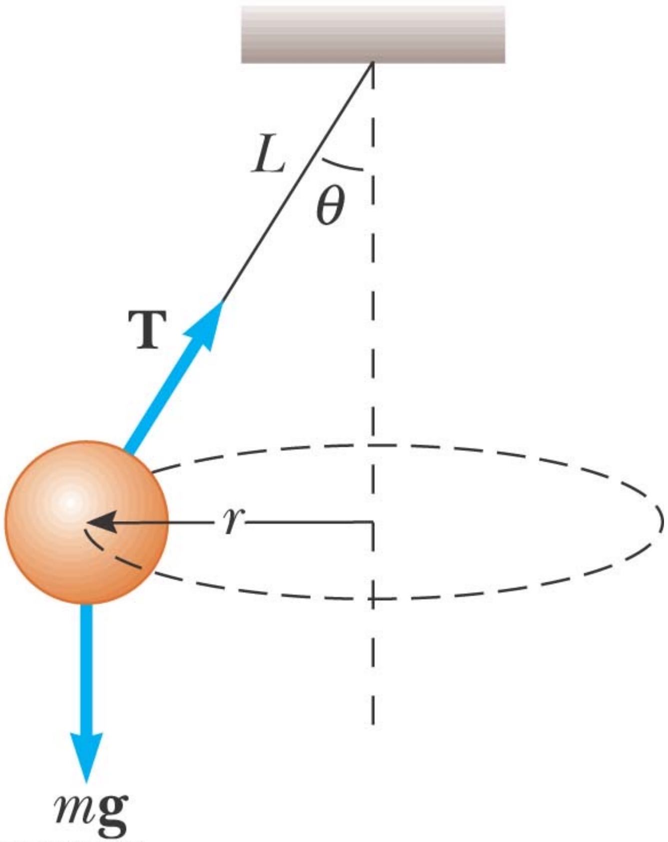
- The force causing the centripetal acceleration is sometimes called the ***centripetal force***
- This is not a new force, it is a new *role* for a force
- It is a force *acting in the role of a force that causes a circular motion*

Conical Pendulum

- The object is in equilibrium in the vertical direction and undergoes uniform circular motion in the horizontal direction

$$v = \sqrt{Lg\sin\theta\tan\theta}$$

- *v* is independent of *m*



Motion in a Horizontal Circle

- The speed at which the object moves depends on the mass of the object and the tension in the cord
- The centripetal force is supplied by the tension

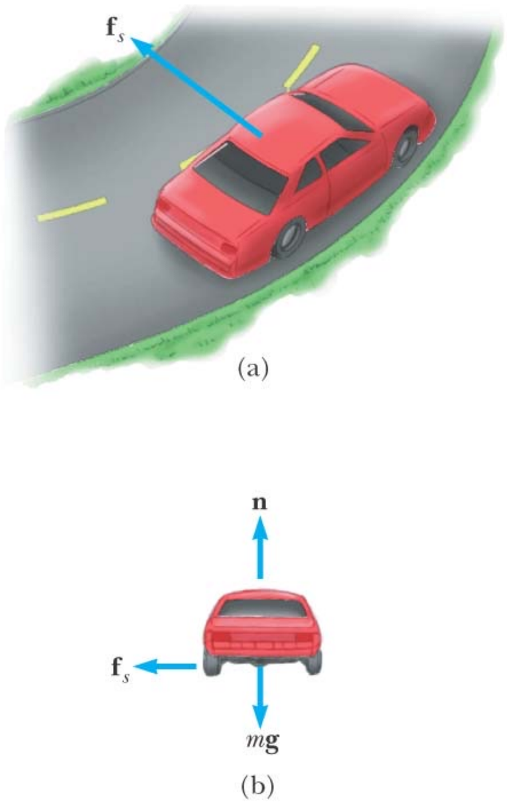
$$v = \sqrt{\frac{Tr}{m}}$$

Horizontal (Flat) Curve

- The force of static friction supplies the centripetal force
- The maximum speed at which the car can negotiate the curve is

$$v = \sqrt{\mu g r}$$

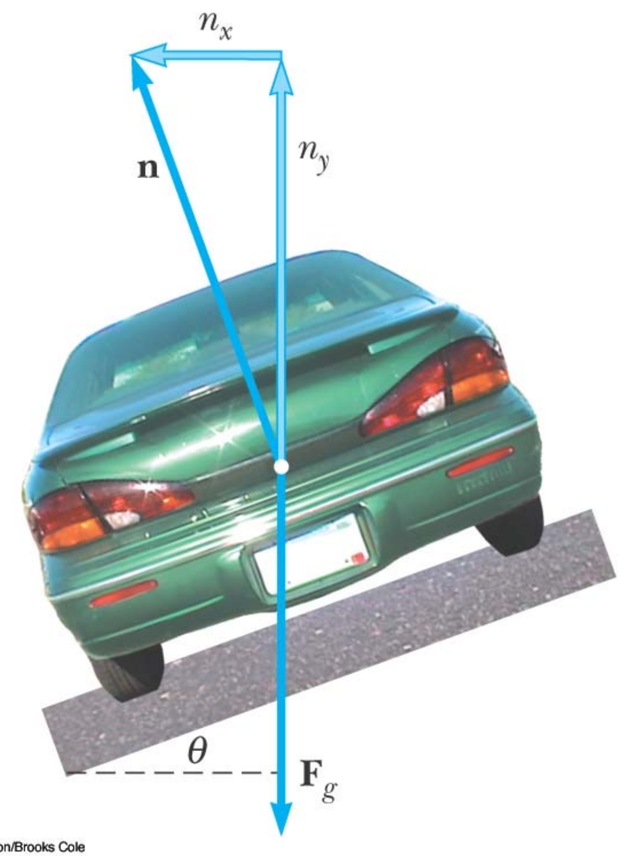
- Note, this does not depend on the mass of the car



Banked Curve

- These are designed with friction equaling zero
- There is a component of the normal force that supplies the centripetal force

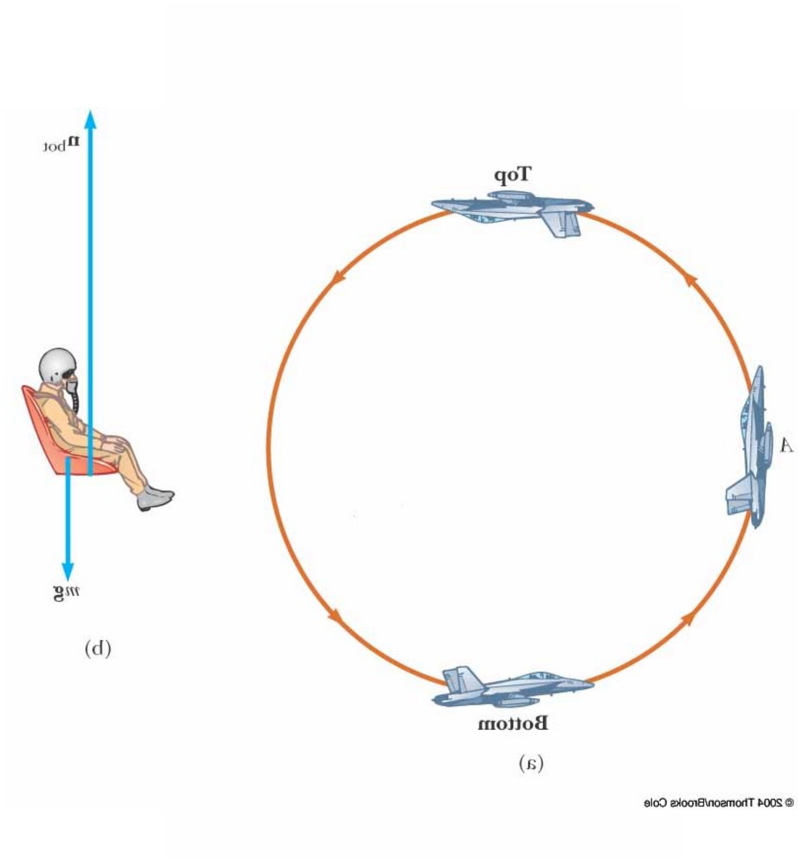
$$\tan\theta = \frac{v^2}{rg}$$



Loop-the-Loop

- This is an example of a vertical circle
- At the bottom of the loop (b), the upward force experienced by the object is greater than its weight

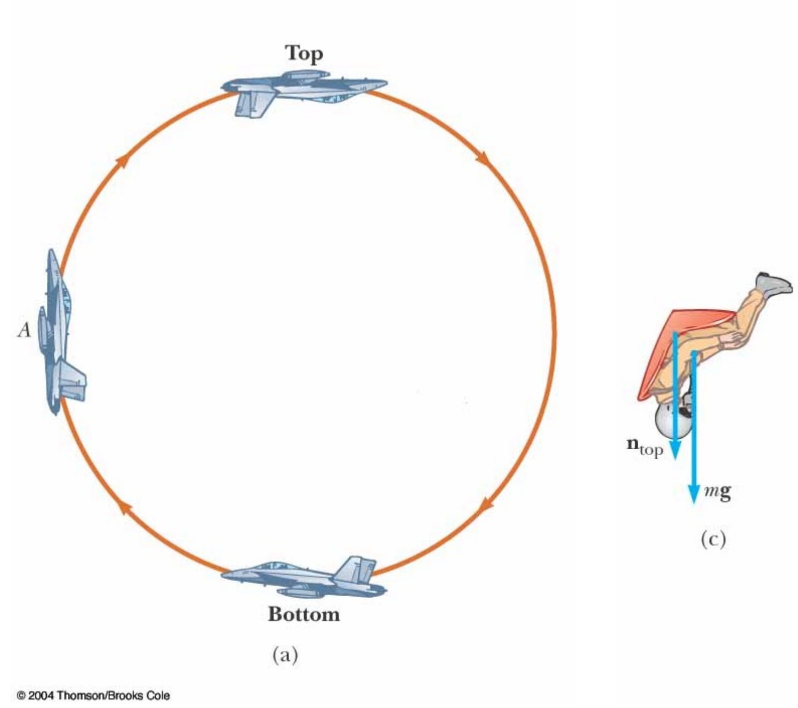
$$n_{bot} = mg\left(1 + \frac{v^2}{rg}\right)$$



Loop-the-Loop, Part 2

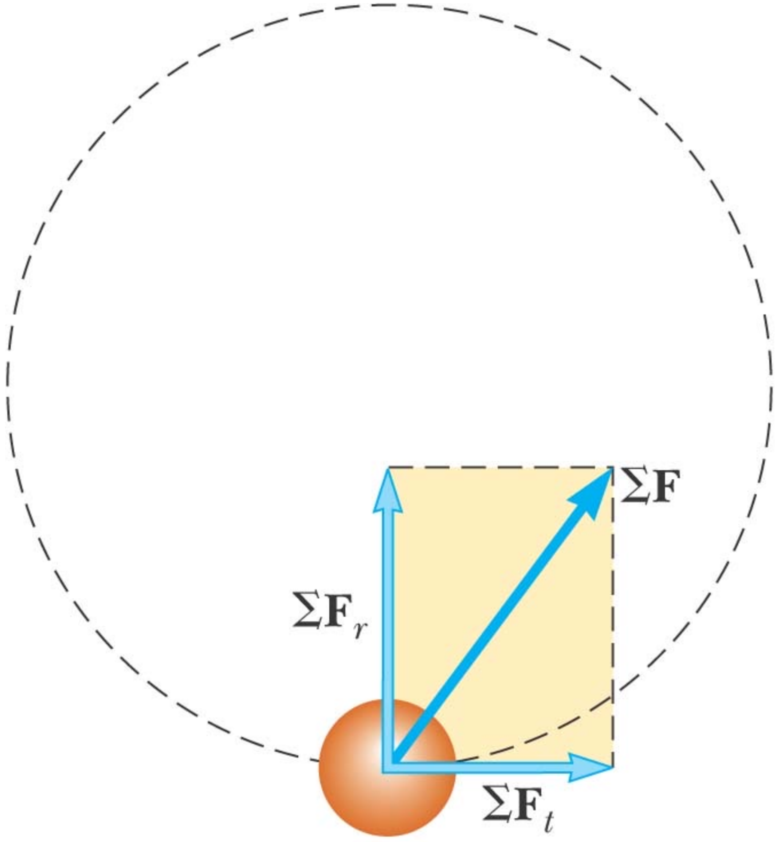
- At the top of the circle (c), the force exerted on the object is less than its weight

$$n_{top} = mg\left(\frac{v^2}{rg} - 1\right)$$



Non-Uniform Circular Motion

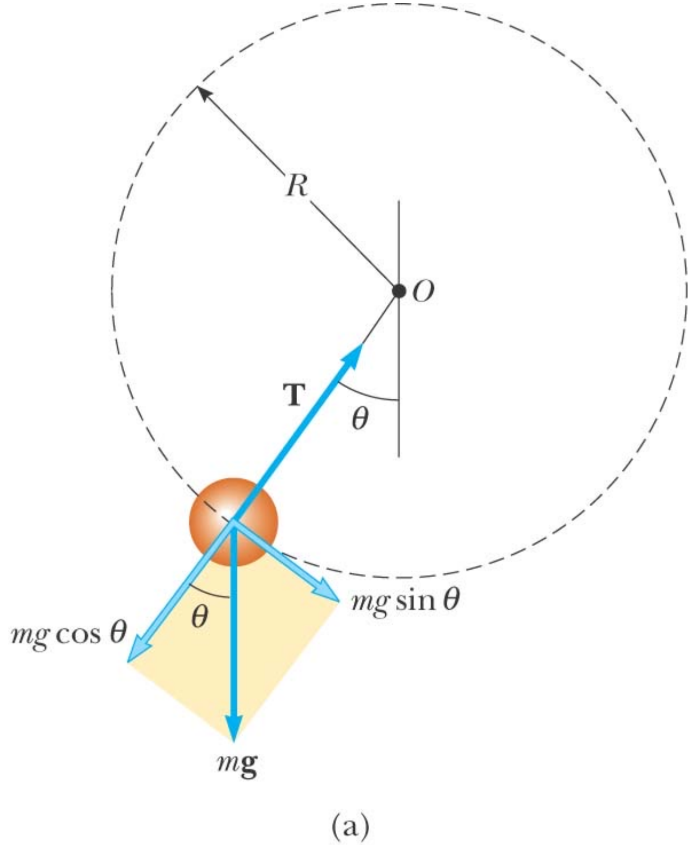
- The acceleration and force have tangential components
- $\mathbf{F}_r$ produces the centripetal acceleration
- $\mathbf{F}_t$ produces the tangential acceleration
- $\Sigma\mathbf{F} = \Sigma\mathbf{F}_r + \Sigma\mathbf{F}_t$



Vertical Circle with Non-Uniform Speed

- The gravitational force exerts a tangential force on the object
  - Look at the components of F_g
- The tension at any point can be found

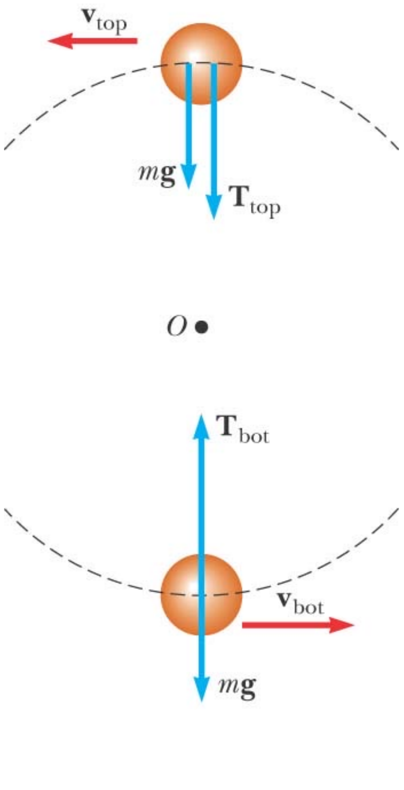
$$T = m\left(\frac{v^2}{R} + g\cos\theta\right)$$



Top and Bottom of Circle

- The tension at the bottom is a maximum
- The tension at the top is a minimum
- If $T_{top} = 0$, then

$$v_{top} = \sqrt{gR}$$



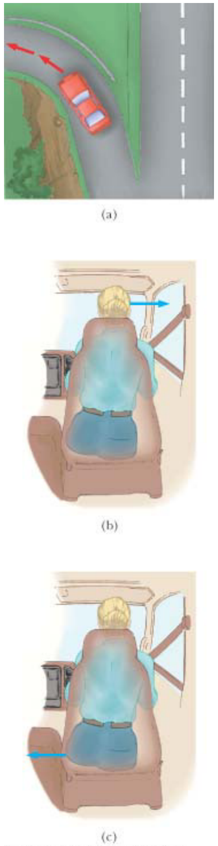
(b)

Motion in Accelerated Frames

- A ***fictitious force*** results from an accelerated frame of reference
  - A fictitious force appears to act on an object in the same way as a real force, but you cannot identify a second object for the fictitious force

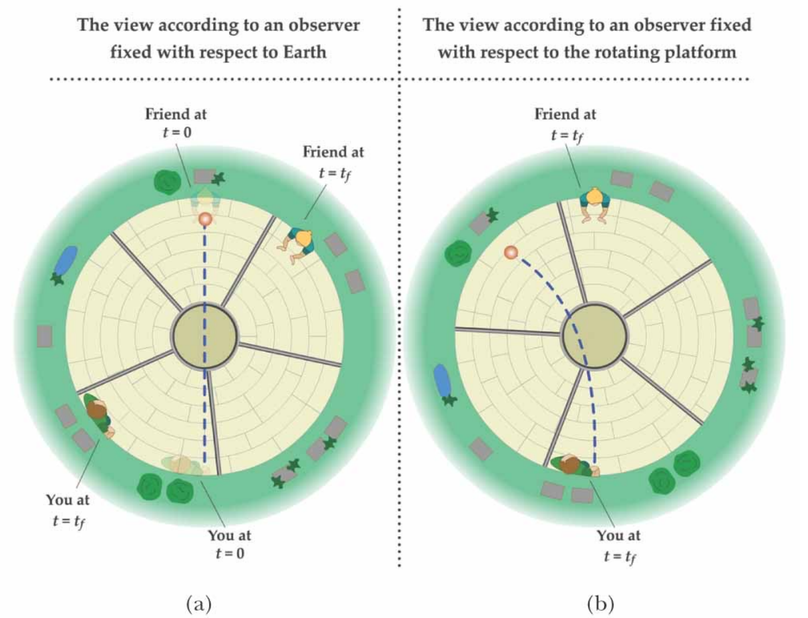
"Centrifugal" Force

- From the frame of the passenger (b), a force appears to push her toward the door
- From the frame of the Earth, the car applies a leftward force on the passenger
- The outward force is often called a ***centrifugal*** force
  - It is a fictitious force due to the acceleration associated with the car's change in direction



“Coriolis Force”

- This is an apparent force caused by changing the radial position of an object in a rotating coordinate system
- The result of the rotation is the curved path of the ball



Fictitious Forces, examples

- Although fictitious forces are not real forces, they can have real effects
- Examples:
  - Objects in the car do slide
  - You feel pushed to the outside of a rotating platform
  - The Coriolis force is responsible for the rotation of weather systems and ocean currents

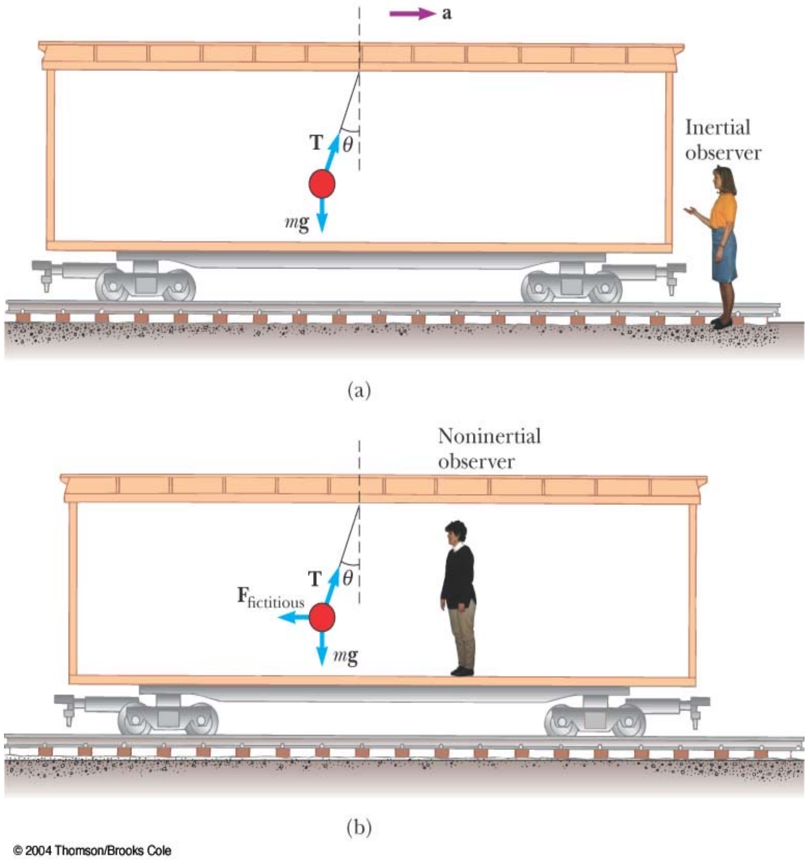
Fictitious Forces in Linear Systems

- The inertial observer (a) sees

$$\sum F_x = T\sin\theta = ma$$
$$\sum F_y = T\cos\theta - mg = 0$$

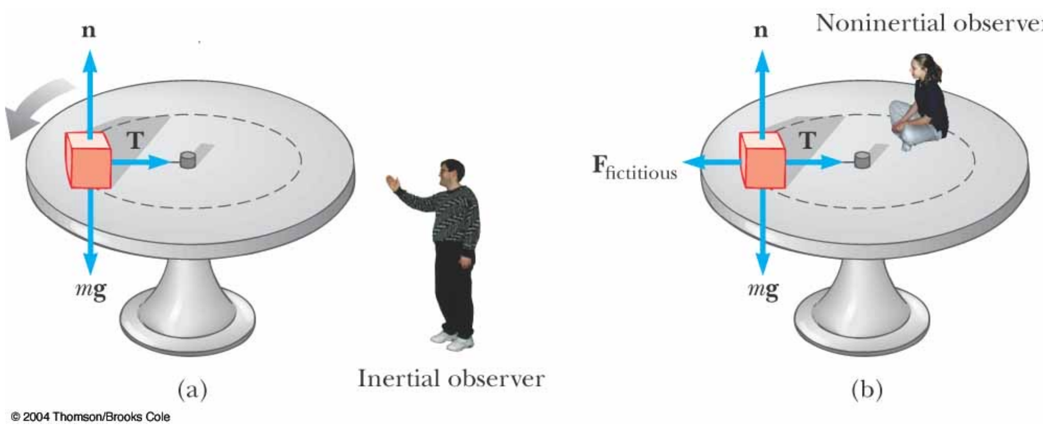
- The noninertial observer (b) sees

$$\sum F'_x = T\sin\theta - F_{fictitious} = ma$$
$$\sum F'_y = T\cos\theta - mg = 0$$



(a) (b)

Fictitious Forces in a Rotating System



- According to the inertial observer (a), the tension is the centripetal force

$$T = \frac{mv^2}{r}$$

- The noninertial observer (b) sees

$$T - F_{fictitious} = T - \frac{mv^2}{r} = 0$$

Motion with Resistive Forces

- Motion can be through a medium
  - Either a liquid or a gas
- The medium exerts a *resistive force*, **R**, on an object moving through the medium
- The magnitude of **R** depends on the medium
- The direction of **R** is opposite the direction of motion of the object relative to the medium
- **R** nearly always increases with increasing speed

Motion with Resistive Forces, cont

- The magnitude of **R** can depend on the speed in complex ways
- We will discuss only two
  - **R** is proportional to **v**
    - Good approximation for slow motions or small objects
  - **R** is proportional to $\mathbf{v}^2$
    - Good approximation for large objects

R Proportional To **v**

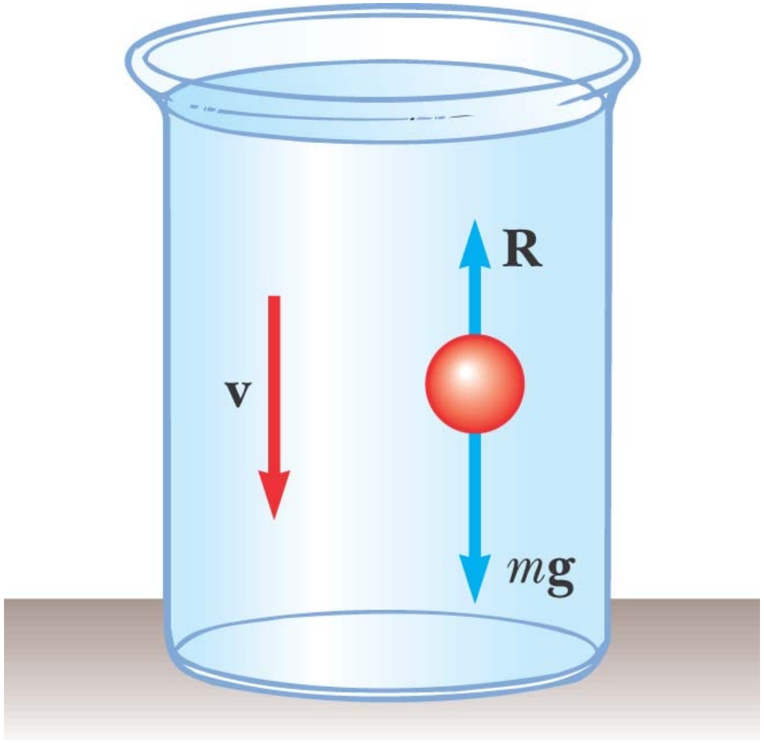
- The resistive force can be expressed as **R** = - *b* **v**
- *b* depends on the property of the medium, and on the shape and dimensions of the object
- The negative sign indicates **R** is in the opposite direction to **v**

R Proportional To **v**, Example

- Analyzing the motion results in

$$mg - bv = ma = m\frac{dv}{dt}$$

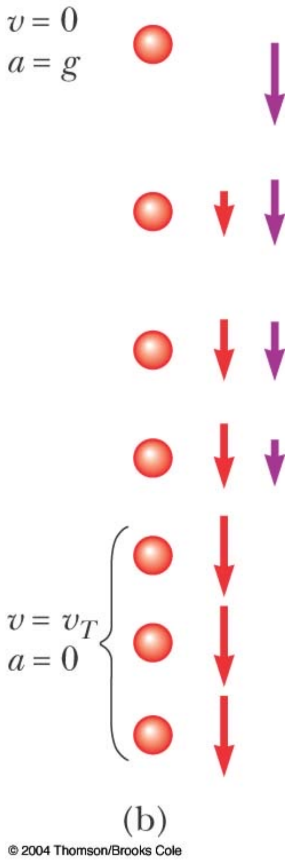
$$a = \frac{dv}{dt} = g - \frac{b}{m}v$$



(a)

R Proportional To **v**, Example, cont

- Initially, $v = 0$ and $dv/dt = g$
- As t increases, R increases and a decreases
- The acceleration approaches 0 when $R \rightarrow mg$
- At this point, v approaches the ***terminal speed*** of the object



Terminal Speed

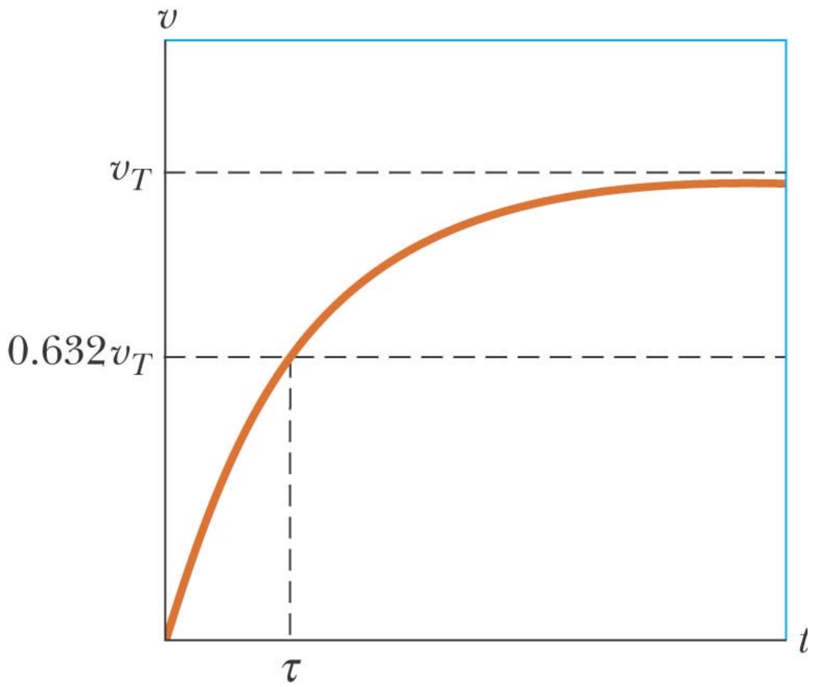
- To find the terminal speed, let *a* = 0

$$v_T = \frac{mg}{b}$$

- Solving the differential equation gives

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_T\left(1 - e^{-t/\tau}\right)$$

- τ is the ***time constant*** and $\tau = m/b$



(c)

R Proportional To $\mathbf{v}^2$

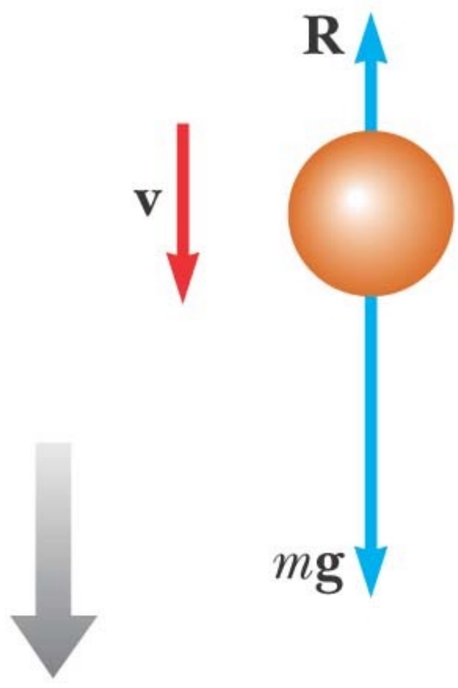
- For objects moving at high speeds through air, the resistive force is approximately equal to the square of the speed
- $R = \frac{1}{2} D\rho A v^2$
  - D is a dimensionless empirical quantity that called the drag coefficient
  - ρ is the density of air
  - A is the cross-sectional area of the object
  - v is the speed of the object

R Proportional To $\mathbf{v}^2$, example

- Analysis of an object falling through air accounting for air resistance

$$\sum F = mg - \frac{1}{2} D \rho A v^2 = ma$$

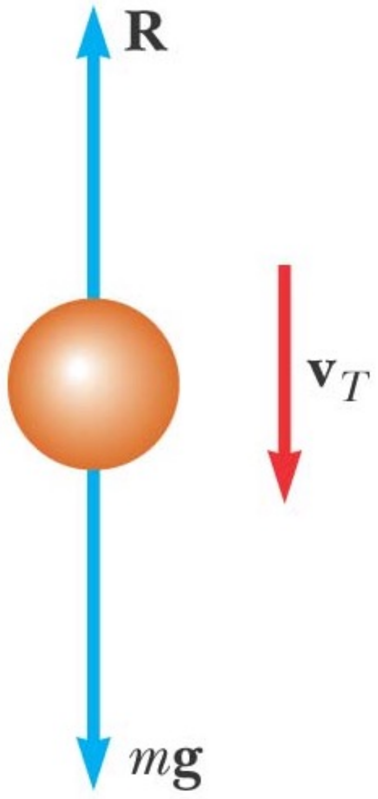
$$a = g - \left(\frac{D \rho A}{2m} \right) v^2$$



R Proportional To **v**², Terminal Speed

- The terminal speed will occur when the acceleration goes to zero
- Solving the equation gives

$$v_T = \sqrt{\frac{2mg}{D\rho A}}$$



Some Terminal Speeds

Table 6.1

Terminal Speed for Various Objects Falling Through Air

Object	Mass (kg)	Cross-Sectional Area (m^2)	v_T (m/s)
Sky diver	75	0.70	60
Baseball (radius 3.7 cm)	0.145	4.2×10^{-3}	43
Golf ball (radius 2.1 cm)	0.046	1.4×10^{-3}	44
Hailstone (radius 0.50 cm)	4.8×10^{-4}	7.9×10^{-5}	14
Raindrop (radius 0.20 cm)	3.4×10^{-5}	1.3×10^{-5}	9.0

Process for Problem-Solving

- Analytical Method
  - The process used so far
  - Involves the identification of well-behaved functional expressions generated from algebraic manipulation or techniques of calculus

Analytical Method

- Apply the method using this procedure:
  - Sum all the forces acting on the particle to find the net force, ΣF
  - Use this net force to determine the acceleration from the relationship $a = \Sigma F / m$
  - Use this acceleration to determine the velocity from the relationship $dv/dt = a$
  - Use this velocity to determine the position from the relationship $dx/dt = v$

Analytic Method, Example

- Applying the procedure:
  - $F_g = ma_y = -mg$
  - $a_y = -g$ and $dv_y / dt = -g$
  - $v_y(t) = v_{yi} - gt$
  - $y(t) = y_i + v_{yi}\, t - \frac{1}{2}\, gt^2$

Numerical Modeling

- In many cases, the analytic method is not sufficient for solving "real" problems
- Numerical modeling can be used in place of the analytic method for these more complicated situations
- The *Euler method* is one of the simplest numerical modeling techniques

Euler Method

- In the Euler Method, derivatives are approximated as ratios of finite differences
- Δt is assumed to be very small, such that the change in acceleration during the time interval is also very small

Equations for the Euler Method

$$a\left(t\right) \approx \frac{\Delta v}{\Delta t} = \frac{v(t+\Delta t) - v(t)}{\Delta t}$$

$$v(t+\Delta t) \approx v(t) + a(t)\Delta t$$

and

$$v(t) \approx \frac{\Delta x}{\Delta t} \approx \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$x\left(t+\Delta t\right) \approx x(t) + v(t)\Delta t$$

Euler Method Continued

- It is convenient to set up the numerical solution to this kind of problem by numbering the steps and entering the calculations into a table
- Many small increments can be taken, and accurate results can be obtained by a computer

Euler Method Set Up

Table 6.3

The Euler Method for Solving Dynamics Problems

Step	Time	Position	Velocity	Acceleration
0	t_0	x_0	v_0	$a_0 = F(x_0, v_0, t_0)/m$
1	$t_1 = t_0 + \Delta t$	$x_1 = x_0 + v_0 \Delta t$	$v_1 = v_0 + a_0 \Delta t$	$a_1 = F(x_1, v_1, t_1)/m$
2	$t_2 = t_1 + \Delta t$	$x_2 = x_1 + v_1 \Delta t$	$v_2 = v_1 + a_1 \Delta t$	$a_2 = F(x_2, v_2, t_2)/m$
3	$t_3 = t_2 + \Delta t$	$x_3 = x_2 + v_2 \Delta t$	$v_3 = v_2 + a_2 \Delta t$	$a_3 = F(x_3, v_3, t_3)/m$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	t_n	x_n	v_n	a_n

Euler Method Final

- One advantage of the method is that the dynamics are not obscured
  - The relationships among acceleration, force, velocity and position are clearly shown
- The time interval must be small
  - The method is completely reliable for infinitesimally small time increments
  - For practical reasons a finite increment must be chosen
  - A time increment can be chosen based on the initial conditions and used throughout the problem
    - In certain cases, the time increment may need to be changed within the problem

Accuracy of the Euler Method

- The size of the time increment influences the accuracy of the results
- It is difficult to determine the accuracy of the result without knowing the analytical solution
- One method of determining the accuracy of the numerical solution is to repeat the solution with a smaller time increment and compare the results
  - If the results agree, the results are correct to the precision of the number of significant figures of agreement

Euler Method, Numerical Example

Table 6.4

The Sphere Begins to Fall in Oil

Step	Time (ms)	Position (cm)	Velocity (cm/s)	Acceleration (cm/s^2)
0	0.0	0.0000	0.0	−980.0
1	0.1	0.0000	−0.10	−960.8
2	0.2	0.0000	−0.19	−942.0
3	0.3	0.0000	−0.29	−923.5
4	0.4	−0.0001	−0.38	−905.4
5	0.5	−0.0001	−0.47	−887.7
6	0.6	−0.0001	−0.56	−870.3
7	0.7	−0.0002	−0.65	−853.2
8	0.8	−0.0003	−0.73	−836.5
9	0.9	−0.0003	−0.82	−820.1
10	1.0	−0.0004	−0.90	−804.0

Euler Method, Numerical Example cont.

Table 6.5

The Sphere Reaches 0.900 v_T

Step	Time (ms)	Position (cm)	Velocity (cm/s)	Acceleration (cm/s^2)
110	11.0	−0.0324	−4.43	−111.1
111	11.1	−0.0328	−4.44	−108.9
112	11.2	−0.0333	−4.46	−106.8
113	11.3	−0.0337	−4.47	−104.7
114	11.4	−0.0342	−4.48	−102.6
115	11.5	−0.0346	−4.49	−100.6
116	11.6	−0.0351	−4.50	−98.6
117	11.7	−0.0355	−4.51	−96.7
118	11.8	−0.0360	−4.52	−94.8
119	11.9	−0.0364	−4.53	−92.9
120	12.0	−0.0369	−4.54	−91.1