Phys 103
Chapter 9

Linear Momentum and Collisions

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LECTURE OUTLINE

- 9.1 Linear Momentum and Its Conservation
- 9.2 Impulse and Momentum
- 9.3 Collisions in One Dimension
- 9.4 Two-Dimensional Collisions

Introduction

Consider what happens when a bowling ball strikes a pin, as in the opening photograph. The pin is given a large velocity as a result of the collision; consequently, it flies away and hits other pins or is projected toward the backstop. Because the average force exerted on the pin during the collision is large, the pin achieves the large velocity very rapidly and experiences the force for a very short time interval. According to Newton’s third law, the pin exerts a reaction force on the ball that is equal in magnitude and opposite in direction to the force exerted by the ball on the pin.

This reaction force causes the ball to accelerate, but because the ball is so much more massive than the pin, the ball’s acceleration is much less than the pin’s acceleration.

Introduction

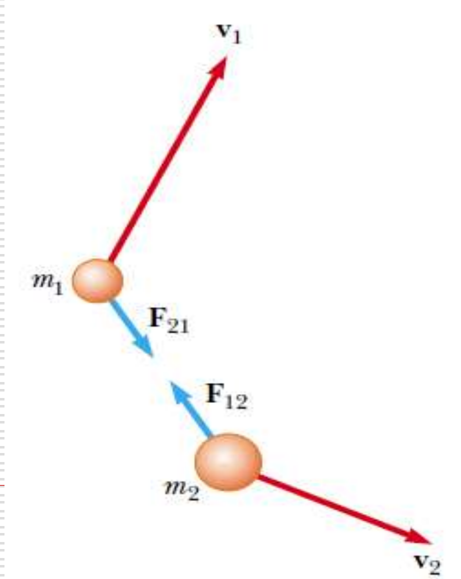
- Momentum Analysis Models Force and acceleration are related by Newton's second law. When force and acceleration vary by time, the situation can be very complicated. The techniques developed in this chapter will enable you to understand and analyze these situations in a simple way. Will develop momentum versions of analysis models for isolated and non-isolated systems These models are especially useful for treating problems that involve collisions and for analyzing rocket propulsion.

9.1 Linear Momentum and Its Conservation

Consider two particles m1and m2with v1and v2collide as in figuer:

If a force from particle 1 acts on particle 2, then there must be a second force—equal in magnitude but opposite in direction—that particle 2 exerts on particle 1. That is, they form a Newton's third law action–reaction pair, so that F_{12}=-F_{21}. We can express this condition as:

$$F_{12} + F_{21} = 0$$



Using Newton's 2nd law:

$$m_1 a_1 + m_2 a_2 = 0$$

$$m_1 \frac{dv_1}{dt} + m_2 \frac{dv_2}{dt} = 0$$

9.1 Linear Momentum and Its Conservation

If the masses m1and m2are constant, we can bring them into the derivatives, which gives:

$$\frac{d(m_1v_1)}{dt} + \frac{d(m_2v_2)}{dt} = 0 \text{ so } \frac{d(m_1v_1 + m_2v_2)}{dt} = 0$$

- To finalize this discussion, note that the derivative of the sum $(m_1v_1 + m_2v_2)$ with respect to time is zero. Consequently, this sum must be constant. We learn from this discussion that the quantity mv for a particle is important, in that the sum of these quantities for an isolated system is conserved. We call this quantity linear momentum linear momentum of a particle or an object is defind as:

p= m**v**

Linear momentum is a vector quantity.

Its direction is the same as the direction of the velocity.

The dimensions of momentum are ML/T. The SI units of momentum are kg · m / s.

9.1 Linear Momentum and Its Conservation

If a particle is moving in 3-D then: $\mathbf{p_x} = m\mathbf{v_x}$

- Using Newton's second law of motion, we can relate the linear momentum of a particle to the resultant force acting on the particle:

$$\sum F_x = ma = m\frac{dv}{dt}$$

- In Newton's second law, the mass m is assumed to be constant. Thus, we can bring m inside the derivative notation to give us:

$$\sum F_x = ma = m\frac{dv}{dt} = \frac{d(mv)}{dt} = \frac{dp}{dt}$$

- *This shows that the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.*

- **This is the form in which Newton presented the Second Law.**
- **It is a more general form than the one we used previously.**
- **This form also allows for mass changes.**

9.1 Linear Momentum and Its Conservation

- Using the definition of momentum, $\frac{d(m_1v_1+m_2v_2)}{dt} = 0$ can be written:

$$\frac{d(p_1+p_2)}{dt} = 0$$

$$\frac{d(p_1+p_2)}{dt} = \frac{dp_{tot}}{dt} = 0$$

$$p_{tot} = p_1 + p_2 = constant$$

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

Conservation of Linear Momentum

This is the mathematical statement of a new analysis model, the isolated system (momentum).

- *Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.*
- This law tells us that the total momentum of an isolated system at all times equals its initial momentum.

9.1 Linear Momentum and Its Conservation

Momentum and Kinetic Energy

Momentum and kinetic energy both involve mass and velocity. There are major differences between them:

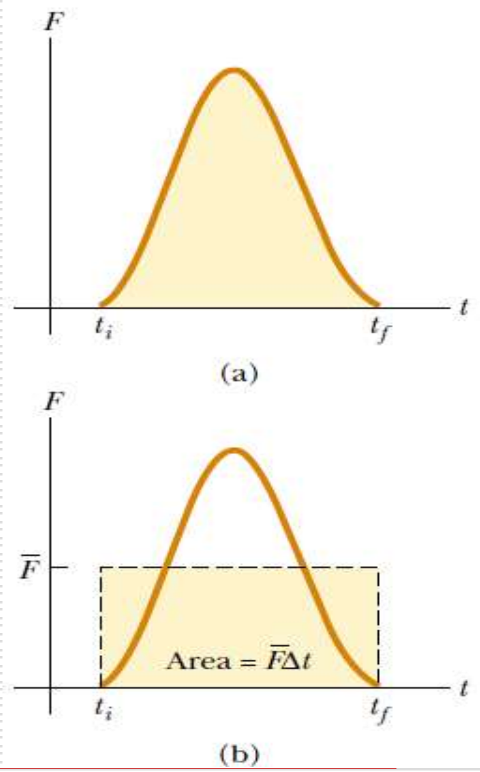
- Kinetic energy is a scalar and momentum is a vector.
- Kinetic energy can be transformed to other types of energy.

There is only one type of linear momentum, so there are no similar transformations.

Analysis models based on momentum are separate from those based on energy. This difference allows an independent tool to use in solving problems.

9.2 Impulse and Momentum

- To build a better, let us assume that a single force **F** acts on a particle and that this force may vary with time. According to Newton's second law:



$$F = \frac{dp}{dt}, \qquad dp = Fdt$$

Inegrating for time t_i to t_f:

$$\Delta p = p_f - p_i = \int_{t_i}^{t_f} Fdt$$

Or

$$I = \int_{t_i}^{t_f} Fdt$$

The integral is called the impulse, I, of the force acting on an object over Δ t.

9.2 Impulse and Momentum

Forces and Conservation of Momentum

- In conservation of momentum, there is no statement concerning the types of forces acting on the particles of the system. The forces are not specified as conservative or non-conservative. There is no indication if the forces are constant or not. The only requirement is that the forces must be internal to the system.
- This gives a hint about the power of this new model.

9.2 Impulse and Momentum

- The quantity in ($I = \int_{t_i}^{t_f} F dt$) is called: Impulse. ($\Delta p = p_f - p_i = \int_{t_i}^{t_f} F dt$) is called: **Impulse-Momentum Theorm**.
- The impulse of the force **F** acting on a particle equals the change in the momentum of the particle.
- Because the force imparting an impulse can generally vary in time, it is convenient to define a time-averaged force:

$$\overline{F} = \frac{1}{\Delta t} \int_{t_i}^{t_f} F dt \quad or \; I = \overline{F} \Delta t$$

In principle, if **F** is known as a function of time, the impulse can be calculated from Equation ($I = \int_{t_i}^{t_f} F dt$). The calculation becomes especially simple if the force acting on the particle is constant. In this case: **I= FΔt**

9.2 Impulse and Momentum

Impulse-Momentum Theorem

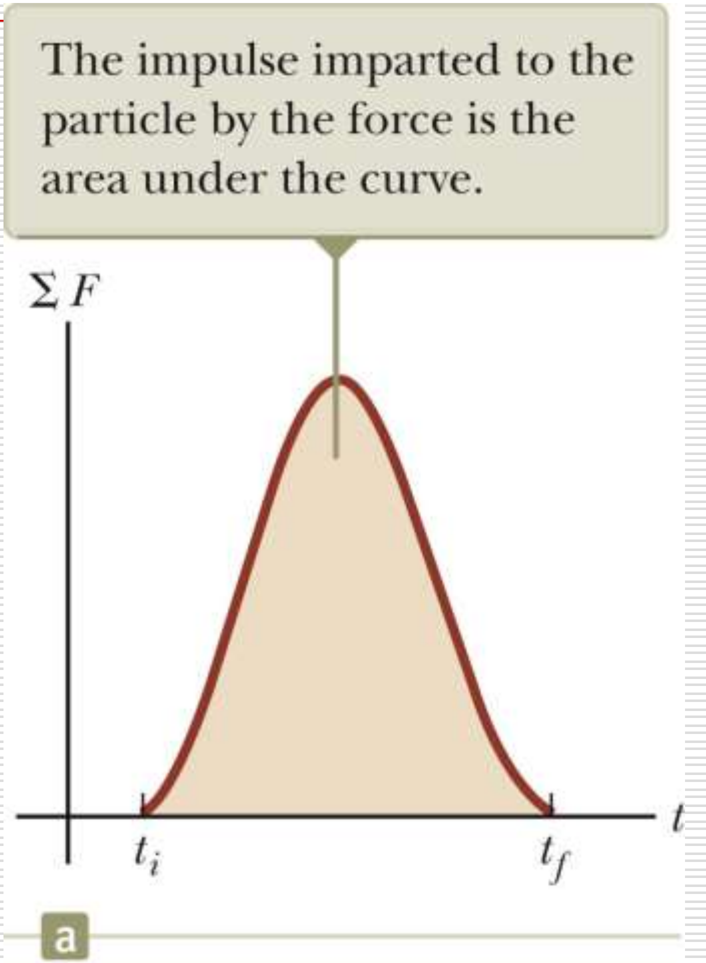
- This equation expresses the **impulse-momentum theorem**: The change in the momentum of a particle is equal to the impulse of the new force acting on the particle.
- This is equivalent to Newton's Second Law.
- This is identical in form to the conservation of energy equation.
- This is the most general statement of the principle of conservation of momentum and is called the conservation of momentum equation.
- This form applies to non-isolated systems.
- This is the mathematical statement of the **non-isolated system (momentum) model.**

Impulse is a vector quantity.

9.2 Impulse and Momentum

More About Impulse

- Impulse is a vector quantity. The magnitude of the impulse is equal to the area under the force-time curve.
- The force may vary with time.
Dimensions of impulse are M L / T
Impulse is not a property of the particle, but a measure of the change in momentum of the particle.



Example 9.1 The Archer

Let us consider the situation proposed at the beginning of this section. A 60-kg archer stands at rest on frictionless ice and fires a 0.50-kg arrow horizontally at 50 m/s (Fig. 9.2). With what velocity does the archer move across the ice after firing the arrow?

$$(m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = 0).$$

$$m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f} = 0$$

$$\mathbf{v}_{1f} = -\frac{m_2}{m_1}\mathbf{v}_{2f} = -\left(\frac{0.50\text{ kg}}{60\text{ kg}}\right)(50\hat{\mathbf{i}}\text{ m/s}) = -0.42\hat{\mathbf{i}}\text{ m/s}$$

Example 9.4 How Good Are the Bumpers?

In a particular crash test, a car of mass 1 500 kg collides with a wall, as shown in Figure 9.6. The initial and final velocities of the car are vi= -15 m/s and vf= 2.6 m/s , respectively. If the collision lasts for 0.150 s, *find the impulse caused by the collision and the average force exerted on the car.*

- ***Solution***:

Conceptualize

- The collision time is short.
- We can image the car being brought to rest very rapidly and then moving back in the opposite direction with a reduced speed.

Categorize

- Assume net force exerted on the car by wall and friction with the ground is large compared with other forces.
- Gravitational and normal forces are perpendicular and so do not effect the horizontal momentum.

Example 9.4 How Good Are the Bumpers?

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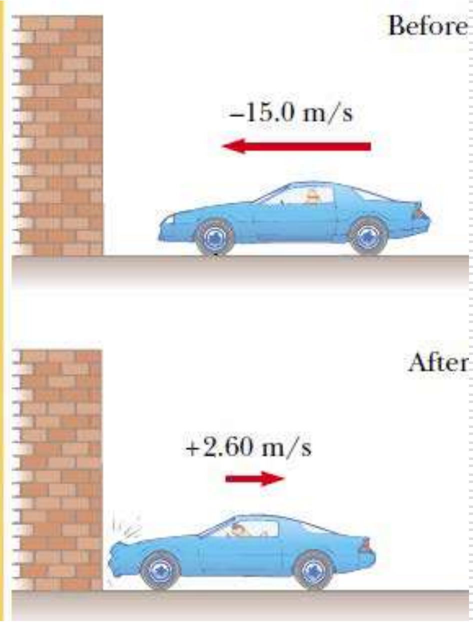


- ***Solution:***

$$\because I = \Delta p = p_f - p_i$$
$$= mv_f - mv_i$$
$$= (1500)(2.6\hat{\imath}) - (1500)(-15\hat{\imath})$$
$$= 2.64 \times 10^4\, \hat{\imath}\; kg.m/s$$
$$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{2.64 \times 10^4}{0.15} = 1.76 \times 10^5$$

9.3 Collisions in One Dimension

The total kinetic energy of the system of particles may or may not be conserved, depending on the type of collision. In fact, whether or not kinetic energy is conserved is used to classify collisions as either ***elastic or inelastic.***

An elastic collision between two objects is one in which the total kinetic energy (as well as total momentum) of the system is the same before and after the collision.

An inelastic collision is one in which the total kinetic energy of the system is not the same before and after the collision (even though the momentum of the system is conserved).

Inelastic collisions are of two types. When the colliding objects stick together after the collision, the collision is called **perfectly inelastic**, When the colliding objects do not stick together, but some kinetic energy is lost, the collision is called **inelastic**.

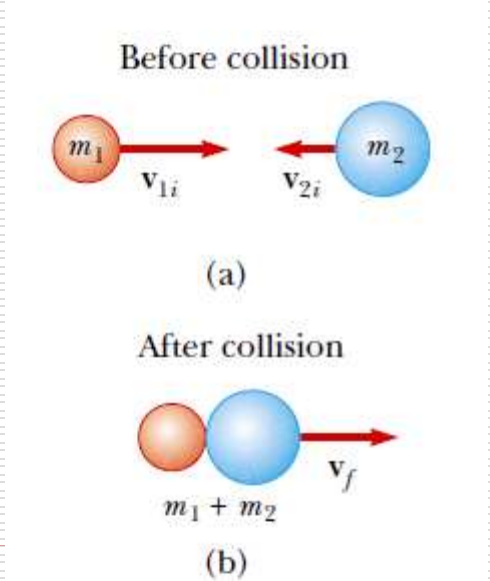
9.3 Collisions in One Dimension

- **Perfectly Inelastic Collisions**

Consider two particles of masses m1 and m2 moving with initial velocities v1i and v2i along the same straight line, as shown in Figure. The two particles collide head-on, stick together, and then move with some common velocity vf after the collision.

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$\Rightarrow v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$



This is true only if the two objects

Stick together in one-object.

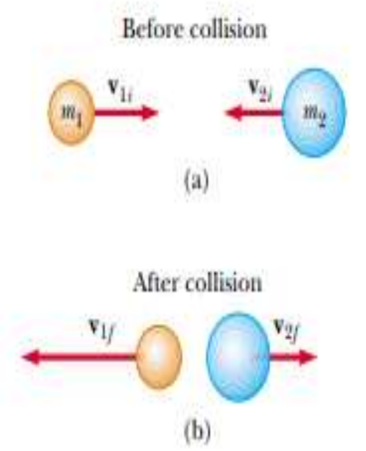
9.3 Collisions in One Dimension

Perfectly Elastic Collisions

For this type of collisions: kinetic energy and liner momentum are conserved:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1\, {v_{1i}}^2 + \frac{1}{2} m_2\, {v_{2i}}^2 = \frac{1}{2} m_1\, {v_{1f}}^2 + \frac{1}{2} m_2\, v_{2f}$$



We can use these equations directly to solve our problems to go directly to some special cases:

$$m_1\, {v_{1i}}^2 + m_2\, {v_{2i}}^2 = m_1\, {v_{1f}}^2 + m_2\, {v_{2f}}^2$$

$$m_1\, {v_{1i}}^2 - m_1\, {v_{1f}}^2 = m_2\, {v_{2f}}^2 - m_2\, {v_{2i}}^2$$

$$m_1 ({v_{1i}}^2 - {v_{1f}}^2) = m_2 ({v_{2f}}^2 - {v_{2i}}^2)$$

9.3 Collisions in One Dimension

Perfectly Elastic Collisions

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \ldots.*$$

$$m_1({v_{1i}}^2 - {v_{1f}}^2) = m_2({v_{2f}}^2 - {v_{2i}}^2)$$

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \ldots.**$$

To obtain our final result, we divide Equation ** by Equation * and obtain:

$$(v_{1i} + v_{1f}) = (v_{2f} + v_{2i})$$

$$(v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$$

9.3 Collisions in One Dimension

Perfectly Elastic Collisions

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \ldots.*$$

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \ldots.**$$

$$(v_{1i} + v_{1f}) = (v_{2f} + v_{2i})$$

$$(v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$$

Suppose that the masses and initial velocities of both particles are known:

$$v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} + \left[\frac{2m_2}{m_1 + m_2}\right] v_{2i}$$

$$v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i} + \left[\frac{m_2 - m_1}{m_1 + m_2}\right] v_{2i}$$

9.3 Collisions in One Dimension

Perfectly Elastic Collisions

$$v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} + \left[\frac{2m_2}{m_1 + m_2}\right] v_{2i}$$

$$v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i} + \left[\frac{m_2 - m_1}{m_1 + m_2}\right] v_{2i}$$

Let us consider some special cases. If $m_1 = m_2$, then tow Equations show us that

$$v_{1f} = v_{2i}$$

And

$$v_{2f} = v_{1i}$$

9.3 Collisions in One Dimension

Perfectly Elastic Collisions

If m_2 is initially at rest $v_{2i} = 0$

So the equations

$$v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} + \left[\frac{2m_2}{m_1 + m_2}\right] v_{2i}$$

$$v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i} + \left[\frac{m_2 - m_1}{m_1 + m_2}\right] v_{2i}$$

becomes:

$$v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} \; and \;\; v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i}$$

9.3 Collisions in One Dimension

Perfectly Elastic Collisions

If m_2 is initially at rest $v_{2i} = 0$

$$v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} \; and \;\; v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i}$$

Now

If m1is much greater than m_2 and $v_{2i} = 0$, we see the tow Equations that $v_{1f} \approx v_{1i}\, and v_{2f} \approx v_{2i}$. That is, when a very heavy particle collides head-on with a very light one that is initially at rest, the heavy particle continues its motion unaltered after the collision and the light particle rebounds with a speed equal to about twice the initial speed of the heavy particle.

9.3 Collisions in One Dimension

Example 9.6 Carry Collision Insurance

- An 1800-kg car stopped at a traffic light is struck from the rear by a 900-kg car, and the two become entangled, moving along the same path as that of the originally moving car. If the smaller car were moving at 20.0 m/s before the collision, *what is the velocity of the entangled cars after the collision?*
- ***Solution:***

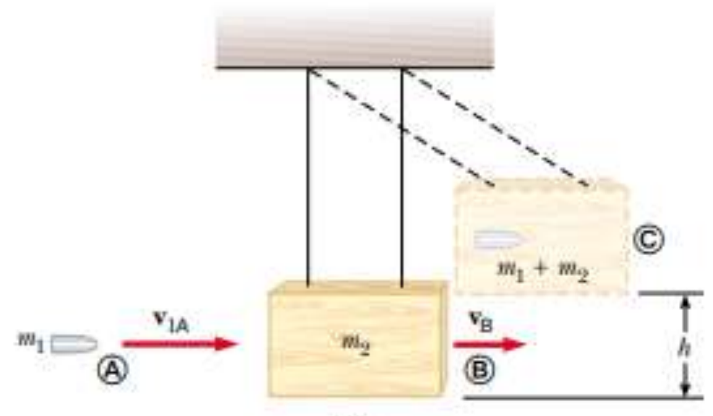
$$\because m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\therefore (1800)(0) + (900)(20) = (1800 + 900) v_f$$

$$\Rightarrow v_f = \frac{900 \times 20}{2700} = 6.67\, m/s$$

Example 9.7 The Ballistic Pendulum

The ballistic pendulum (Fig. 9.11) is an apparatus used to measure the speed of a fast-moving projectile, such as a bullet. A bullet of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The bullet embeds in the block, and the entire system swings through a height h. How can we determine the speed of the bullet from a measurement of h?



(a)

(1) $$v_B = \frac{m_1 v_{1A}}{m_1 + m_2}$$

$$K_B + U_B = K_C + U_C$$

(2) $$K_B = \frac{1}{2}(m_1 + m_2) v_B^2$$

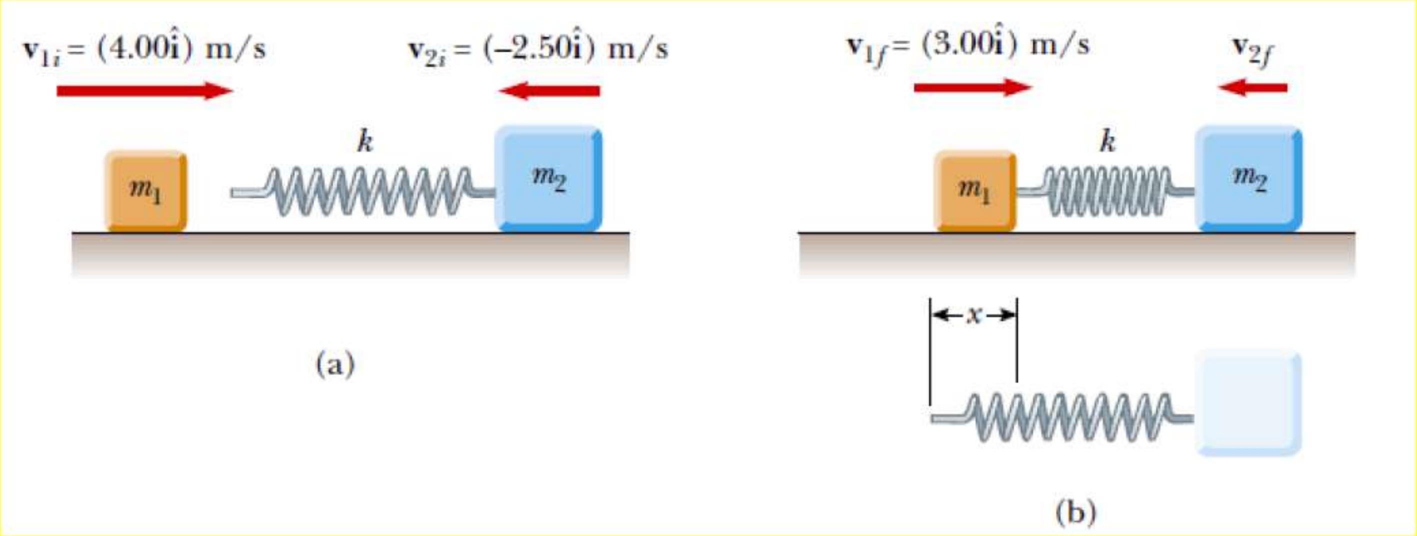
$$\frac{m_1^2 v_{1A}^2}{2(m_1 + m_2)} + 0 = 0 + (m_1 + m_2) g h$$

$$v_{1A} = \left(\frac{m_1 + m_2}{m_1}\right)\sqrt{2gh}$$

9.3 Collisions in One Dimension

Example 9.8 A Two-Body Collision with a Spring

- A block of mass m_1 = 1.60 kg initially moving to the right with a speed of 4.00 m/s on a frictionless horizontal track collides with a spring attached to a second block of mass m_2 =2.10 kg initially moving to the left with a speed of 2.50 m/s. The spring constant is 600 N/m.
- *(A) Find the velocities of the two blocks after the collision*



9.3 Collisions in One Dimension

Example 9.8 (Continued)

- ***Solution:***

$$\because m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$\therefore (1.6)(4)+(2.10)(-2.5)=(1.6)v_{1f} +(2.10)v_{2f} \qquad (1)$$

$$(9.19) \rightarrow v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

$$\therefore (4)-(-2.5) = -v_{1f} + v_{2f}$$

$$\therefore 6.5 = -v_{1f} + v_{2f} \qquad (2)$$

$$(2)\times 1.6 \rightarrow 10.4 = (1.6)(-v_{1f}) + (1.6)(v_{2f}) \qquad (3)$$

$$(1)+(3): 11.55 = 3.7v_{2f}$$

$$\Rightarrow v_{2f} = \frac{11.55}{3.7} = 3.12\, m/s \qquad (4)$$

$$(4)\text{ in }(2): v_{1f} = -3.38\, m/s \qquad (5)$$

9.3 Collisions in One Dimension

Example 9.8 (Continued)

- *(B) During the collision, at the instant block 1 is moving to the right with a velocity of +3.00 m/s, determine the velocity of block 2.*

$$\because m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\therefore (1.6)(4)+(2.10)(-2.5)=(1.6)(3)+(2.10)v_{2f} \Rightarrow v_{2f} = -1.74\, m/s$$

- *(C) Determine the distance the spring is compressed at that instant.*

$$\because K_i + U_i = K_f + U_f$$

$$\therefore \frac{1}{2}m_1 v_{1i}^{\ 2} + \frac{1}{2}m_2 v_{2i}^{\ 2} + 0 = \frac{1}{2}m_1 v_{1f}^{\ 2} + \frac{1}{2}m_2 v_{2f}^{\ 2} + \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{2}(1.6)(4)^2 + \frac{1}{2}(2.1)(-2.5)^2 = \frac{1}{2}(1.6)(3)^2 + \frac{1}{2}(2.1)(-1.74)^2 + \frac{1}{2}(600)x^2$$

$$\therefore x = \sqrt{\frac{8.98 \times 2}{600}} = 0.173\, m$$

9.3 Collisions in One Dimension

PROBLEM-SOLVING HINTS

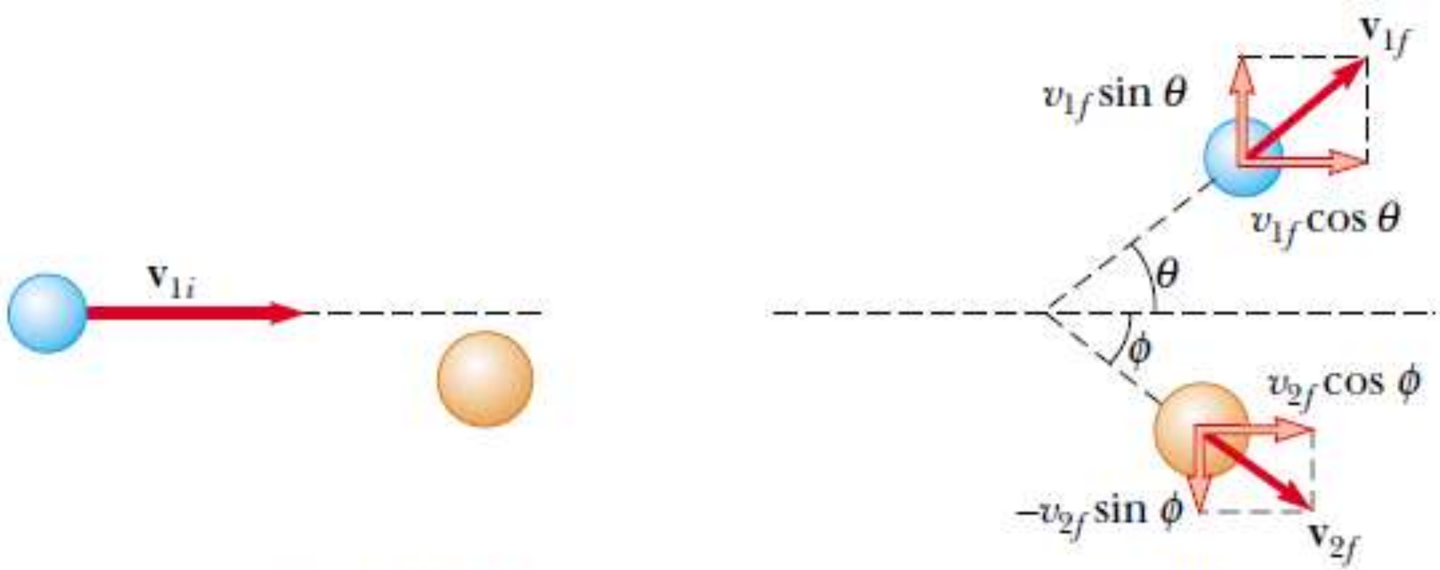
- Set up a coordinate system and define your velocities with respect to that system.
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.
- Write expressions for the *x and y components of the momentum of each* object before and after the collision.
- Write expressions for the total momentum of the system in the *x direction* before and after the collision and equate the two.
- If the collision is inelastic, kinetic energy of the system is *not conserved, and* additional information is probably required.
- If the collision is *perfectly* inelastic, the final velocities of the two objects are equal. Solve the momentum equations for the unknown quantities.
- If the collision is *elastic, kinetic energy of the system is conserved, and you can* equate the total kinetic energy before the collision to the total kinetic energy after the collision to obtain an additional relationship between the velocities.

9.4 Two-Dimensional Collisions

- For two dimensional collisions, we obtain two component equations for conservation of momentum:

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$
$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

consider a 2-D problem in which particle 1 of mass *m_1 collides with particle 2 of mass m_2, where particle 2 is initially at rest, as in Figure*



(a) Before the collision (b) After the collision

9.4 Two-Dimensional Collisions

Applying the law of conservation of momentum in component form and noting that the initial y component of the momentum of the two-particle system is zero, we obtain:

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\varphi$$
$$0 = m_1 v_{1f}\, sin\,\theta - m_2 v_{2f}\, sin\,\varphi$$

- where the minus sign in last Equation comes from the fact that after the collision, particle 2 has a y component of velocity that is downward.
- If the collision is elastic, we can also use Equation 9.16 (conservation of kinetic energy) with v_{2i}= 0 to give:

$$\frac{1}{2} m_1\, {v_{1i}}^2 = \frac{1}{2} m_1\, {v_{1f}}^2 + \frac{1}{2} m_2\, {v_{2f}}^2$$

9.4 Two-Dimensional Collisions

PROBLEM-SOLVING HINTS

- Set up a coordinate system and define your velocities with respect to that system.
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.
- Write expressions for the x and y components of the momentum of each object before and after the collision.
- Write expressions for the total momentum of the system in the x and y directions before and after the collision and equate the two..
- If the collision is inelastic, kinetic energy of the system is not conserved, and additional information is probably required.
- If the collision is perfectly inelastic, the final velocities of the two objects are equal. Solve the momentum equations for the unknown quantities.
- If the collision is elastic, kinetic energy is conserved, and you can equate the total kinetic energy before and after the collision.

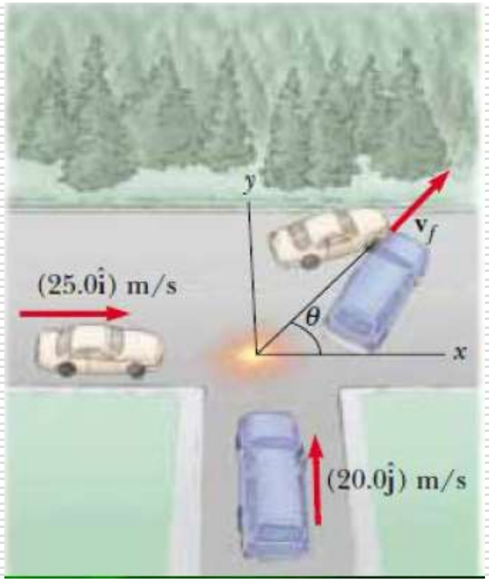
9.4 Two-Dimensional Collisions

Example 9.10 Collision at an Intersection

- A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg van traveling north at a speed of 20.0 m/s, as shown in Figure 9.14. *Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together).*
- ***Solution***:
- We shall apply the conservation of momentum in each direction.

$$x:\ m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx} \qquad (1)$$

$$y:\ m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy} \qquad (2)$$



9.4 Two-Dimensional Collisions

Example 9.10 (continued)

- Solving to find final velocity and direction:

$$(1) \rightarrow: \ (1500)(25) + (2500)(0) = (1500 + 2500)v_{fx} \qquad (3)$$

$$\therefore v_{fx} = \frac{37500}{4000} = 9.37\, m/s$$

$$(2) \rightarrow: \ (1500)(0) + (2500)(20) = (1500 + 2500)v_{fy} \qquad (4)$$

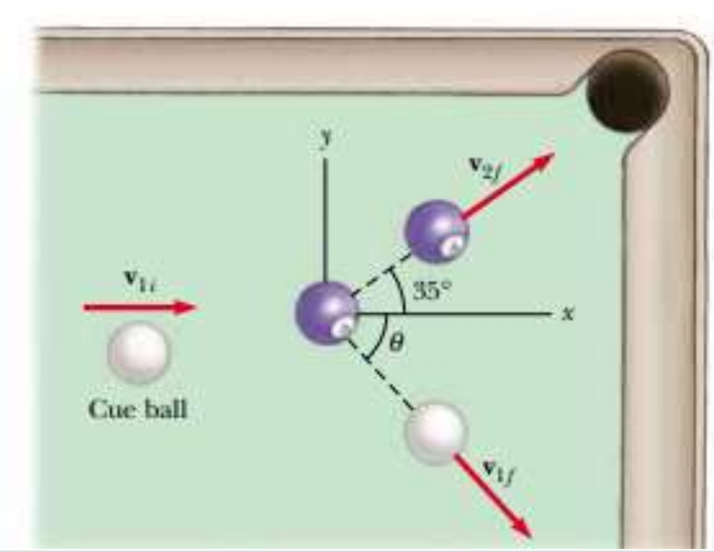
$$\therefore v_{fy} = \frac{50000}{4000} = 12.5\, m/s$$

$$(1) + (2) \rightarrow: v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{9.37^2 + 12.5^2} = 15.6\, m/s$$

$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) = \tan^{-1}\left(\frac{12.5}{9.37}\right) = 53.1^{o}$$

Example 9.12 Billiard Ball Collision

In a game of billiards, a player wishes to sink a target ball in the corner pocket, as shown in Figure 9.15. If the angle to the corner pocket is 35°, at what angle θ is the cue ball deflected? Assume that friction and rotational motion are unimportant and that the collision is elastic. Also assume that all billiard balls have the same mass m.



$$\frac{1}{2}m_1{v_{1i}}^2 = \frac{1}{2}m_1{v_{1f}}^2 + \frac{1}{2}m_2{v_{2f}}^2$$

But $m_1 = m_2 = m$, so that

$$(1) \qquad {v_{1i}}^2 = {v_{1f}}^2 + {v_{2f}}^2$$

$$(2) \qquad m_1\mathbf{v}_{1i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

$${v_{1i}}^2 = (\mathbf{v}_{1f} + \mathbf{v}_{2f})\cdot(\mathbf{v}_{1f} + \mathbf{v}_{2f}) = {v_{1f}}^2 + {v_{2f}}^2 + 2\mathbf{v}_{1f}\cdot\mathbf{v}_{2f}$$

Because the angle between $\mathbf{v}_{1f}$ and $\mathbf{v}_{2f}$ is $\theta + 35°$, $\mathbf{v}_{1f}\cdot\mathbf{v}_{2f} = v_{1f}v_{2f}\cos(\theta + 35°)$, and so

$$(3) \qquad {v_{1i}}^2 = {v_{1f}}^2 + {v_{2f}}^2 + 2v_{1f}v_{2f}\cos(\theta + 35°)$$

Subtracting Equation (1) from Equation (3) gives

$$0 = 2v_{1f}v_{2f}\cos(\theta + 35°)$$

$$0 = \cos(\theta + 35°)$$

$$\theta + 35° = 90° \quad \text{or} \quad \theta = 55°$$

PROBLEMS

Section 9.1 Linear Momentum and its Conservation

1. A 3.00-kg particle has a velocity of (3.00**i** ˆ - 4.00**j** ˆ) m/s.

(a)Find its *x* and *y* components of momentum.

(b) Find the magnitude and direction of its momentum.

SOLUTIONS TO PROBLEM:

$m = 3.00\ \text{kg}, \qquad \mathbf{v} = \left(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right)\ \text{m/s}$

(a) $\mathbf{p} = m\mathbf{v} = \left(9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}}\right)\ \text{kg}\cdot\text{m/s}$

Thus, $\boxed{p_x = 9.00\ \text{kg}\cdot\text{m/s}}$

and $\boxed{p_y = -12.0\ \text{kg}\cdot\text{m/s}}$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0\ \text{kg}\cdot\text{m/s}}$

$\theta = \tan^{-1}\left(\frac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$

PROBLEMS

Section 9.1 Linear Momentum and its Conservation

2. A 0.100-kg ball is thrown straight up into the air with an initial speed of 15.0 m/s. Find the momentum of the ball

(a) at its maximum height and (b) halfway up to its maximum height.

SOLUTIONS TO PROBLEM:

(a) At maximum height $\mathbf{v} = 0$, so $\mathbf{p} = \boxed{0}$.

(b) Its original kinetic energy is its constant total energy,

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(0.100)\text{kg}(15.0\ \text{m/s})^2 = 11.2\ \text{J}.$$

At the top all of this energy is gravitational. Halfway up, one-half of it is gravitational and the other half is kinetic:

$$K = 5.62\ \text{J} = \frac{1}{2}(0.100\ \text{kg})v^2$$

$$v = \sqrt{\frac{2 \times 5.62\ \text{J}}{0.100\ \text{kg}}} = 10.6\ \text{m/s}$$

Then $\mathbf{p} = m\mathbf{v} = (0.100\ \text{kg})(10.6\ \text{m/s})\hat{\mathbf{j}}$

$$\mathbf{p} = \boxed{1.06\ \text{kg} \cdot \text{m/s}\ \hat{\mathbf{j}}}.$$

PROBLEMS

Section 9.1 Linear Momentum and its Conservation

4. Two blocks of masses M and $3M$ are placed on a horizontal, frictionless surface. A light spring is attached to one of them, and the blocks are pushed together with the spring between them (Fig. P9.4). A cord initially holding the blocks together is burned; after this, the block of mass $3M$ moves to the right with a speed of 2.00 m/s.

(a) What is the speed of the block of mass M? (b) Find the original elastic potential energy in the spring if M = 0.350 kg.

SOLUTIONS TO PROBLEM:

(a) For the system of two blocks $\Delta p = 0$,

or $p_i = p_f$

Therefore, $0 = Mv_m + (3M)(2.00 \text{ m/s})$

Solving gives $v_m = \boxed{-6.00 \text{ m/s}}$ (motion toward the left).

(b) $\frac{1}{2}kx^2 = \frac{1}{2}Mv_M^2 + \frac{1}{2}(3M)v_{3M}^2 = \boxed{8.40 \text{ J}}$

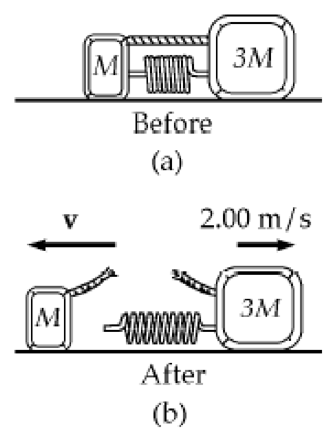


FIG. P9.4

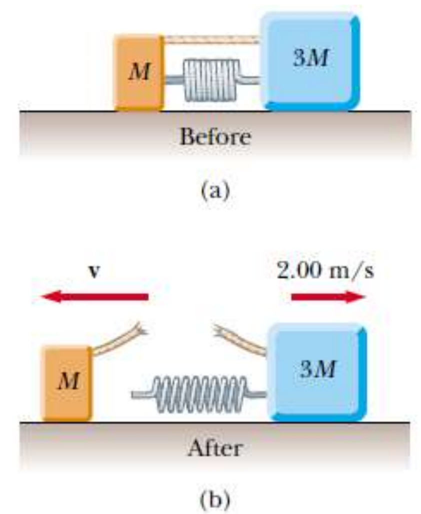


Figure P9.4

PROBLEMS

Section 9.1 Linear Momentum and its Conservation

5. (a) A particle of mass *m* moves with momentum *p*. Show that the kinetic energy of the particle is $K = p^2/2m$.

(b) Express the magnitude of the particle's momentum in terms of its kinetic energy and mass.

SOLUTIONS TO PROBLEM:

(a) The momentum is $p = mv$, so $v = \frac{p}{m}$ and the kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$.

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$, so $p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}$.

PROBLEMS

Section 9.2 Impulse and Momentum

7. An estimated force–time curve for a baseball struck by a bat is shown in Figure P9.7. From this curve, determine

(a) the impulse delivered to the ball, (b) the average force exerted on the ball, and (c) the peak force exerted on the ball.

SOLUTIONS TO PROBLEM:

(a) $I = \int F dt = \text{area under curve}$

$$I = \frac{1}{2}\left(1.50 \times 10^{-3}\ \text{s}\right)\left(18\,000\ \text{N}\right) = \boxed{13.5\ \text{N}\cdot\text{s}}$$

(b) $$F = \frac{13.5\ \text{N}\cdot\text{s}}{1.50 \times 10^{-3}\ \text{s}} = \boxed{9.00\ \text{kN}}$$

(c) From the graph, we see that $F_{\text{max}} = \boxed{18.0\ \text{kN}}$

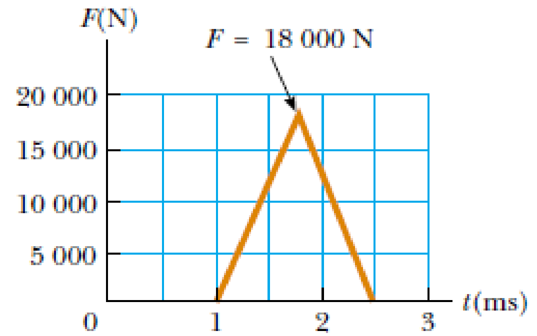


Figure P9.7

PROBLEMS

Section 9.2 Impulse and Momentum

8. A ball of mass 0.150 kg is dropped from rest from a height of 1.25 m. It rebounds from the floor to reach a height of 0.960 m. What impulse was given to the ball by the floor?

SOLUTIONS TO PROBLEM:

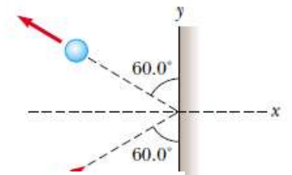
The impact speed is given by $\frac{1}{2}mv_1^2 = mgy_1$. The rebound speed is given by $mgy_2 = \frac{1}{2}mv_2^2$. The impulse of the floor is the change in momentum,

$$\begin{aligned} mv_2 \text{ up} - mv_1 \text{ down} &= m(v_2 + v_1) \text{ up} \\ &= m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right) \text{ up} \\ &= 0.15 \text{ kg}\sqrt{2(9.8 \text{ m/s}^2)}\left(\sqrt{0.960 \text{ m}} + \sqrt{1.25 \text{ m}}\right) \text{ up} \\ &= \boxed{1.39 \text{ kg}\cdot\text{m/s upward}} \end{aligned}$$

PROBLEMS

Section 9.2 Impulse and Momentum

9. A 3.00-kg steel ball strikes a wall with a speed of 10.0 m/s at an angle of 60.0° with the surface. It bounces off with the same speed and angle (Fig. P9.9). If the ball is in contact with the wall for 0.200 s, what is the average force exerted by the wall on the ball ?



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SOLUTIONS TO PROBLEM:

$$\Delta p = F\Delta t$$

$$\Delta p_y = m\left(v_{fy} - v_{iy}\right) = m(v\cos 60.0°) - mv\cos 60.0° = 0$$

$$\Delta p_x = m(-v\sin 60.0° - v\sin 60.0°) = -2mv\sin 60.0°$$

$$= -2(3.00\ \text{kg})(10.0\ \text{m/s})(0.866)$$

$$= -52.0\ \text{kg}\cdot\text{m/s}$$

$$F_{\text{ave}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0\ \text{kg}\cdot\text{m/s}}{0.200\ \text{s}} = \boxed{-260\ \text{N}}$$

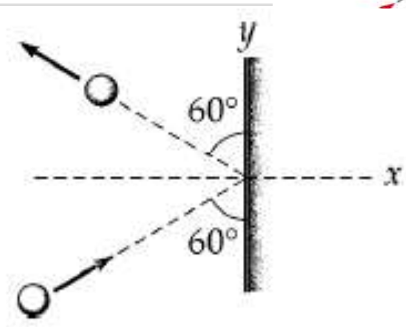

FIG. P9.9

PROBLEMS

Section 9.2 Impulse and Momentum

10. A tennis player receives a shot with the ball (0.060 0 kg) traveling horizontally at 50.0 m/s and returns the shot with the ball traveling horizontally at 40.0 m/s in the opposite direction.

(a) What is the impulse delivered to the ball by the racquet?

(b) What work does the racquet do on the ball?

SOLUTIONS TO PROBLEM:

Assume the initial direction of the ball in the $-x$ direction.

(a) Impulse, $\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = (0.060\,0)(40.0)\hat{\mathbf{i}} - (0.060\,0)(50.0)(-\hat{\mathbf{i}}) = \boxed{5.40\hat{\mathbf{i}}\ \text{N}\cdot\text{s}}$

(b) $\text{Work} = K_f - K_i = \frac{1}{2}(0.060\,0)\left[(40.0)^2 - (50.0)^2\right] = \boxed{-27.0\ \text{J}}$

PROBLEMS

Section 9.2 Impulse and Momentum

13. A garden hose is held as shown in Figure P9.13. The hose is originally full of motionless water. What additional force is necessary to hold the nozzle stationary after the water flow is turned on, if the discharge rate is 0.600 kg/s with a speed of 25.0 m/s?

SOLUTIONS TO PROBLEM:

Figure P9.13

The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600 \text{ kg})(25.0 \text{ m/s}) - 0}{1.00 \text{ s}} = \boxed{15.0 \text{ N}}.$$

According to Newton's third law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0 N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

PROBLEMS

Section 9.3 Collisions in One Dimension

15. High-speed stroboscopic photographs show that the head of a golf club of mass 200 g is traveling at 55.0 m/s just before it strikes a 46.0-g golf ball at rest on a tee. After the collision, the club head travels (in the same direction) at 40.0 m/s. Find the speed of the golf ball just after impact.

SOLUTIONS TO PROBLEM:

$$(200\ \text{g})(55.0\ \text{m/s}) = (46.0\ \text{g})v + (200\ \text{g})(40.0\ \text{m/s})$$

$$v = \boxed{65.2\ \text{m/s}}$$

PROBLEMS

Section 9.3 Collisions in One Dimension

16. An archer shoots an arrow toward a target that is sliding toward her with a speed of 2.50 m/s on a smooth, slippery surface. The 22.5-g arrow is shot with a speed of 35.0 m/s and passes through the 300-g target, which is stopped by the impact. What is the speed of the arrow after passing through the target?

SOLUTIONS TO PROBLEM:

$$(m_1v_1 + m_2v_2)_i = (m_1v_1 + m_2v_2)_f$$

$$22.5\text{ g}(35\text{ m/s}) + 300\text{ g}(-2.5\text{ m/s}) = 22.5\text{ g}v_{1f} + 0$$

$$v_{1f} = \frac{37.5\text{ g}\cdot\text{m/s}}{22.5\text{ g}} = \boxed{1.67\text{ m/s}}$$

PROBLEMS

Section 9.3 Collisions in One Dimension

17. A 10.0-g bullet is fired into a stationary block of wood (*m* !5.00 kg). The relative motion of the bullet stops inside the block. The speed of the bullet-plus-wood combination immediately after the collision is 0.600 m/s. What was the original speed of the bullet?

SOLUTIONS TO PROBLEM:

Momentum is conserved

$$\left(10.0 \times 10^{-3}\ \text{kg}\right) v = \left(5.01\ \text{kg}\right)\left(0.600\ \text{m/s}\right)$$

$$v = \boxed{301\ \text{m/s}}$$

PROBLEMS

Section 9.3 Collisions in One Dimension

18. A railroad car of mass 2.50 * 104 kg is moving with a speed of 4.00 m/s. It collides and couples with three other coupled railroad cars, each of the same mass as the single car and moving in the same direction with an initial speed of 2.00 m/s. (a) What is the speed of the four cars after the collision?

SOLUTIONS TO PROBLEM:

(a) $mv_{1i} + 3mv_{2i} = 4mv_f$ where $m = 2.50 \times 10^4$ kg

$$v_f = \frac{4.00 + 3(2.00)}{4} = \boxed{2.50 \text{ m/s}}$$

(b) $$K_f - K_i = \frac{1}{2}(4m)v_f^2 - \left[\frac{1}{2}mv_{1i}^2 + \frac{1}{2}(3m)v_{2i}^2\right] = \left(2.50 \times 10^4\right)(12.5 - 8.00 - 6.00) = \boxed{-3.75 \times 10^4 \text{ J}}$$

PROBLEMS

Section 9.3 Collisions in One Dimension

21. A 45.0-kg girl is standing on a plank that has a mass of 150 kg. The plank, originally at rest, is free to slide on a frozen lake, which is a flat, frictionless supporting surface. The girl begins to walk along the plank at a constant speed of 1.50 m/s relative to the plank. (a) What is her speed relative to the ice surface? (b) What is the speed of the plank relative to the ice surface?

SOLUTIONS TO PROBLEM:

(a), (b) Let v_g and v_p be the velocity of the girl and the plank relative to the ice surface. Then we may say that $v_g - v_p$ is the velocity of the girl relative to the plank, so that

$$v_g - v_p = 1.50 \qquad (1)$$

But also we must have $m_g v_g + m_p v_p = 0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore

$$45.0 v_g + 150 v_p = 0\text{, or } v_g = -3.33 v_p$$

Putting this into the equation (1) above gives

$$-3.33 v_p - v_p = 1.50 \text{ or } v_p = \boxed{-0.346 \text{ m/s}}$$

Then $v_g = -3.33(-0.346) = \boxed{1.15 \text{ m/s}}$

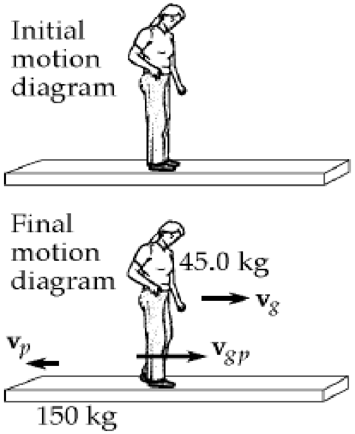

FIG. P9.21

PROBLEMS

Section 9.3 Collisions in One Dimension

25. A 12.0-g wad of sticky clay is hurled horizontally at a 100-g wooden block initially at rest on a horizontal surface. The clay sticks to the block. After impact, the block slides 7.50 m before coming to rest. If the coefficient of friction between the block and the surface is 0.650, what was the speed of the clay immediately before impact?

SOLUTIONS TO PROBLEM:

At impact, momentum of the clay-block system is conserved, so:

$$mv_1 = (m_1 + m_2)v_2$$

After impact, the change in kinetic energy of the clay-block-surface system is equal to the increase in internal energy:

$$\frac{1}{2}(m_1 + m_2)v_2^2 = f_f d = \mu(m_1 + m_2)gd$$

$$\frac{1}{2}(0.112\ \text{kg})v_2^2 = 0.650(0.112\ \text{kg})(9.80\ \text{m/s}^2)(7.50\ \text{m})$$

$$v_2^2 = 95.6\ \text{m}^2/\text{s}^2 \qquad v_2 = 9.77\ \text{m/s}$$

$$(12.0\times10^{-3}\ \text{kg})v_1 = (0.112\ \text{kg})(9.77\ \text{m/s}) \qquad v_1 = \boxed{91.2\ \text{m/s}}$$

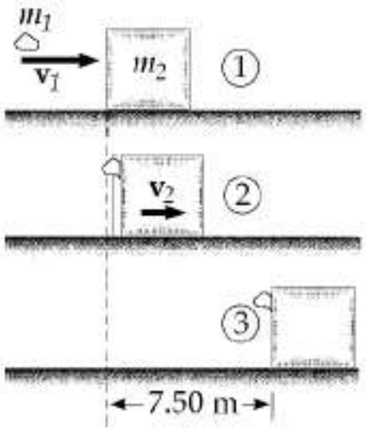

FIG. P9.25

PROBLEMS

Section 9.3 Collisions in One Dimension

27. (a) Three carts of masses 4.00 kg, 10.0 kg, and 3.00 kg move on a frictionless horizontal track with speeds of 5.00 m/s, 3.00 m/s, and 4.00 m/s, as shown in Figure P9.27. Velcro couplers make the carts stick together after colliding. Find the final velocity of the train of three carts. (b) **What If?** Does your answer require that all the carts collide and stick together at the same time? What if they collide in a different order?

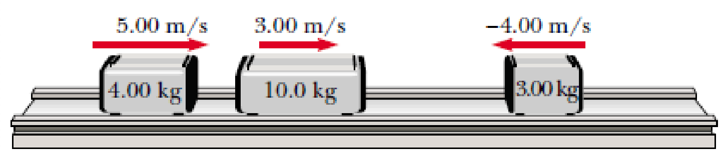

Figure P9.27

SOLUTIONS TO PROBLEM:

(a) Using conservation of momentum, $\left(\sum p\right)_{after} = \left(\sum p\right)_{before}$, gives

$$[(4.0 + 10 + 3.0)\ \text{kg}]v = (4.0\ \text{kg})(5.0\ \text{m/s}) + (10\ \text{kg})(3.0\ \text{m/s}) + (3.0\ \text{kg})(-4.0\ \text{m/s}).$$

Therefore, $v = +2.24$ m/s, or $\boxed{2.24\ \text{m/s toward the right}}$.

(b) $\boxed{\text{No}}$. For example, if the 10-kg and 3.0-kg mass were to stick together first, they would move with a speed given by solving

$$(13\ \text{kg})v_1 = (10\ \text{kg})(3.0\ \text{m/s}) + (3.0\ \text{kg})(-4.0\ \text{m/s}),\ \text{or}\ v_1 = +1.38\ \text{m/s}.$$

Then when this 13 kg combined mass collides with the 4.0 kg mass, we have

$$(17\ \text{kg})v = (13\ \text{kg})(1.38\ \text{m/s}) + (4.0\ \text{kg})(5.0\ \text{m/s}),\ \text{and}\ v = +2.24\ \text{m/s}$$

just as in part (a). Coupling order makes no difference.

PROBLEMS

Section 9.4 Two-Dimensional Collisions

32. Two automobiles of equal mass approach an intersection. One vehicle is traveling with velocity 13.0 m/s toward the east, and the other is traveling north with speed *v2i* . Neither driver sees the other. The vehicles collide in the intersection and stick together, leaving parallel skid marks at an angle of 55.0° north of east. The speed limit for both roads is 35 mi/h, and the driver of the northward-moving vehicle claims he was within the speed limit when the collision occurred. Is he telling the truth?

SOLUTIONS TO PROBLEM:

We use conservation of momentum for the system of two vehicles for both northward and eastward components.

For the eastward direction:

$$M(13.0\ \text{m/s}) = 2MV_f \cos 55.0°$$

For the northward direction:

$$Mv_{2i} = 2MV_f \sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0\ \text{m/s})\tan 55.0° = 18.6\ \text{m/s} = \boxed{41.5\ \text{mi/h}}$$

Thus, the driver of the north bound car was untruthful.

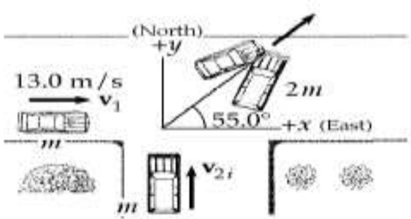

FIG. P9.32

PROBLEMS

Section 9.4 Two-Dimensional Collisions

33. A billiard ball moving at 5.00 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves, at 4.33 m/s, at an angle of 30.0° with respect to the original line of motion. Assuming an elastic collision (and ignoring friction and rotational motion), find the struck ball's velocity after the collision.

SOLUTIONS TO PROBLEM:

By conservation of momentum for the system of the two billiard balls (with all masses equal),

$$5.00 \text{ m/s} + 0 = (4.33 \text{ m/s})\cos 30.0° + v_{2fx}$$

$$v_{2fx} = 1.25 \text{ m/s}$$

$$0 = (4.33 \text{ m/s})\sin 30.0° + v_{2fy}$$

$$v_{2fy} = -2.16 \text{ m/s}$$

$$\mathbf{v}_{2f} = \boxed{2.50 \text{ m/s at } -60.0°}$$

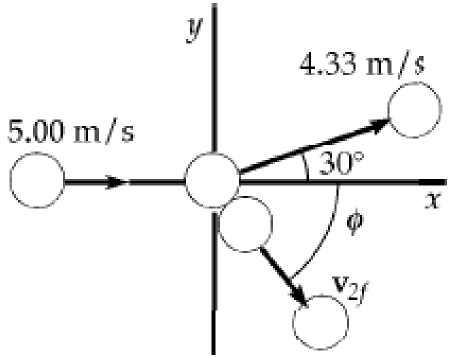

FIG. P9.33

Note that we did not need to use the fact that the collision is perfectly elastic.

PROBLEMS

Section 9.4 Two-Dimensional Collisions

35. An object of mass 3.00 kg, moving with an initial velocity of 5.00ˆi m/s, collides with and sticks to an object of mass 2.00 kg with an initial velocity of "3.00ˆj m/s. Find the final velocity of the composite object.

SOLUTIONS TO PROBLEM:

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f: \qquad 3.00(5.00)\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}} = 5.00\mathbf{v}$$

$$\mathbf{v} = \boxed{\left(3.00\hat{\mathbf{i}} - 1.20\hat{\mathbf{j}}\right)\ \text{m/s}}$$