Discrete Mathematics
Chapter 03
Sets

إعداد وتقديم

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كلية العلوم– قسم الرياضيات –جامعة الملك سعود

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٠٥٥٩٥٩٦٧٢٠


بسم الله الرحمن الرحيم

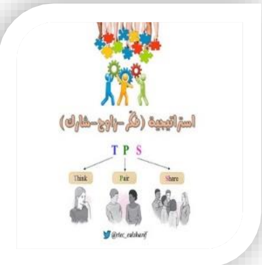
استراتيجيات التعلم

المناقشة

العرض التقدمي

العصف الذهني

التطبيق العملي



فكر – زاوج –شارك

المحاضرة

الاكتشاف

البحث والاستقصاء

د. أحمد السيد

1- Course Syllabus (Credit Hours: 4 (3+2))

No	List of Topics
1	Introduction to Number Systems: • Binary System (Binary to Decimal Conversion - Decimal to Binary Conversion – Arithmetic: addition, subtraction, multiplication) • Octal Number System (Conversions and Arithmetic) • Hexadecimal Number System (Conversions and Arithmetic)
2	Logic: • Proposition calculus and connectives • Truth tables • Propositional Equivalence.
3	Sets: • Set operations
4	Boolean Algebra: • Boolean Functions • Representation Boolean Functions • Logic Gates • Minimization of Circuit
5	Basic Concepts of Graph Theory: • Graph Terminology and Special Types of Graphs • Connectivity

King Saud University
College of Applied Studies & Community Service
Department of Computer Science & Engineering

	خطة تدريس المقرر (مقترح) **Course plan**
رمز ورقم المقرر: 153 ريض Math. 153	مـقرر: الرياضيات المحددة Discrete Mathematics

Course Objectives

- Learn how to think mathematically.
- Grasp the basic logical and reasoning mechanisms
- of mathematical thought.
- Acquire logic and proof as the basics for abstract
- thinking.
- Improve problem-solving skills.
- Grasp the basic elements of induction, recursion, combination and discrete structures.

Chapter 2: Sets

- Sets.
- Functions.
- Sequences, and Summations.
- Matrices.

Sets (1/24)

A **set** is an unordered collection of objects.

The objects in a set are called the *elements*, or *members*, of the set. A set is said to contain its elements.

Sets (2/24)

$S = \{a, b, c, d\}$

We write $a \in S$ to denote that a is an element of the set S. The notation $e \notin S$ denotes that e is not an element of the set S.

Sets (3/24)

The set O of odd positive integers less than 10 can be expressed by $O = \{1, 3, 5, 7, 9\}$.

The set of positive integers less than 100 can be denoted by $\{1, 2, 3, \ldots, 99\}$.

ellipses (…)

Sets (4/24)

Another way to describe a set is to use **set builder** notation.

The set O of odd positive integers less than 10 can be expressed by $O = \{1, 3, 5, 7, 9\}$.

$$O = \{x \mid x \text{ is an odd positive integer less than } 10\},$$

$$O = \{x \in \mathbf{Z}^+ \mid x \text{ is odd and } x < 10\}.$$

Sets (5/24)

$\mathbf{N} = \{0, 1, 2, 3, \ldots\}$, the set of all **natural numbers**

$\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of all **integers**

$\mathbf{Z}^+ = \{1, 2, 3, \ldots\}$, the set of all **positive integers**

$\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, \text{ and } q \neq 0\}$,
the set of all **rational numbers**

$\mathbf{R}$, the set of all **real numbers**

$\mathbf{R}^+$, the set of all **positive real numbers**

$\mathbf{C}$, the set of all **complex numbers**.

Sets (6/24)

Interval Notation

Closed interval $[a, b]$

Open interval (a, b)

$$[a, b] = \{x \mid a \leq x \leq b\}$$

$$[a, b) = \{x \mid a \leq x < b\}$$

$$(a, b] = \{x \mid a < x \leq b\}$$

$$(a, b) = \{x \mid a < x < b\}$$

Sets (7/24)

If A and B are sets, then A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$. We write $A = B$, if A and B are equal sets.

- The sets $\{1, 3, 5\}$ and $\{3, 5, 1\}$ are equal, because they have the same elements.

- $\{1, 3, 3, 5, 5, 5\}$ is the same as the set $\{1, 3, 5\}$ because they have the same elements.

Empty Set

There is a special set that has no elements. This set is called the empty set, or null set, and is denoted by $\emptyset$.

The empty set can also be denoted by { }

Cardinality

The cardinality is the number of distinct elements in S.
The cardinality of S is denoted by $|S|$.

Sets (10/24)

Example1

$S = \{a, b, c, d\}$

$|S| = 4$

$A = \{1, 2, 3, 7, 9\}$

$\emptyset = \{\ \}$

Sets (10/24)

Example1

$$S = \{a, b, c, d\}$$
$$|S| = 4$$

$$A = \{1, 2, 3, 7, 9\}$$
$$|A| = 5$$

$$\emptyset = \{\ \}$$
$$|\emptyset| = 0$$

Example2

$$S = \{a, b, c, d, \{2\}\}$$

$$|S| =$$

$$A = \{1, 2, 3, \{2,3\}, 9\}$$

$$|A| =$$

$$\{\emptyset\} = \{\{\ \}\}$$

$$|\{\emptyset\}| =$$

Sets (11/24)

Example2

$$S = \{a, b, c, d, \{2\}\}$$

$$|S| = 5$$

$$A = \{1, 2, 3, \{2,3\}, 9\}$$

$$|A| = 5$$

$$\{\emptyset\} = \{\{\ \}\}$$

$$|\{\emptyset\}| = 1$$

Sets (12/24)

Infinite

A set is said to be **infinite** if it is not finite.
The set of positive integers is infinite.

$$Z^+ = \{1,2,3, \ldots\}$$

Sets (13/24)

Subset

The set A is said to be a subset of B if and only if every element of A is also an element of B .

We use the notation $A \subseteq B$ to indicate that A is a subset of the set B .

$$A \subseteq B \leftrightarrow \forall x(x \in A \rightarrow x \in B)$$

Sets (13/24)

Subset

The set A is said to be a subset of B if and only if every element of A is also an element of B .

We use the notation $A \subseteq B$ to indicate that A is a subset of the set B .

$$(\boldsymbol{A} \subseteq \boldsymbol{B}) \equiv (\boldsymbol{B} \supseteq \boldsymbol{A})$$

$$A \subseteq B \;\leftrightarrow\; \forall x(x \in A \rightarrow x \in B)$$

Sets (13/24)

Subset

For every set S,

$(i)\ \emptyset \subseteq S$ and $(ii)\ S \subseteq S$.

To show that two sets A and B are equal, show that $A \subseteq B$ and $B \subseteq A$.

Sets (14/24)

Proper Subset

The set A is a subset of the set B but that $A \neq B$,
we write $A \subset B$
and say that A is a **proper subset** of B.

$$A \subset B \leftrightarrow \big(\forall x(x \in A \rightarrow x \in B) \land \exists x(x \in B \land x \notin A)\big)$$

Sets (15/24)

Example

For each of the following sets,
determine whether 3 is an element of that set.

$\{1,2,3,4\}$

$\{\{1\},\{2\},\{3\},\{4\}\}$

$\{1,2,\{1,3\}\}$

Venn Diagram

$A = \{1,2,3,4,7\}$

$B = \{0,3,5,7,9\}$

$C = \{1,2\}$

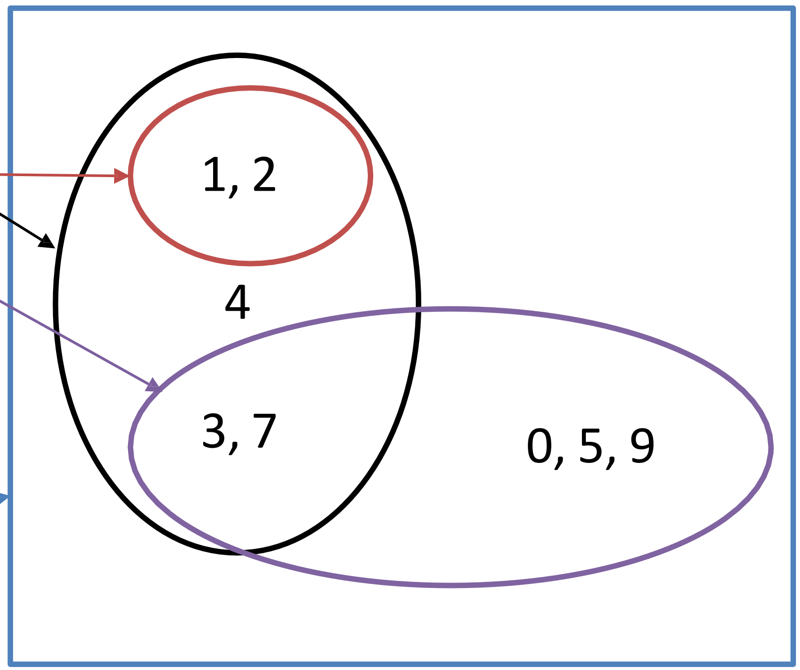
Sets (17/24)

Venn Diagram

$A = \{1,2,3,4,7\}$

$B = \{0,3,5,7,9\}$

$C = \{1,2\}$



Universal Set

Sets (18/24)

Power Set

The set of all subsets.

If the set is S. The power set of S is denoted by $P(S)$.

The number of elements in the power set is $2^{|S|}$

Power Set

The set of all subsets.

If the set is S. The power set of S is denoted by $P(S)$.

The number of elements in the power set is $2^{|S|}$

$S = \{1,2,3\}$

$|P(S)| = 2^3 = 8$ **elements**

$P(S) = 2^S$

$= \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Sets (19/24)

Example1

What is the power set of the empty set?

Sets (19/24)

Example1

What is the power set of the empty set?

$$\mathcal{P}(\emptyset) = \{\emptyset\}.$$

Sets (20/24)

Example2

What is the power set of the set $\{\emptyset\}$?

Sets (20/24)

Example2

What is the power set of the set $\{\emptyset\}$?

$$\mathcal{P}(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}.$$

Sets (21/24)

The ordered n-tuple

The ordered n-tuple $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, … , and a_n as its nth element.

In particular, ordered 2-tuples are called ordered pairs (e.g., the ordered pairs (a, b))

Cartesian Products

Let A and B be sets.

The Cartesian product of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence, $A \times B = \{(a, b) \mid a \in A \land b \in B\}$.

Cartesian Products - Example

Let $A = \{1,2\}$, and $B = \{a, b, c\}$

$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}.$

$|A \times B| = |A| * |B| = 2 * 3 = 6$

Cartesian Products - Example

Let $A = \{1,2\}$, and $B = \{a, b, c\}$

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}.$$

$$|A \times B| = |A| * |B| = 2 * 3 = 6$$

Find $B \times A$?

The Cartesian product of more than two sets.

The Cartesian product of the sets $A_1, A_2, \ldots,$ A_n, denoted by $A_1 \times A_2 \times \cdots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \ldots, a_n)$, where a_i belongs to A_i for $i = 1, 2, \ldots, n$. In other words,

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \ldots, n\}.$$

Example:

$A \times B \times C$, where $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{0, 1, 2\}$

$$A \times B \times C = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2)\}.$$

Set Operations (1/7)

Union

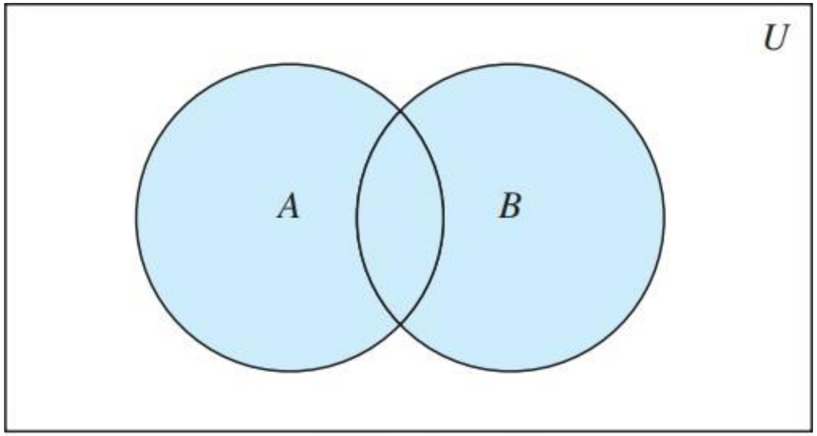
Let A and B be sets. The union of the sets A and B , denoted by $A \cup B$, is the set that contains those elements that are either in A or in B , or in both.

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

Set Operations (1/7)

Union

Let A and B be sets. The **union** of the sets A and B , denoted by $A \cup B$, is the set that contains those elements that are either in A or in B , or in both.



$A \cup B$ is shaded.

Set Operations (1/7)

Union

Let A and B be sets. The union of the sets A and B , denoted by $A \cup B$, is the set that contains those elements that are either in A or in B , or in both.

The union of the sets $\{1, 3, 5\}$ and $\{1, 2, 3\}$ is the set $\{1, 2, 3, 5\}$

Set Operations (2/7)

intersection

Intersection

Let A and B be sets. The intersection of the sets A and

B , denoted by $A \cap B$, is the set that contains those elements that

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

are in both A and B.

Set Operations (2/7)

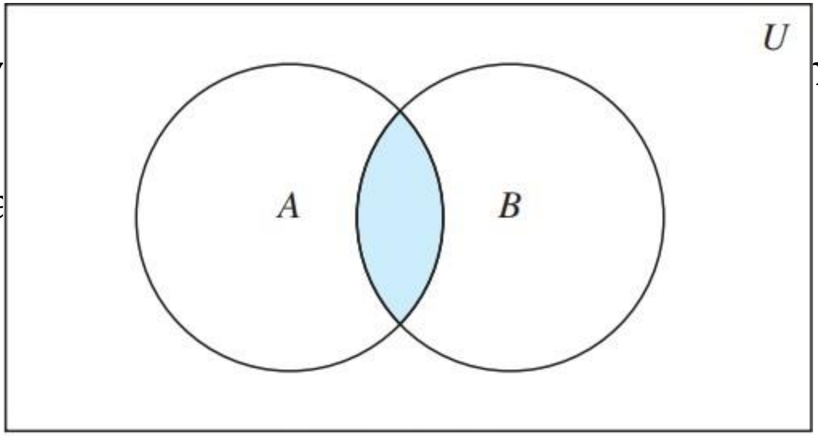
intersection

Intersection

Let A and B be sets. The intersection of the sets A and

B , denoted by ments that

are in both A a



$A \cap B$ is shaded.

Set Operations (2/7)

intersection

Intersection

Let A and B be sets. The intersection of the sets A and B , denoted by $A \cap B$, is the set that contains those elements that are in both A and B.

The intersection of the sets $\{1, 3, 5\}$ and $\{1, 2, 3\}$ is the set $\{1, 3\}$

Set Operations (3/7)

Disjoint

Two sets are called disjoint if their intersection is the empty set.

$$A \cap B = \emptyset$$

Set Operations (4/7)

Difference

Let A and B be sets. The difference of A and B , denoted by $A - B$, is the set containing those elements that are in A but not in B.

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

Set Operations (4/7)

Difference

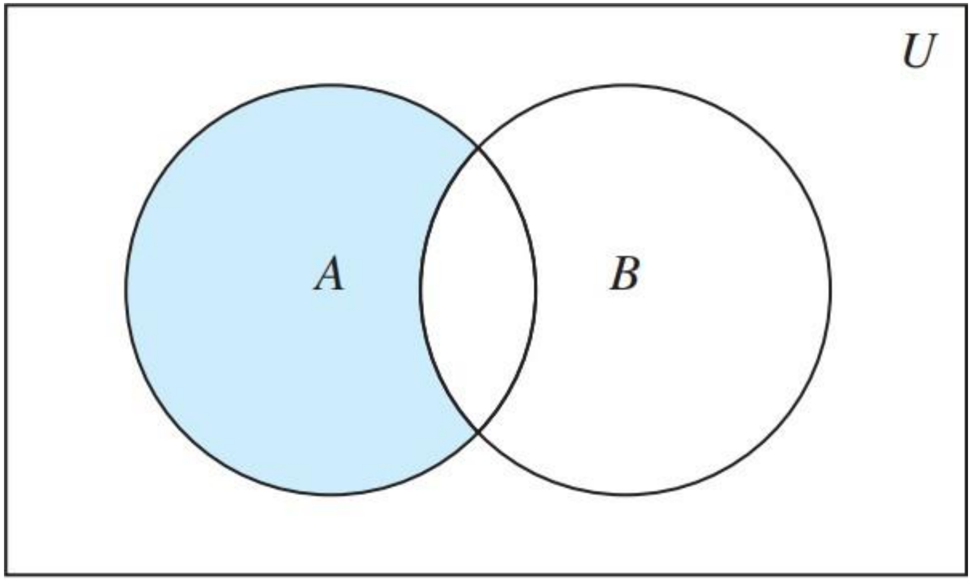
Let A and B be sets. The difference of A and B , denoted by $A - B$, is the set containing those elements that are in A but not in B.

$$A = \{1,3,5\}, \qquad B = \{1,2,3\}$$

$$A - B = \{5\}$$

Set Operations (4/7)

Difference



$A - B$ is shaded.

Set Operations (5/7)

Complement

Let U be the universal set.

The complement of the set A , denoted by AЂ

An element x belongs to U if and only if $x \notin A$.

$$\bar{A} = \{x \in U \mid x \notin A\}$$

Set Operations (5/7)

Complement

Let U be the universal set.

The complement of the set A , denoted by AЪ

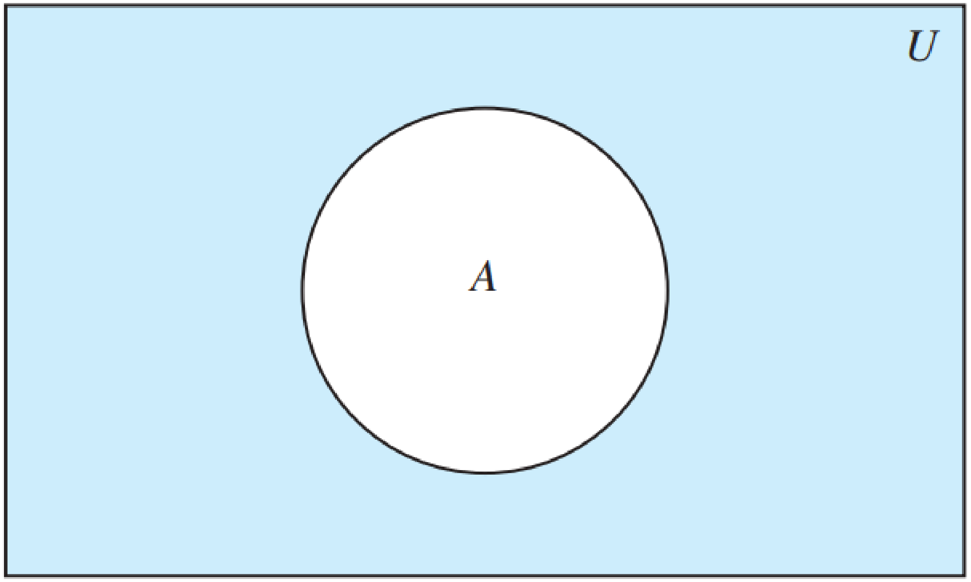
An element x belongs to U if and only if $x \notin A$.

$$U = \{1,2,3,4,5\}, \qquad A = \{1,3\}$$

$$A\text{Ъ} = \{ \quad 2,4,5 \}$$

Set Operations (5/7)

Complement



$\overline{A}$ is shaded.

Set Operations (6/7)

Generalized Unions

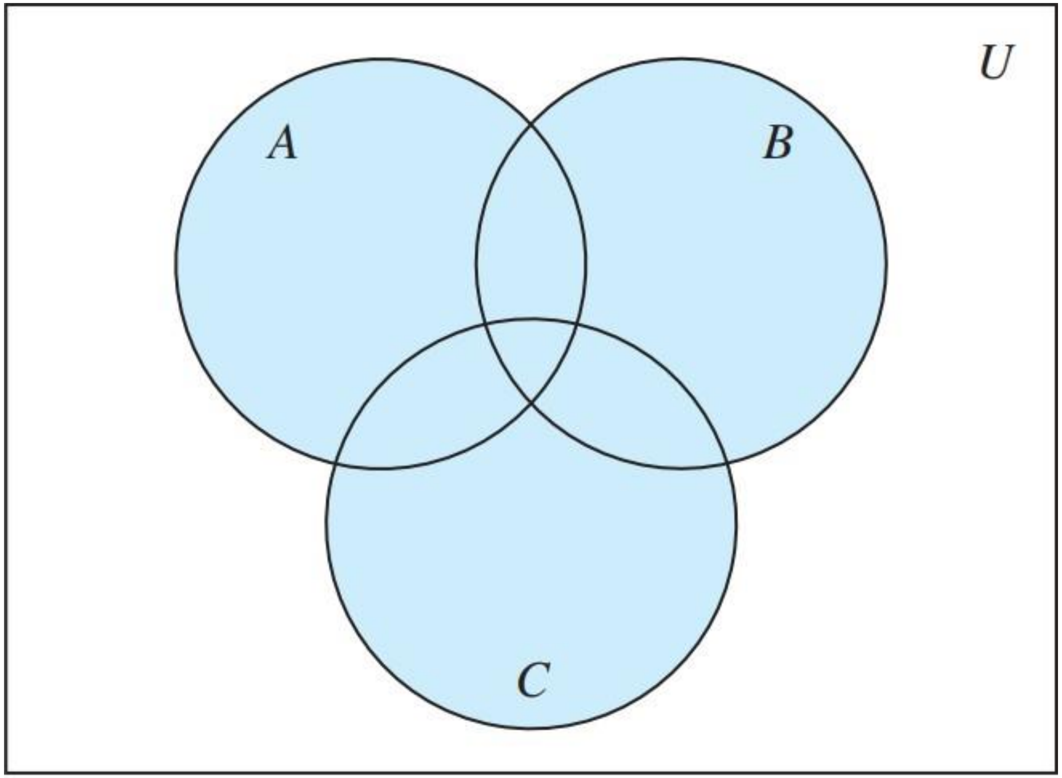
We use the notation

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$$

to denote the union of the sets $A_1, A_2, \ldots, A_n$.

Set Operations (6/7)

Generalized Unions



$A \cup B \cup C$ is shaded.

Set Operations (7/7)

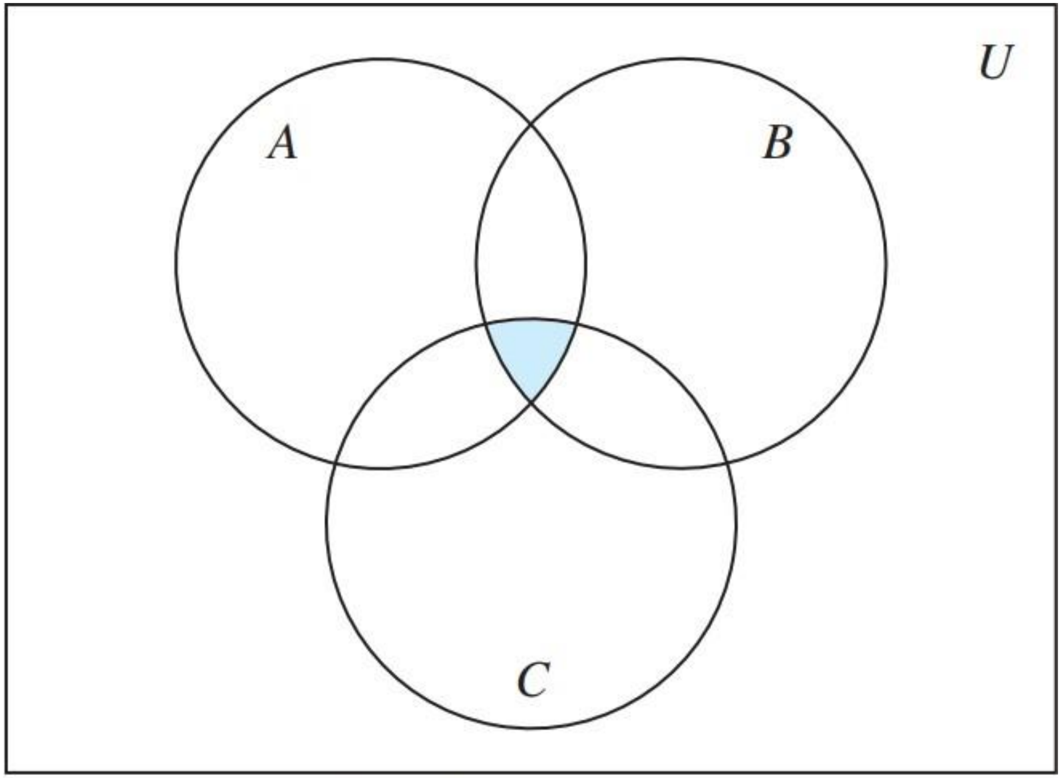
Generalized Intersections

We use the notation

$$A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$$

to denote the intersection of the sets $A_1, A_2, \ldots, A_n$.

Generalized Intersections



$A \cap B \cap C$ is shaded.

Set Identities (1/8)

TABLE Set Identities.

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws

Set Identities (2/8)

TABLE Set Identities.

$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

Set Identities (3/8)

Example1

Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

Set Identities (4/8)

Example1 – Answer

Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

First, we will show that $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$.

Next, we will show that $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$.

Set Identities (5/8)

First, we will show that $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$.

$x \in \overline{A \cap B}$	by assumption
$x \notin A \cap B$	defn. of complement
$\neg((x \in A) \wedge (x \in B))$	defn. of intersection
$\neg(x \in A) \vee \neg(x \in B)$	1st De Morgan Law for Prop Logic
$x \notin A \vee x \notin B$	defn. of negation
$x \in \overline{A} \vee x \in \overline{B}$	defn. of complement
$x \in \overline{A} \cup \overline{B}$	defn. of union

Set Identities (6/8)

Next, we will show that $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$.

$x \in \overline{A} \cup \overline{B}$	by assumption
$(x \in \overline{A}) \vee (x \in \overline{B})$	defn. of union
$(x \notin A) \vee (x \notin B)$	defn. of complement
$\neg(x \in A) \vee \neg(x \in B)$	defn. of negation
$\neg((x \in A) \wedge (x \in B))$	by 1st De Morgan Law for Prop Logic
$\neg(x \in A \cap B)$	defn. of intersection
$x \in \overline{A \cap B}$	defn. of complement

Set Identities (7/8)

Example2

Use set builder notation and logical equivalences to establish the first De Morgan law $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

Set Identities (8/8)

Example2 – Answer

Use set builder notation and logical equivalences to establish the first De Morgan law $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

$\overline{A \cap B} = \{x \mid x \notin A \cap B\}$	by definition of complement
$= \{x \mid \neg(x \in (A \cap B))\}$	by definition of does not belong symbol
$= \{x \mid \neg(x \in A \wedge x \in B)\}$	by definition of intersection
$= \{x \mid \neg(x \in A) \vee \neg(x \in B)\}$	by the first De Morgan law for logical equivalences
$= \{x \mid x \notin A \vee x \notin B\}$	by definition of does not belong symbol
$= \{x \mid x \in \overline{A} \vee x \in \overline{B}\}$	by definition of complement
$= \{x \mid x \in \overline{A} \cup \overline{B}\}$	by definition of union
$= \overline{A} \cup \overline{B}$	by meaning of set builder notation

Functions (1/21)

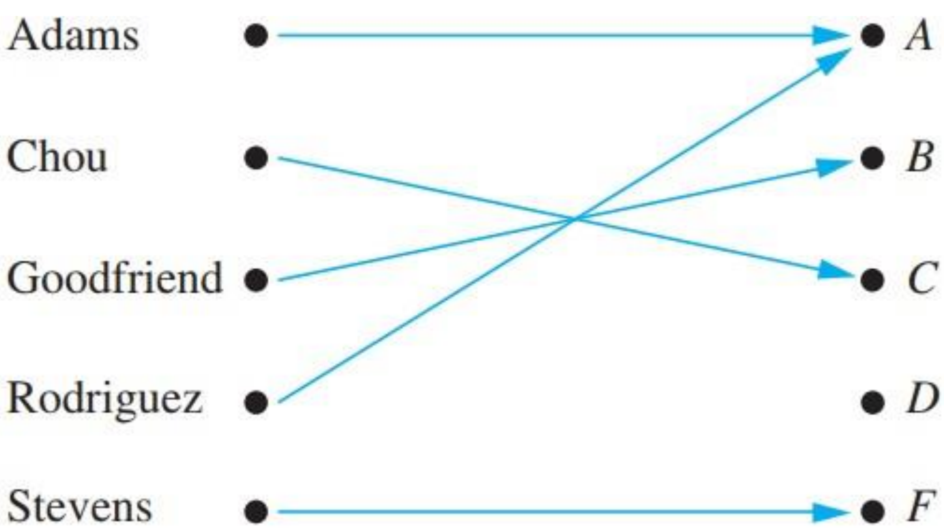
Function

Let A and B be nonempty sets. A function f from A to B is an assignment of exactly one element of B to each element of A.

We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A.

If f is a function from A to B, we write $f: A \to B$.

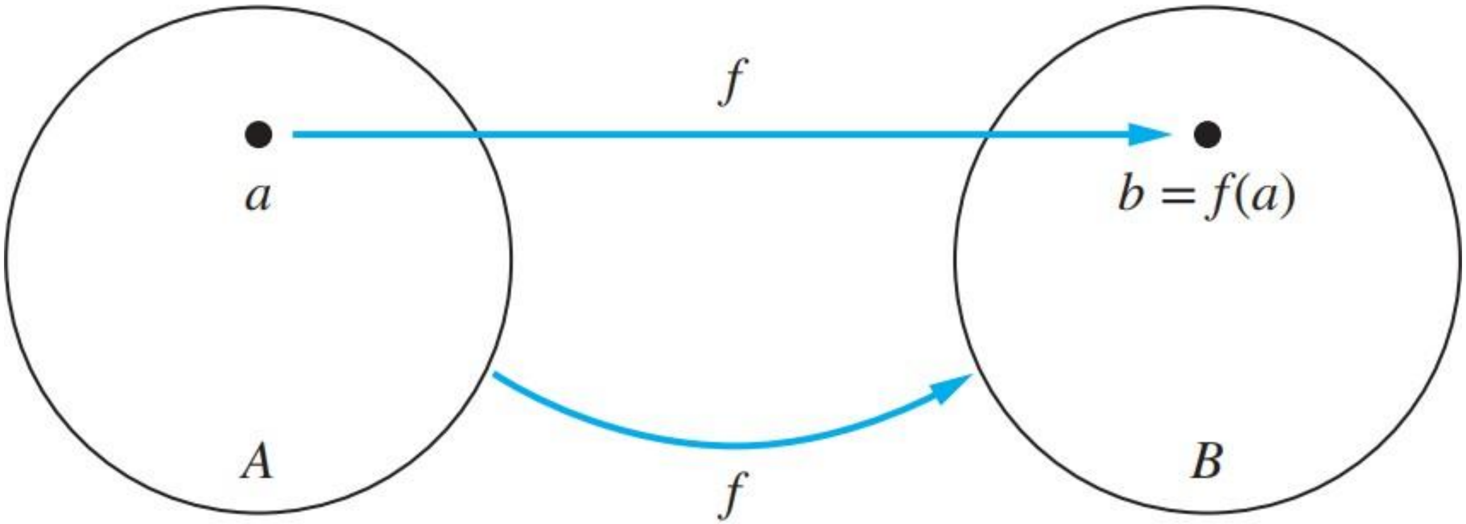
Function



Assignment of grades in a discrete mathematics class.

Functions (3/21)

The Function $f: A \to B$



The function f maps A to B.

Functions (3/21)

The Function $f: A \to B$

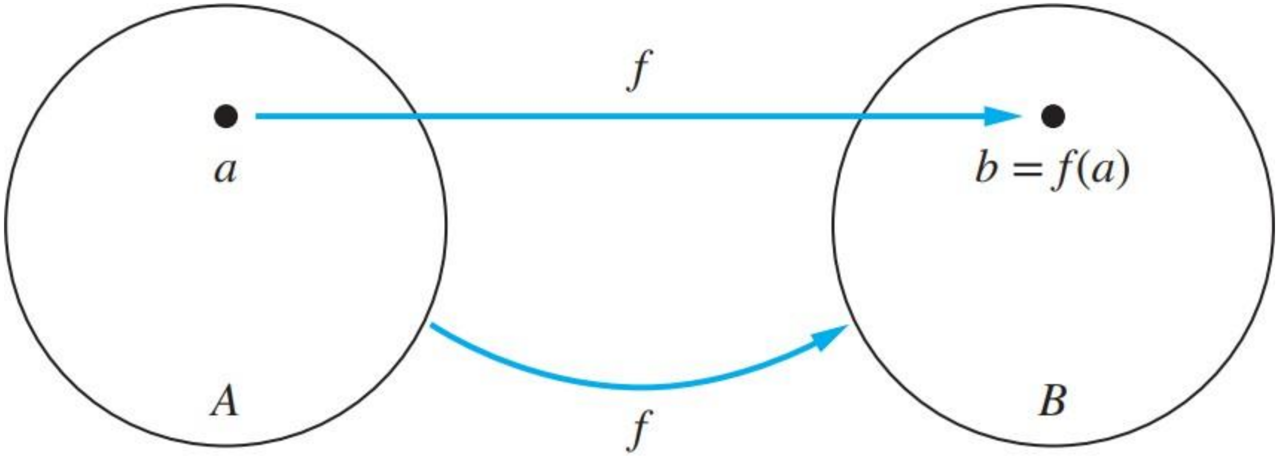
Domain: A

Co-Domain: B

$f(a) = b$

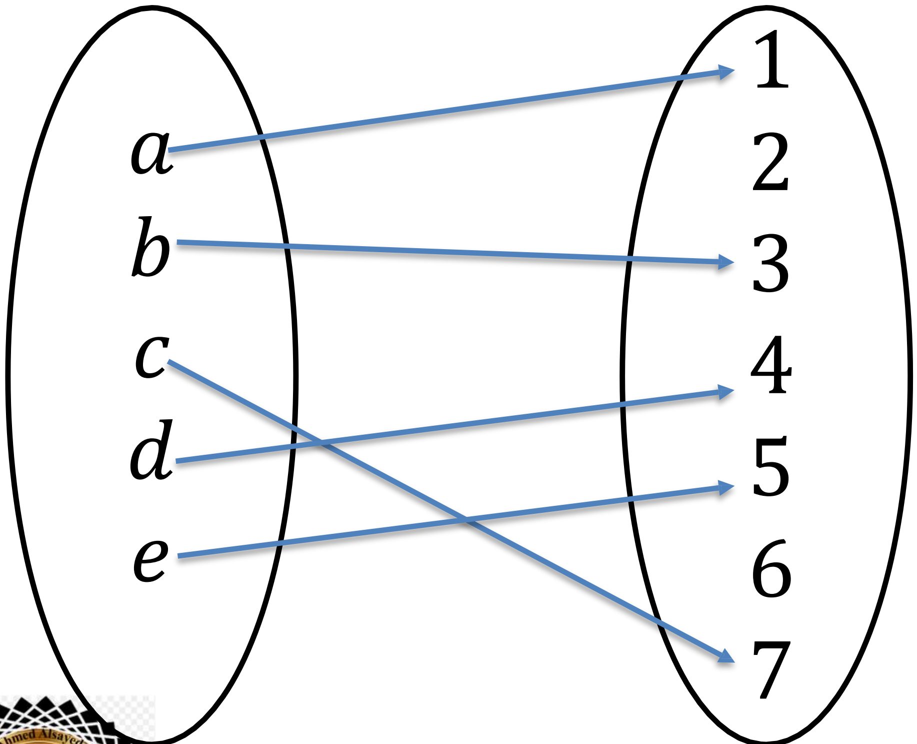
b is the ***image*** of a
a is a ***preimage*** of b

The **range**, or image, of f is the ***set of all images*** of elements of A.



The function f maps A to B.

The Function $f: A \to B$



$A \quad \to \quad B$

Domain $= \{a, b, c, d, e\}$

Co-Domain $= \{1,2,3,4,5,6,7\}$

Range $= \{1,3,4,5,7\}$

Functions (5/21)

Definition

Let f_1 and f_2 be functions from A to **R**. Then $f_1 + f_2$ and $f_1 f_2$ are also functions from A to **R** defined for all $x \in A$ by

$$(f_1 + f_2)(x) = f_1(x) + f_2(x),$$

$$(f_1 f_2)(x) = f_1(x) f_2(x).$$

Functions (6/21)

Example

Let f_1 and f_2 be functions from **R** to **R** such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$. What are the functions $f_1 + f_2$ and $f_1 f_2$?

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x^2 + (x - x^2) = x,$$

$$(f_1 f_2)(x) = f_1(x) f_2(x) = x^2(x - x^2) = x^3 - x^4.$$

Functions (7/21)

Definition

Let f be a function from A to B and let S be a subset of A.

The image of S under the function f is the subset of B that consists of the images of the elements of S.

We denote the image of S by $f(S)$, so

$$f(S) = \{t \mid \exists s \in S\ (t = f(s))\}.$$

or shortly $\{f(s) \mid s \in S\}$.

Functions (8/21)

Example

Let $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4\}$ with $f(a) = 2$, $f(b) = 1$, $f(c) = 4$, $f(d) = 1$, and $f(e) = 1$.

$S = \{b, c, d\} \subseteq A$

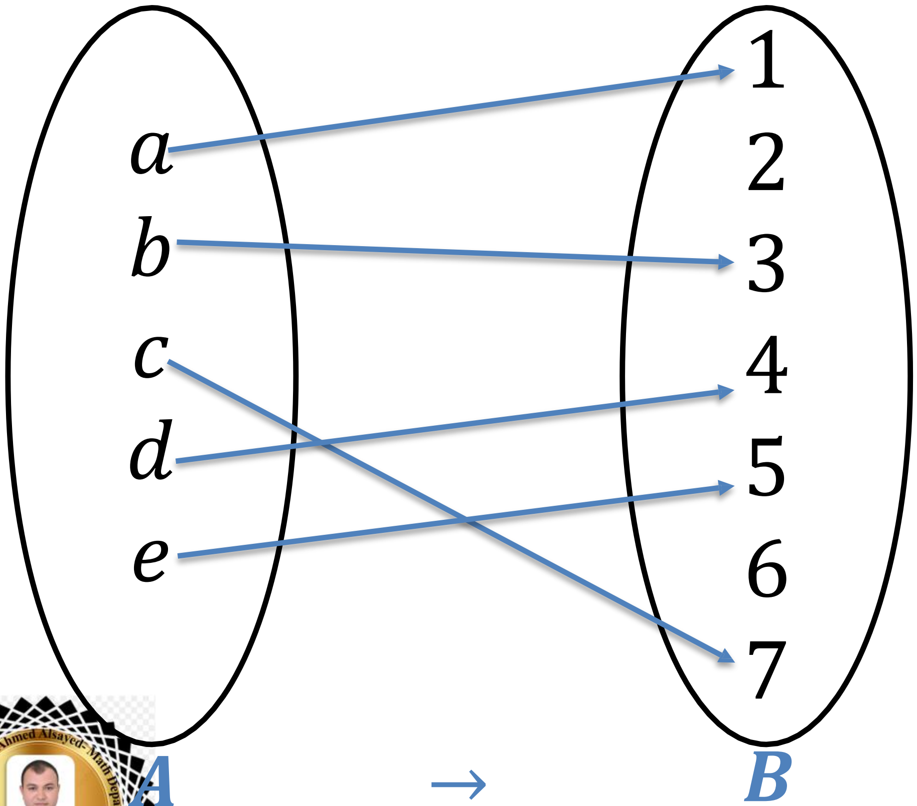
The image of the subset $S = \{b, c, d\}$ is the set $f(S) = \{1, 4\}$

Functions (9/21)

One-to-One function (injective)

A function f is said to be **one-to-one**, or **injective**, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.

One-to-One function (injective)



$f(a) = 1$

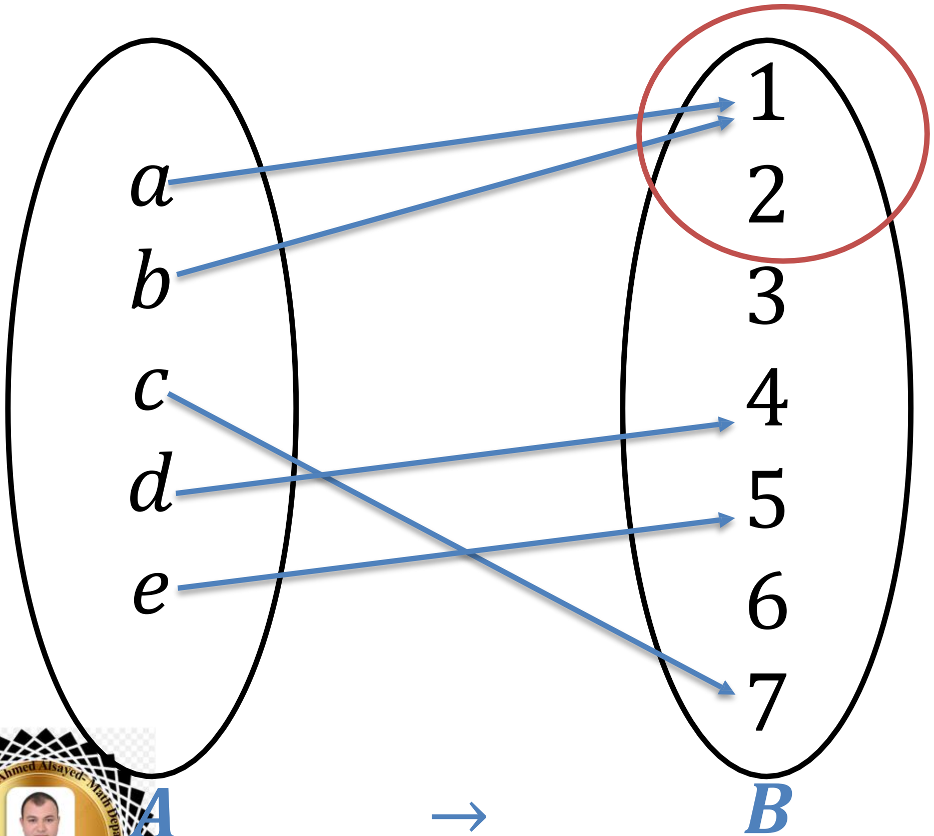
$f(b) = 3$

$f(c) = 7$

$f(d) = 4$

$f(e) = 5$

NOT *One-to-One* function (Not injective)



$$f(a) = 1$$

$$f(b) = 1$$

$$f(c) = 7$$

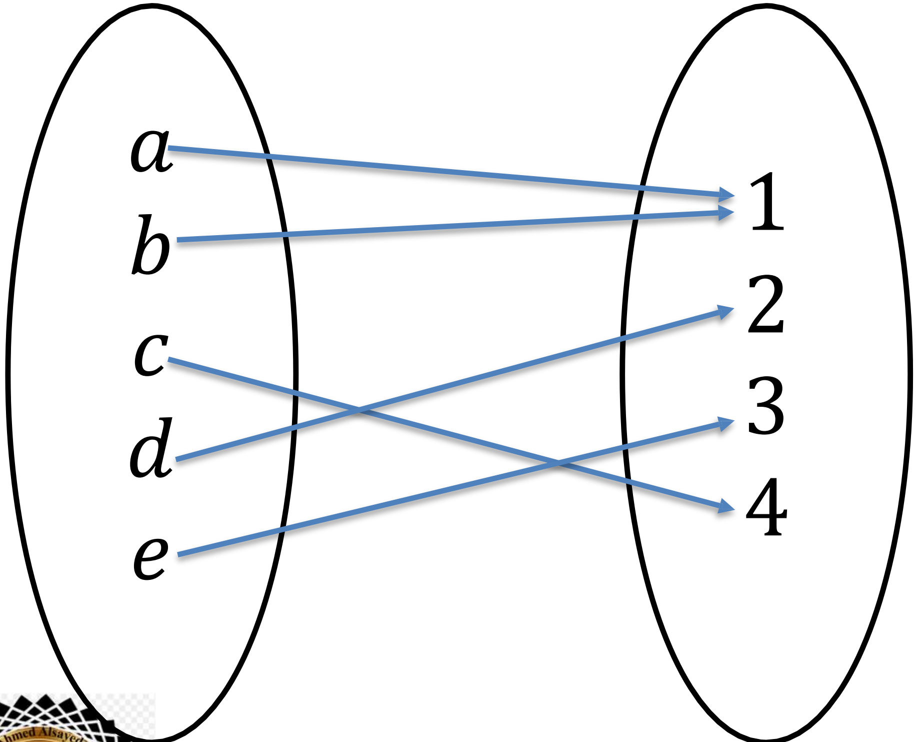
$$f(d) = 4$$

$$f(e) = 5$$

onto function (surjective)

A function f from A to B is called **onto**, or **surjective**, if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$.

onto function (surjective)



$A \rightarrow B$

$f(a) = 1$

$f(b) = 1$

$f(c) = 4$

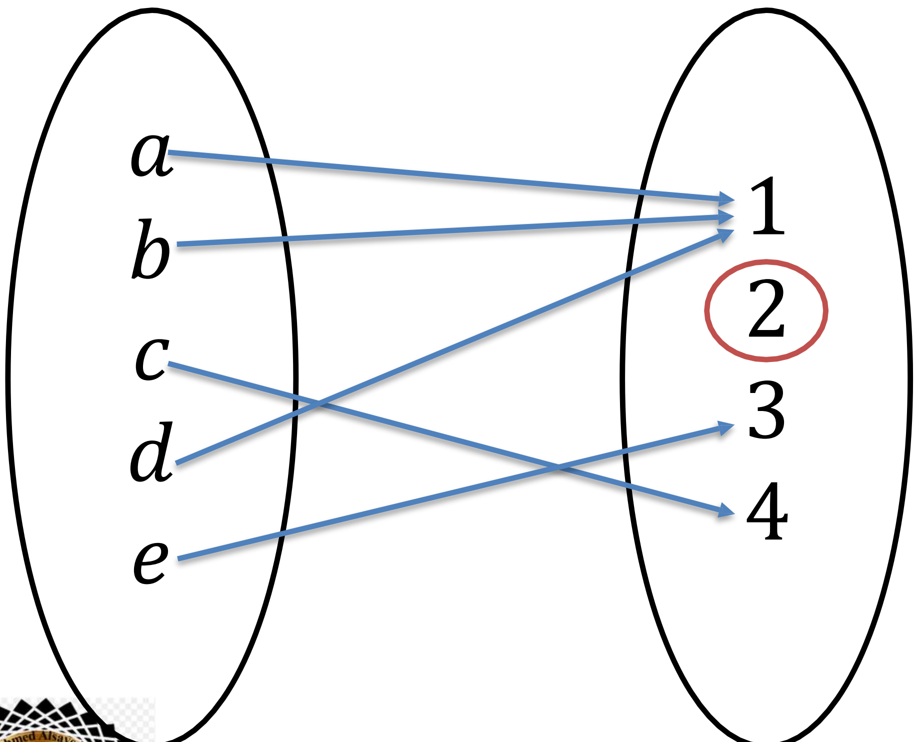
$f(d) = 2$

$f(e) = 3$

Co-Domain = {1,2,3,4}

Range = {1,2,3,4}

NOT *onto* function (Not surjective)



A → *B*

(∋ 1*af*

(∋ 1*bf*

(∋ 4*cf*

(∋ 1*df*

(∋ 3*ef*

1,2,3,4Co-D{main =}

Range = {1,3,4}

Functions (11/21)

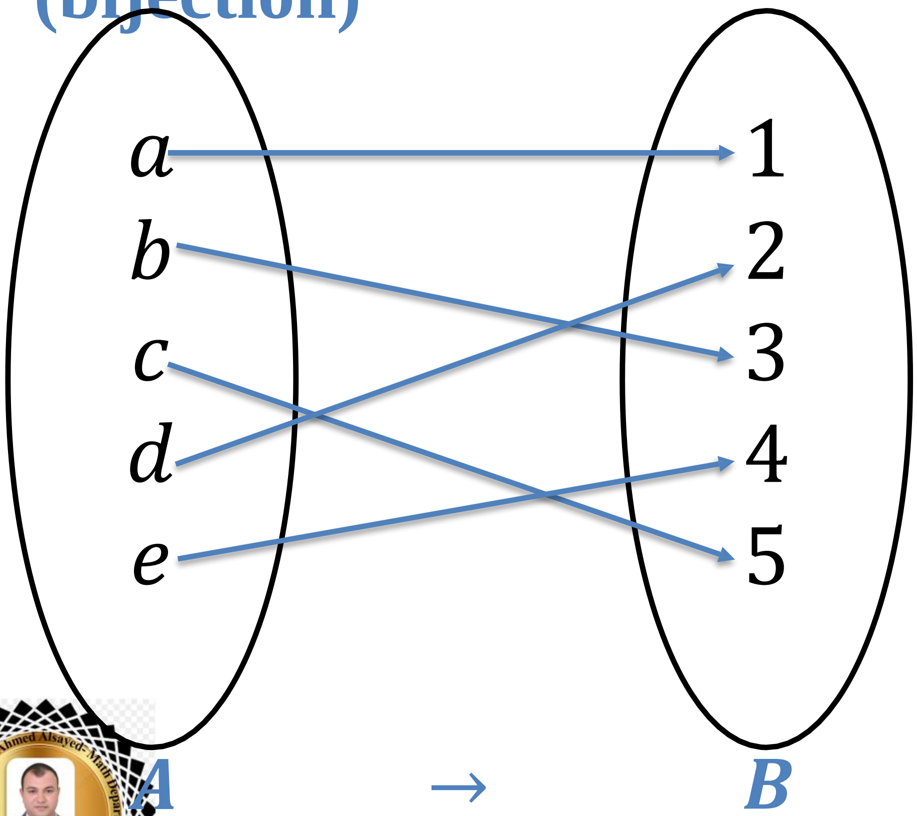
One-to-one correspondence (bijection)

The function *f* is a **one-to-one correspondence**, or a **bijection**, if it is both one-to-one and onto.

Functions (11/21)

One-to-one correspondence (bijection)

$$|A| = |B|$$



$f(a) = 1$

$f(b) = 3$

$f(c) = 5$

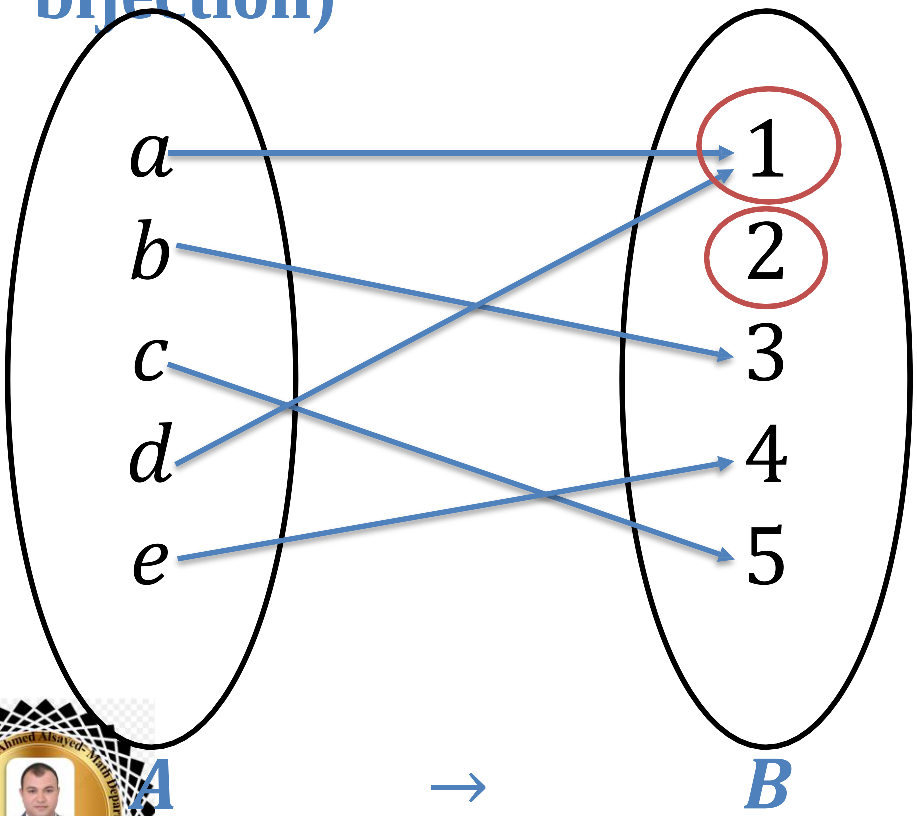
$f(d) = 2$

$f(e) = 4$

Co-Domain = {1,2,3,4,5}

Range = {1,2,3,4,5}

NOT *One-to-one correspondence* **(Not bijection)**



$f(a) = 1$

$f(b) = 3$

$f(c) = 5$

$f(d) = 1$

$f(e) = 4$

NOT one-to-one

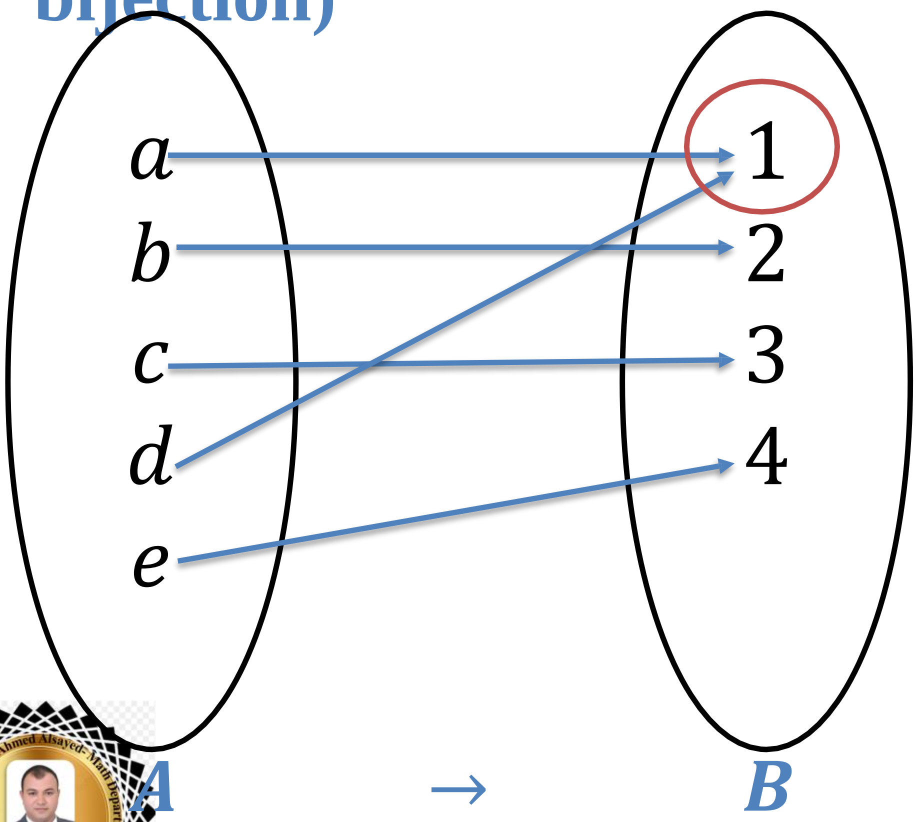
NOT onto

Co-Domain $= \{1,2,3,4,5\}$

Range $= \{1,3,4,5\}$

Functions (11/21)

NOT ***One-to-one correspondence*** **(Not bijection)**



$f(a) = 1$

$f(b) = 2$

$f(c) = 3$

$f(d) = 1$

$f(e) = 4$

Onto

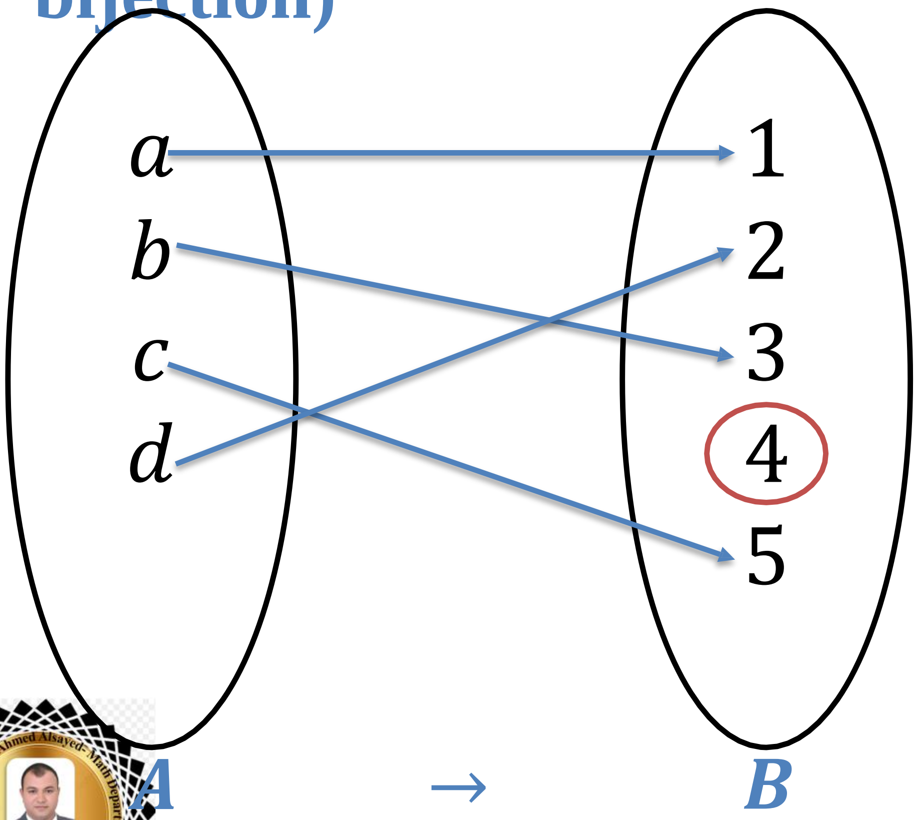
NOT one-to-one

Co-Domain = $\{1,2,3,4\}$

Range = $\{1,2,3,4\}$

Functions (11/21)

NOT *One-to-one correspondence* (Not bijection)



$f(a) = 1$

$f(b) = 3$

$f(c) = 5$

$f(d) = 2$

One-to-one

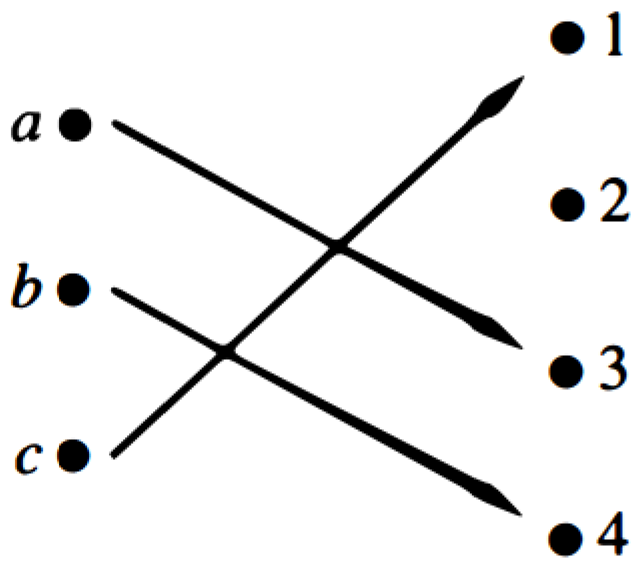
NOT onto

Co-Domain = $\{1,2,3,4,5\}$

Range = $\{1,2,3,5\}$

Functions (12/21)

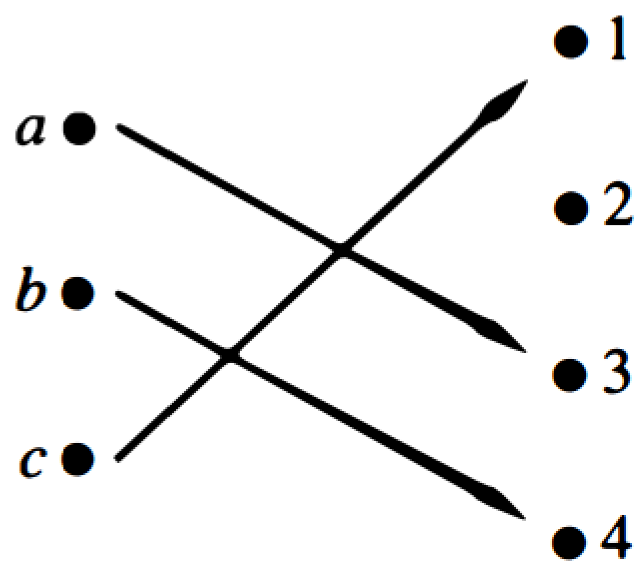
Examples



A → *B*

Functions (12/21)

Examples

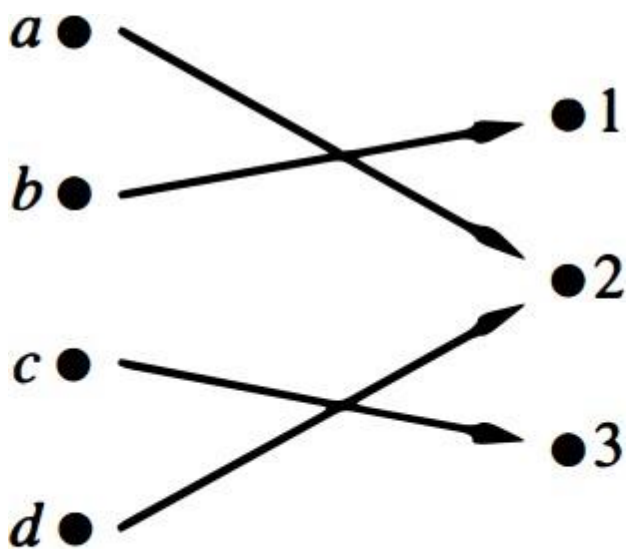


$A \rightarrow B$

One-to-one

NOT onto

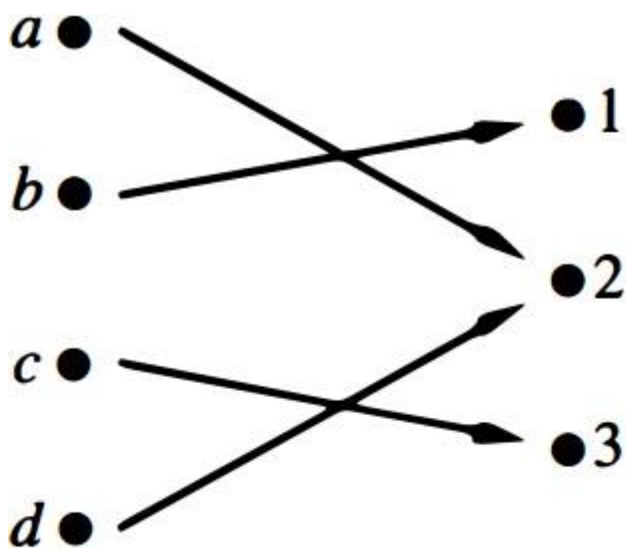
Examples



$A \rightarrow B$

Functions (12/21)

Examples



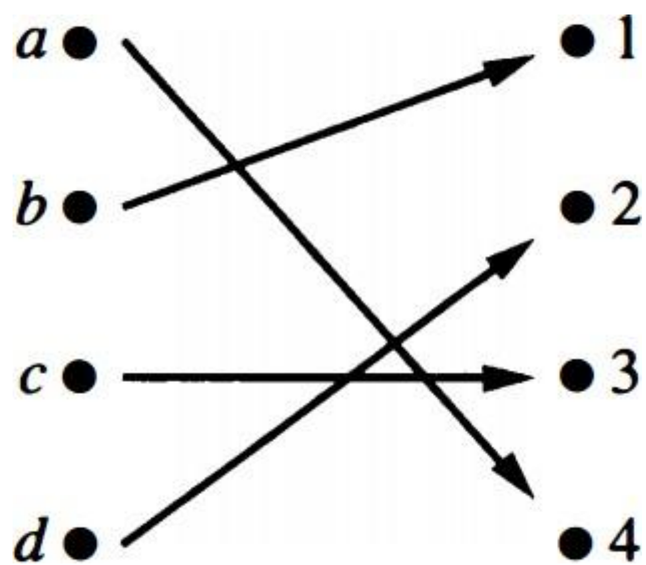
A → ***B***

NOT One-to-one

Onto

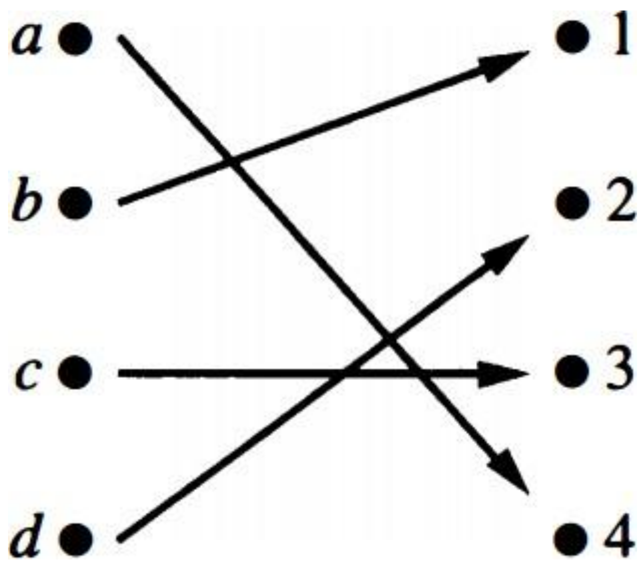
Functions (12/21)

Examples



$A \rightarrow B$

Examples



One-to-one

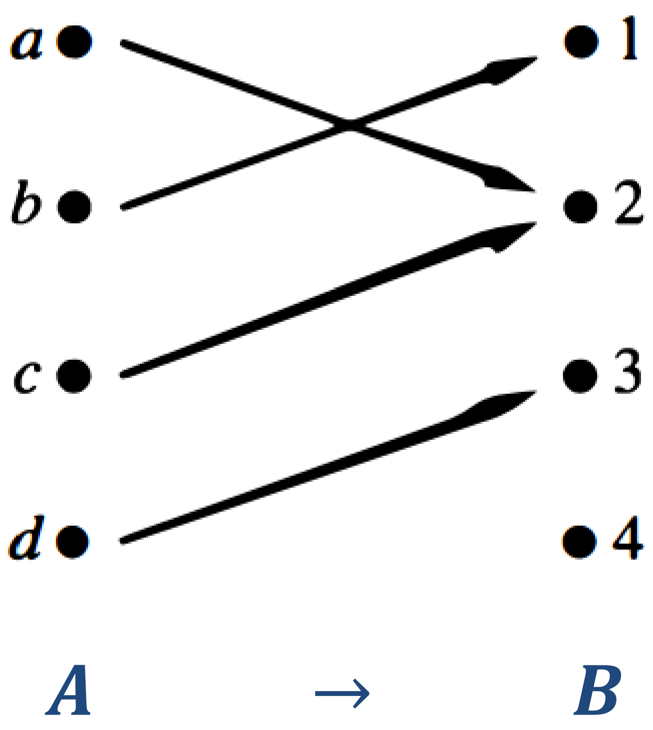
Onto

∴ bijection

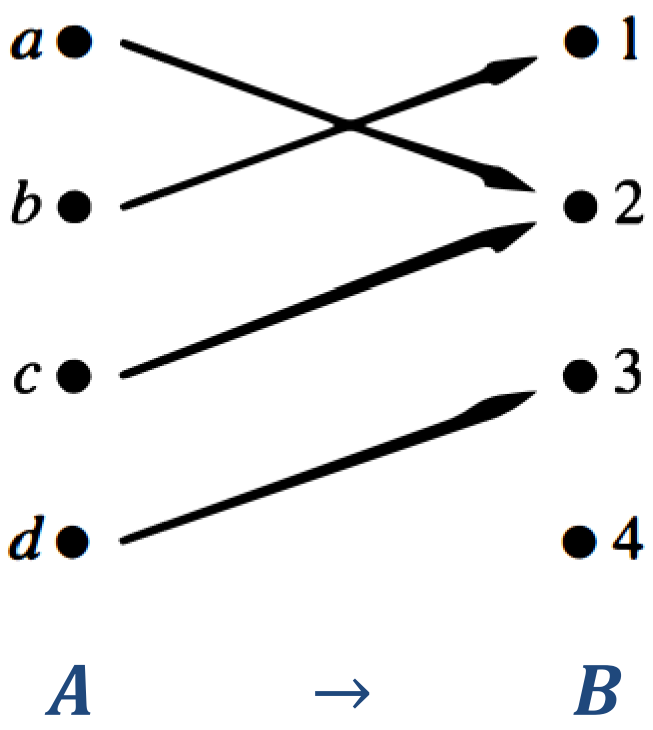
$A \rightarrow B$

Functions (12/21)

Examples



Examples

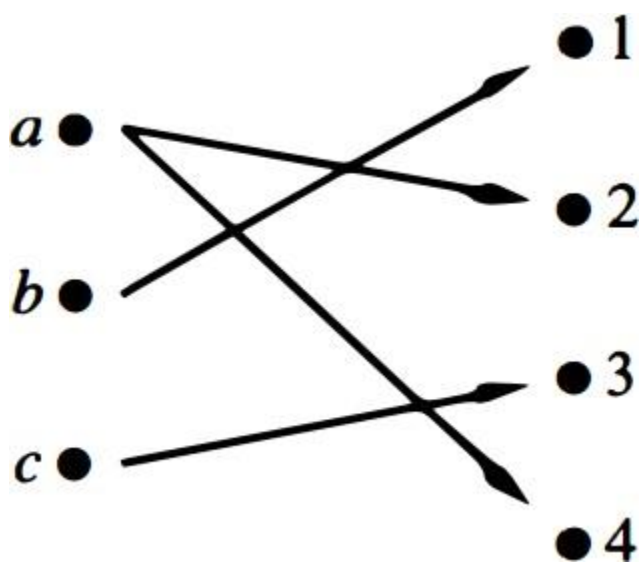


NOT One-to-one

NOT Onto

Functions (12/21)

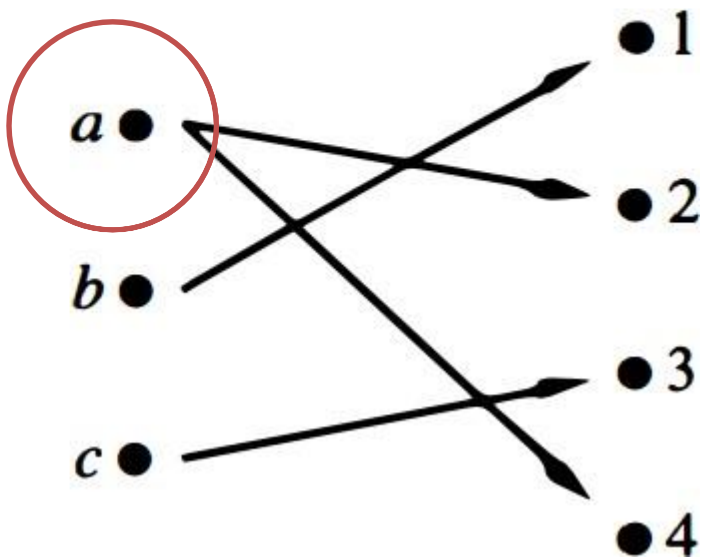
Examples



$A \rightarrow B$

Functions (12/21)

Examples



$A \rightarrow B$

NOT a function from *A* to *B*

Examples

xDetermine whether the function$(f) = x + 1$ from the set of integers to the set of integers is one-to-one.

Examples (Answer)

- Determine whether the function $f(x) = x + 1$ from the set of integers to the set of integers is one-to-one.

$f(a) = a + 1$ and $f(b) = b + 1$

$f(a) = f(b)$ (if f is one-to-one, then a equal b).

$$a + 1 = b + 1$$

$$a = b$$

$$\therefore f(x) \text{ is one-to-one}$$

Functions (14/21)

Examples

Determine whether the function $f(x) = x^2$ from the set of integers to the set of integers is one-to-one.

Functions (14/21)

Examples (Answer)

Determine whether the function $f(x) = x^2$ from the set of integers to the set of integers is one-to-one.

$f(a) = a^2$ and $f(b) = b^2$

$f(x)$ is one–to–one (if $f(a) = f(b)$ and a equal b then).

$$a^2 = b^2$$

$$\pm a = \pm b$$

$$a \text{ may be not equal } b$$

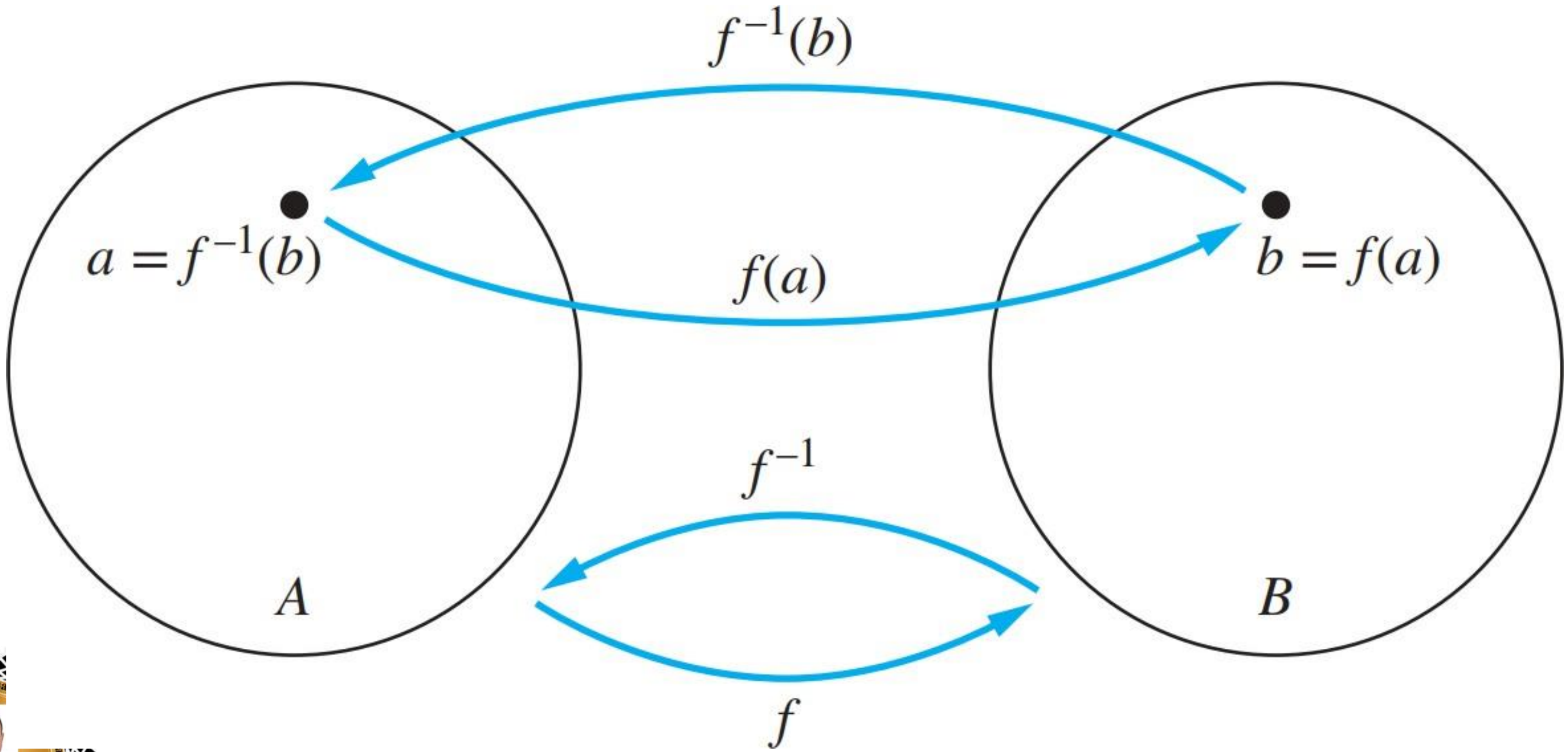
$$\therefore f(x) \text{ is NOT one–to–one}$$

Functions (15/21)

Inverse Functions

Let f be a *one-to-one correspondence* from the set A to the set B. The **inverse** function of f is the function that assigns to an element b belonging to B the unique element a in A such that $f(a) = b$. The inverse function of f is denoted by $\boldsymbol{f^{-1}}$. Hence, $f^{-1}(b) = a$ when $f(a) = b$.

Inverse Functions



Invertible

A one-to-one correspondence is called **invertible** because we can define an inverse of this function. A function is **not invertible** if it is not a one-to-one correspondence, because the inverse of such a function does not exist.

Functions (17/21)

Invertible – Example

Let f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is f invertible, and if it is, what is its inverse?

Functions (17/21)

Invertible – Example

Let f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is f invertible, and if it is, what is its inverse?

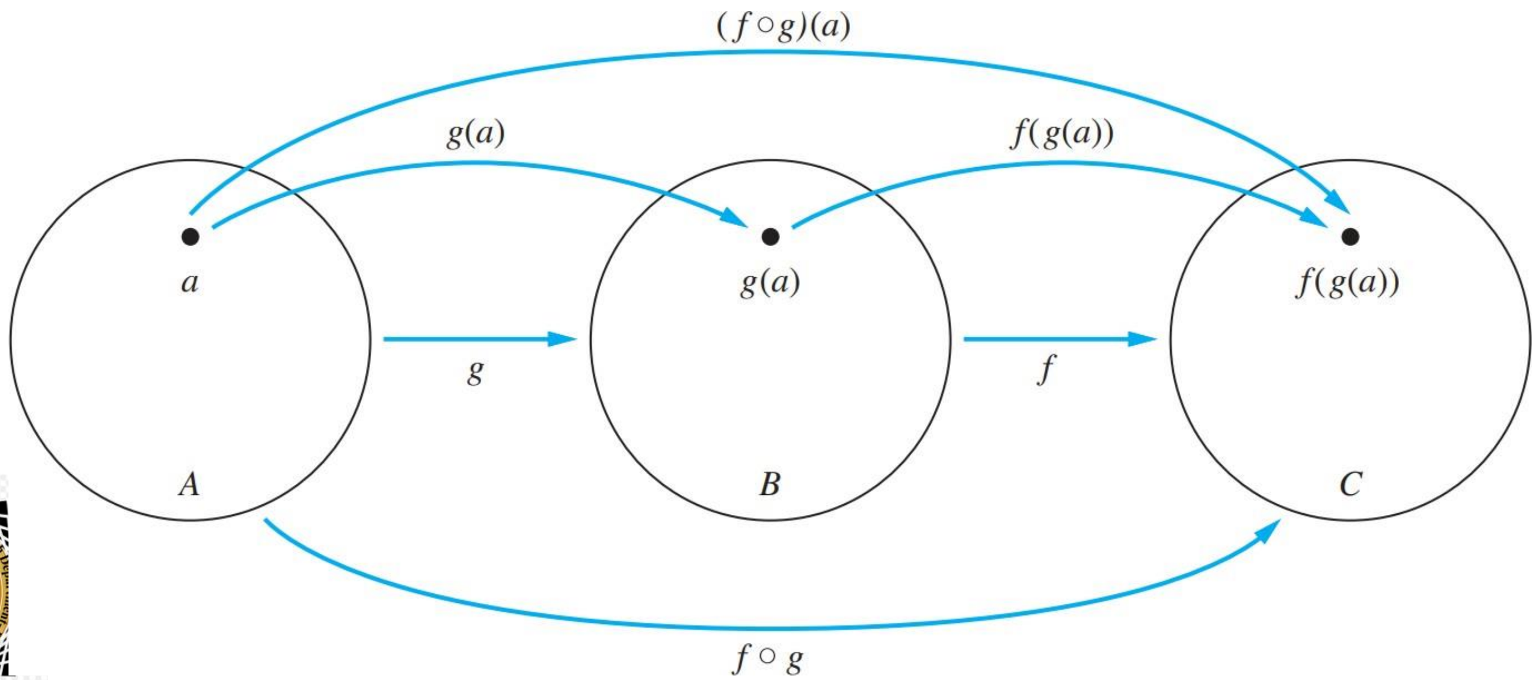
Answer:

The function f is invertible because it is a one-to-one correspondence. The inverse function f^{-1} reverses the correspondence given by f, so $f^{-1}(1) = c$, $f^{-1}(2) = a$, and $f^{-1}(3) = b$.

Functions (18/21)

Composition of the Functions f and g

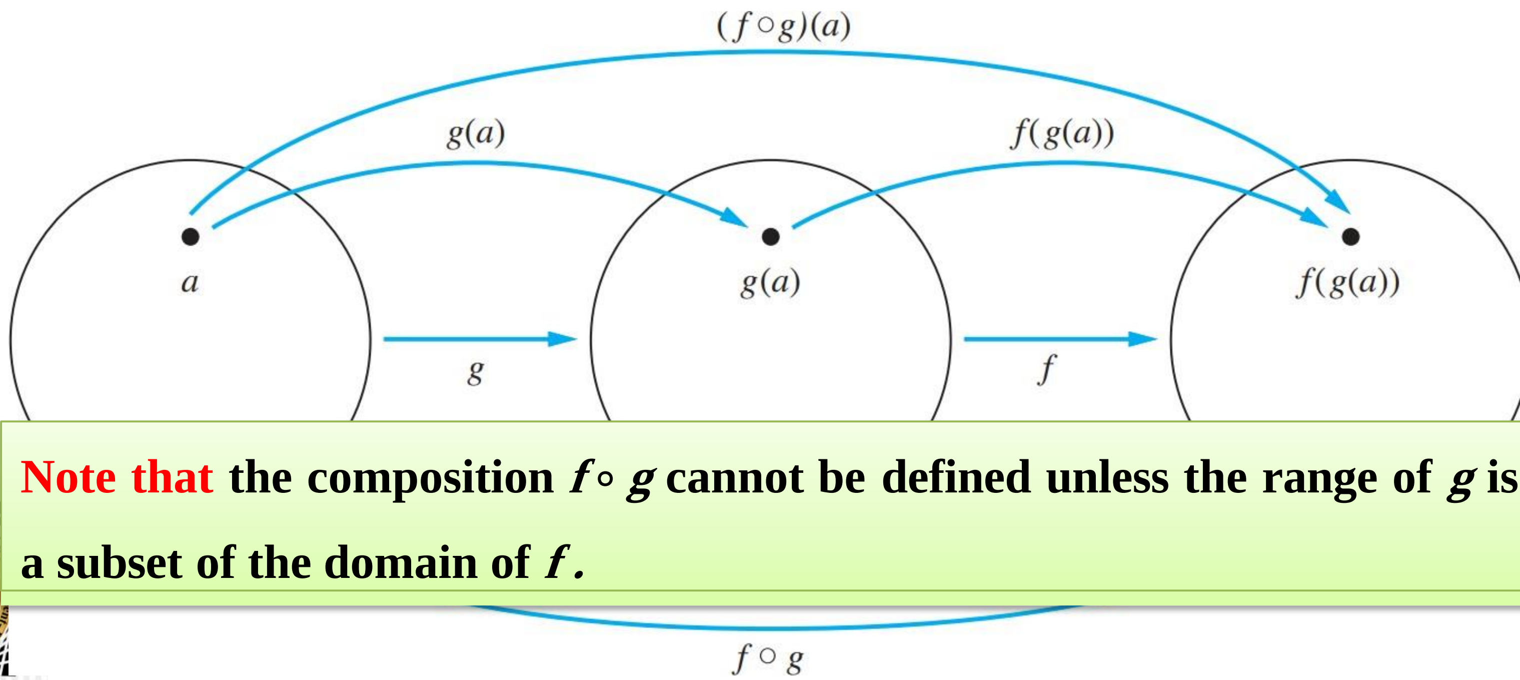
Let g be a function from the set A to the set B and let f be a function from the set B to the set C. The composition of the functions f and g, denoted by $f \circ g$, is defined by $(f \circ g)(a) = f(g(a))$.



Functions (18/21)

Composition of the Functions f and g

Let g be a function from the set A to the set B and let f be a function from the set B to the set C. The composition of the functions f and g, denoted by $f \circ g$, is defined by $(f \circ g)(a) = f(g(a))$.



Note that the composition $f \circ g$ cannot be defined unless the range of g is a subset of the domain of f.

Functions (19/21)

Composition Example 1

Let g be the function from the set $\{a, b, c\}$ to itself such that $g(a) = b$, $g(b) = c$, and $g(c) = a$. Let f be the function from the set $\{a, b, c\}$ to the set $\{1, 2, 3\}$ such that $f(a) = 3$, $f(b) = 2$, and $f(c) = 1$. What is the composition of f and g, and what is the composition of g and f?

Functions (19/21)

Composition Example 1

Let g be the function from the set $\{a, b, c\}$ to itself such that $g(a) = b$, $g(b) = c$, and $g(c) = a$. Let f be the function from the set $\{a, b, c\}$ to the set $\{1, 2, 3\}$ such that $f(a) = 3$, $f(b) = 2$, and $f(c) = 1$.

Answer:

1)The composition of f and g (i.e., $(f \circ g)$)

$(f \circ g)(a) = 2$, $(f \circ g)(b) = 1$, $(f \circ g)(c) = 3$

2) The composition of g and f (i.e., $(g \circ f)$) **cannot be defined** because the range of f is NOT a subset of the domain of g.

Composition Example 2

Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? What is the composition of g and f?

Composition Example 2

Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$.

Answer:

1) The composition of f and g (i.e., $(f \circ g)$)

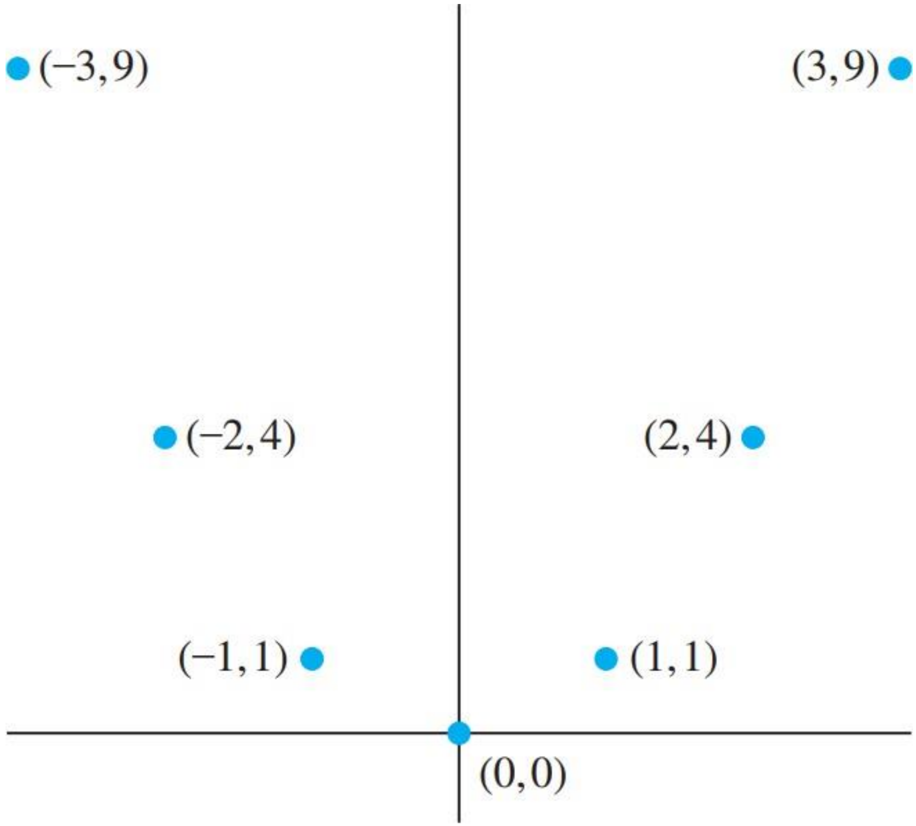
$$(f \circ g)(x) = f\big(g(x)\big) = 2(3x + 2) + 3 = 6x + 7$$

2) The composition of g and f (i.e., $(g \circ f)$)

$$(g \circ f)(x) = g\big(f(x)\big) = 3(2x + 3) + 2 = 6x + 11$$

The Graphs of Functions

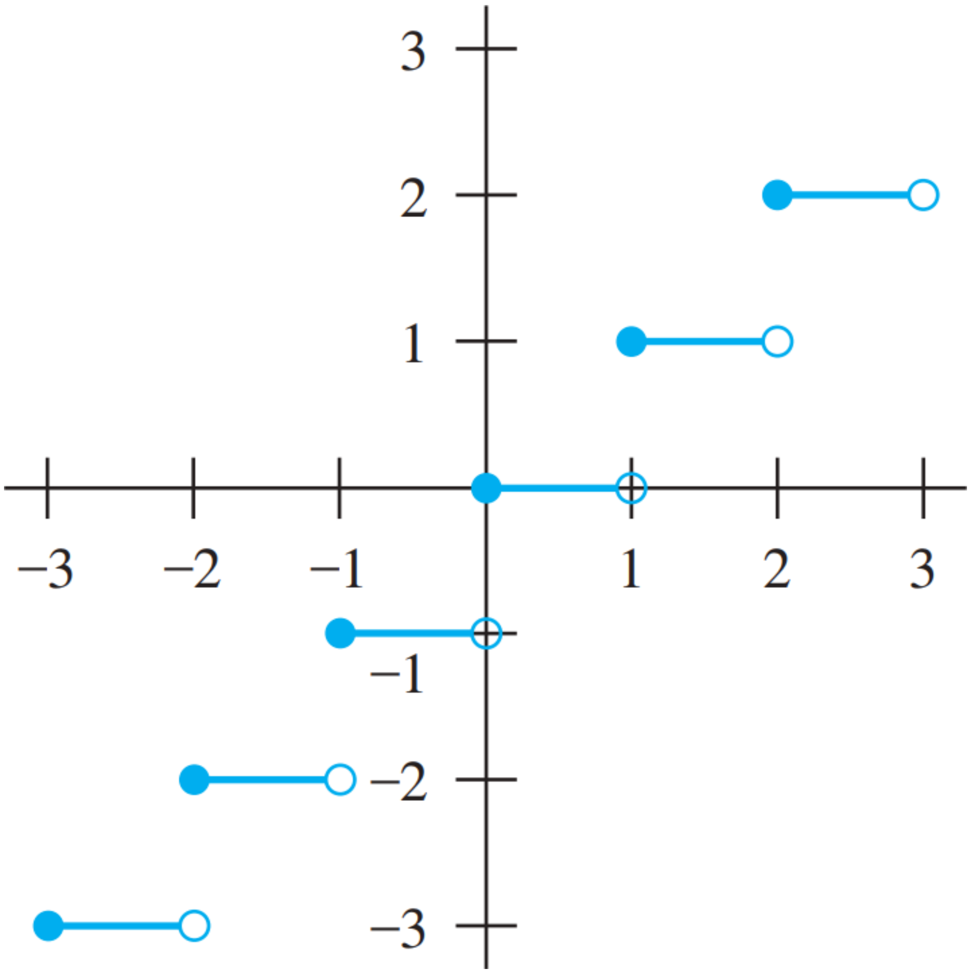
Let f be a function from A to B. The graph of the function f is the set of ordered pairs $\{(a, b) \mid a \in A \text{ and } b \in B\}$.



The graph of $f(x) = x^2$ from Z to Z.

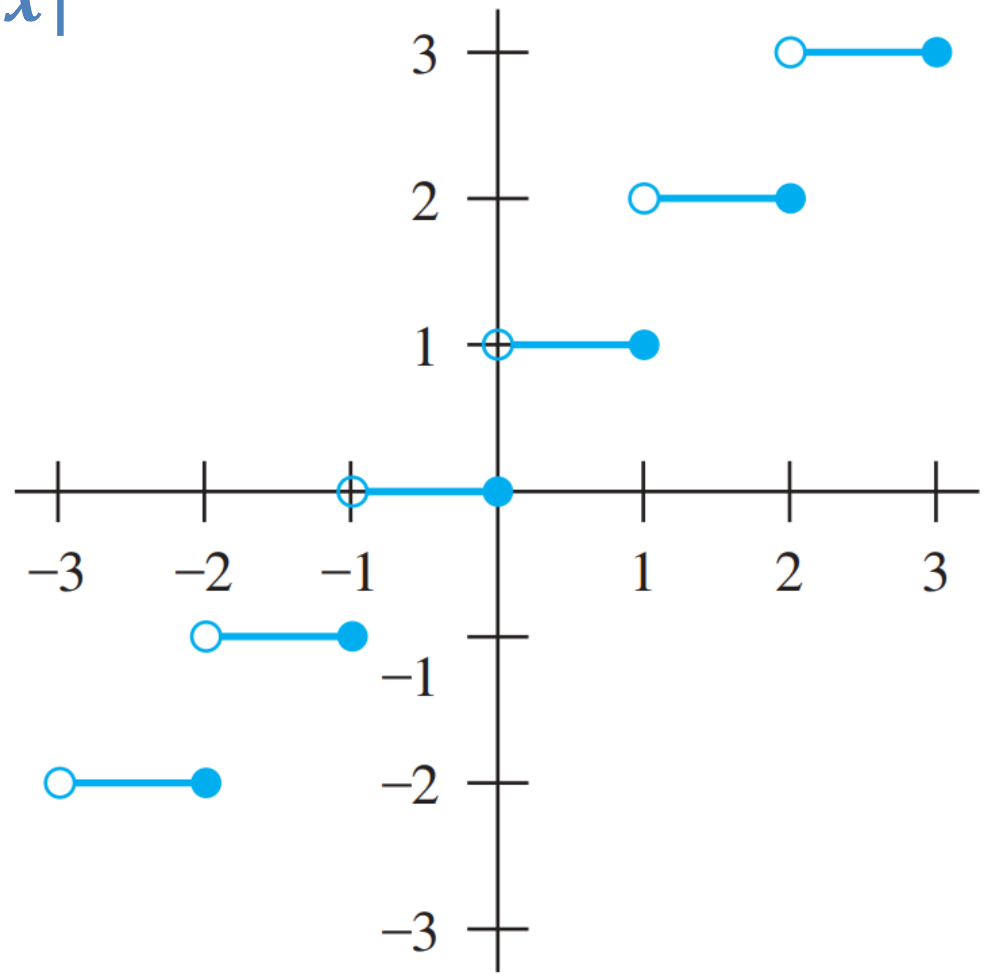
Some Important Functions (1/4)

Floor function $y = \lfloor x \rfloor$



Some Important Functions (2/4)

Ceiling function $y = \lceil x \rceil$



Some Important Functions (3/4)

Useful Properties

$$\lfloor -x \rfloor = -\lceil x \rceil$$

$$\lceil -x \rceil = -\lfloor x \rfloor$$

$$\lfloor x + n \rfloor = \lfloor x \rfloor + n$$

$$\lceil x + n \rceil = \lceil x \rceil + n$$

Some Important Functions (4/4)

Examples

$\lfloor 0.5 \rfloor =$

$\lceil 0.5 \rceil =$

$\lceil 3 \rceil =$

$\lfloor -0.5 \rfloor =$

$\lceil -1.2 \rceil =$

$\lfloor 1.1 \rfloor =$

$\lfloor 0.3 + 2 \rfloor =$

$\left\lceil 1.1 + \lceil 0.5 \rceil \right\rceil =$

Some Important Functions (4/4)

Examples-Answer

$\lfloor 0.5 \rfloor = 0$

$\lceil 0.5 \rceil = 1$

$\lceil 3 \rceil = 3$

$\lfloor -0.5 \rfloor = -\lceil 0.5 \rceil = -1$

$\lceil -1.2 \rceil = -1$

$\lfloor 1.1 \rfloor = 1$

$\lfloor 0.3 + 2 \rfloor = 2$

$\lceil 1.1 + \lceil 0.5 \rceil \rceil = 3$

Sequences (1/13)

Definition

- A sequence is a set of things (usually numbers) that are in order.
  - ➢ For example, 1 , 2, 3, 5, 8 is a sequence with five terms and 1, 3, 9, 27, 81, . . . , 30, . . . is an infinite sequence.
- We use the notation a_n to denote the image of the integer n. We call a_n a term of the sequence.
- We use the notation $\{a_n\}$ to describe the sequence.

$$\{a_n\} = a_1, a_2, a_3, \ldots \qquad \{ \qquad\qquad \}$$

Example

- Consider the sequence $\{a_n\}$, where

$$a_n = \frac{1}{n}$$

The list of the terms of this sequence, beginning with a_1, namely,

$$a_1, a_2, a_3, a_4, \ldots,$$

Starts with

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$$

Sequences (3/13)

Geometric

A *geometric progression* is a sequence of the form

$$a, ar, ar^2, \ldots, ar^n, \ldots$$

where the *initial term a* and the *common ratio r* are real numbers.

$$2, 10, 50, 250, \ldots$$

Sequences (4/13)

Geometric – Example1

$$1, -1, 1, -1, 1, \ldots;$$

$$\{ar^n\}, \qquad n = 0,1,2, \ldots$$

$$a = 1$$

$$r = -1$$

Sequences (5/13)

Geometric – Example2

2, 10, 50, 250, 1250, . . . ;

$\{ar^n\}, \qquad n = 0,1,2, \ldots$

$a = 2$

$r = 5$

Geometric – Example3

$$6, 2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \ldots$$

$\{ar^n\}, \qquad n = 0,1,2,\ldots$

$a = 6$

$r = 1/3$

Geometric – Example4

Find $\boldsymbol{a}, \boldsymbol{r}?\ \{\mathbf{3} * \mathbf{4}^{\boldsymbol{n}}\},\ \ \boldsymbol{n} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \ldots$

$$\{ar^n\}, \qquad n = 0,1,2, \ldots$$

$$a = 3$$

$$r = 4$$

Sequences (8/13)

Geometric – Example5

Find a, r? $\{3 * 4^n\}, \quad n = 1, 2, 3, \ldots$

$$a = 12$$
$$r = 4$$

Sequences (9/13)

Arithmetic

An *arithmetic progression* is a sequence of the form

$$a, a + d, a + 2d, \ldots, a + nd, \ldots$$

where the *initial term a* and the *common difference d* are real numbers.

Sequences (10/13)

Arithmetic – Example1

$$-1, 3, 7, 11, \ldots,$$

$$\{a + nd\}, \qquad n = 0,1,2, \ldots$$

$$a = -1$$
$$d = 4$$

Sequences (11/13)

Arithmetic – Example2

$$7, 4, 1, -2, \ldots$$

$$\{a + nd\}, \qquad n = 0,1,2, \ldots$$

$$a = 7$$

$$d = -3$$

Sequences (12/13)

Notes:

- Are terms obtained from previous terms by adding the same amount or an amount that depends on the position in the sequence?
- Are terms obtained from previous terms by multiplying by a particular amount?
- Are terms obtained by combining previous terms in a certain way?
- Are there cycles among the terms?

Fibonacci Sequence

The *Fibonacci sequence*, $f_0, f_1, f_2, \ldots,$
is defined by the initial conditions $f_0 = 0, f_1 = 1$, and
the recurrence relation

$$f_n = f_{n-1} + f_{n-2}$$

for $n = 2, 3, 4, \ldots$.

0, 1, 1, 2, 3, 5, 8, ...

Summations (1/8)

Next, we introduce **summation notation.**
We begin by describing the notation used to express the sum of the terms

$$a_m, a_{m+1}, \ldots, a_n$$

from the sequence $\{a_n\}$. We use the notation

$$\sum_{j=m}^{n} a_j, \qquad \sum\nolimits_{j=m}^{n} a_j, \qquad \text{or} \qquad \sum\nolimits_{m \le j \le n} a_j$$

(read as the sum from $j = m$ to $j = n$ of a_j)

to represent

Here, the variable j is called the **index of summation**

$$a_m + a_{m+1} + \cdots + a_n.$$

Summations (1/8)

$$\sum_{j=m}^{n} a_j = \sum_{i=m}^{n} a_i = \sum_{k=m}^{n} a_k$$

Here, the index of summation runs through all integers starting with its **lower limit** m and ending with its **upper limit** n. A large uppercase Greek letter sigma, $\sum$, is used to denote summation.

Summations (2/8)

Example 1

Express the sum of the first 100 terms of the sequence $\{a_n\}$, where $a_n = 1/n$ for $n = 1, 2, 3, \ldots$

Example 1

Express the sum of the first 100 terms of the sequence $\{a_n\}$, where $a_n = 1/n$ for $n = 1, 2, 3, \ldots$

Answer

$$\sum_{n=1}^{100} 1/n$$

Summations (4/8)

Example 2

What is the value of $\sum_{j=1}^{5} j^2$?

Summations (4/8)

Example 2

What is the value of $\sum_{j=1}^{5} j^2$?

Answer

$$\sum_{j=1}^{5} j^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$= 1 + 4 + 9 + 16 + 25$$

$$= 55.$$

Summations (5/8)

Example 3

What is the value of $\sum_{s \in \{0,2,4\}} s$?

Summations (5/8)

Example 3

What is the value of $\sum_{s \in \{0,2,4\}} s$?

$$\sum_{s \in \{0,2,4\}} s = 0 + 2 + 4 = 6.$$

Summations (6/8)

Example 4

Suppose we have the sum

$$\sum_{j=1}^{5} j^2$$

but want the index of summation to run between 0 and 4

$$\sum_{j=1}^{5} j^2 = \sum_{k=0}^{4} (k+1)^2$$

It is easily checked that both sums are $1 + 4 + 9 + 16 + 25 = 55$.

Double Summation

Find

$$\sum_{i=1}^{4}\sum_{j=1}^{3} ij$$

Summations (8/8)

Double Summation

Find

$$\sum_{i=1}^{4}\sum_{j=1}^{3} ij = \sum_{i=1}^{4}(i + 2i + 3i)$$

$$= \sum_{i=1}^{4} 6i$$

$$= 6 + 12 + 18 + 24 = 60.$$

Matrices (1/14)

Definition:

A ***matrix*** is a rectangular array of numbers. A matrix with m rows and n columns is called an $m \times n$ matrix. A matrix with the same number of rows as columns is called ***square***.

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$$

Matrices (1/14)

Definition:

A ***matrix*** is a rectangular array of numbers. A matrix with m rows and n columns is called an $m \times n$ matrix. A matrix with the same number of rows as columns is called ***square***.

$$\text{The matrix } \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix} \text{ is a } 3 \times 2 \text{ matrix.}$$

Matrices (2/14)

$m \times n$ matrix

Let m and n be positive integers and let

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

Matrices (3/14)

$m \times n$ matrix

The (2, 1)th ***element*** or ***entry*** of **A** is the element $\boldsymbol{a_{21}}$, means 2nd *row* and 1st *column* of **A**.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

Matrix Arithmetic (Sum.)

Let **A** = $[a_{ij}]$ and **B** = $[b_{ij}]$ be $m \times n$ matrices.
The sum of **A** and **B**, denoted by **A** + **B**, is the $m \times n$ matrix that has $a_{ij} + b_{ij}$ as its (i, j)th element. In other words, **A** + **B** = $[a_{ij} + b_{ij}]$.

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 2 & -3 \\ 3 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 4 & -1 \\ 1 & -3 & 0 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -2 \\ 3 & -1 & -3 \\ 2 & 5 & 2 \end{bmatrix}.$$

A **B** **A+B**

Matrices (4/14)

Matrix Arithmetic (Sum.)

Note: matrices of *different sizes* can **not** be added.

Let $\mathbf{A} = [a_{ij}]$ and $\mathbf{B} = [b_{ij}]$ be $m \times n$ matrices.

The sum of **A** and **B**, denoted by $\mathbf{A} + \mathbf{B}$, is the $m \times n$ matrix that has $a_{ij} + b_{ij}$ as its (i, j)th element. In other words, $\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$.

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 2 & -3 \\ 3 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 4 & -1 \\ 1 & -3 & 0 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -2 \\ 3 & -1 & -3 \\ 2 & 5 & 2 \end{bmatrix}.$$

A **B** **A+B**

Matrices (5/14)

Matrix Arithmetic (Product/Multiplication)

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ a_{21} & a_{22} & \cdots & a_{2k} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ik} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mk} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2j} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{k1} & b_{k2} & \cdots & b_{kj} & \cdots & b_{kn} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & c_{ij} & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}$$

$$\mathbf{A}_{mk} \qquad \mathbf{B}_{kn} \qquad \mathbf{AB} = \mathbf{C}_{mn}$$

Matrix Arithmetic (Product/Multiplication)

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ a_{21} & a_{22} & \cdots & a_{2k} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ik} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mk} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2j} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{k1} & b_{k2} & \cdots & b_{kj} & \cdots & b_{kn} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & c_{ij} & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}$$

$\mathbf{A}_{mk}$ $\quad$ $\mathbf{B}_{kn}$ $\quad$ $\mathbf{AB} = \mathbf{C}_{mn}$

Matrices (6/14)

Example1 (1/2)

$$\mathbf{A}_{3\times3} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & -1 \end{bmatrix}_{3\times3} \qquad \mathbf{M}_{3\times2} = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 1 & 1 \end{bmatrix}_{3\times2}$$

$$\mathbf{A}_{3\times3} \times \mathbf{M}_{3\times2} = \mathbf{B}_{3\times2}$$

Rows: 1, 2, 3; Columns: 1, 2

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \quad & \quad \end{bmatrix}$$

Example1 (2/2)

$$\mathbf{A}_{3\times3} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & -1 \end{bmatrix}_{3\times3} \qquad \mathbf{M}_{3\times2} = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 1 & 1 \end{bmatrix}_{3\times2}$$

$$\mathbf{A}_{3\times3} \times \mathbf{M}_{3\times2} = \mathbf{B}_{3\times2}$$

$$\boldsymbol{a_{11} = 6}$$
$$= (\mathbf{1 \times 1 + 1 \times 3 + 2 \times 1})$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 10 & 3 \\ 9 & -2 \end{bmatrix}$$

Rows: 1, 2, 3 — Columns: 1, 2

Matrices (6/14)

Example1 (2/2)

$$\mathbf{A}_{3\times3} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & -1 \end{bmatrix}_{3\times3} \qquad \mathbf{M}_{3\times2} = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 1 & 1 \end{bmatrix}_{3\times2}$$

$$\mathbf{A}_{3\times3} \times \mathbf{M}_{3\times2} = \mathbf{B}_{3\times2}$$

$$\boldsymbol{a_{31} = 9}$$
$$\boldsymbol{= (1\times1 + 3\times3 + (-1)\times1)}$$

Rows: 1, 2, 3; Columns: 1, 2

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 10 & 3 \\ 9 & -2 \end{bmatrix}$$

Matrices (7/14)

Example2 (1/2)

Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

Does $\mathbf{AB} = \mathbf{BA}$?

Example2 (2/2)

Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

Solution: We find that

$$\mathbf{AB} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{BA} = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}.$$

Hence, $\mathbf{AB} \neq \mathbf{BA}$.

Identity matrix ($\mathbf{I}_n$)

The *identity matrix* of order n is the $n \times n$ matrix $\mathbf{I}_n = [\delta_{ij}]$, where $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$.

$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

A is an $m \times n$ matrix, we have

$$\mathbf{A}\mathbf{I}_n = \mathbf{I}_m\mathbf{A} = \mathbf{A}.$$

Matrices (9/14)

Powers of square matrices (A^r)

When $\mathbf{A}$ is an $n \times n$ matrix, we have

$$\mathbf{A}^0 = \mathbf{I}_n, \qquad \mathbf{A}^r = \underbrace{\mathbf{AAA} \cdots \mathbf{A}}_{r \text{ times}}.$$

Matrices (10/14)

Transpose of A (A^t)

Interchanging the rows and columns of **A**

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad \mathbf{A}^t = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

Matrices (10/14)

Transpose of A (A^t)

Interchanging the rows and columns of **A**

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

A

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$\mathbf{A}^t$

Matrices (11/14)

Symmetric

A square matrix $\mathbf{A}$ is called *symmetric* if $\mathbf{A} = \mathbf{A}^t$

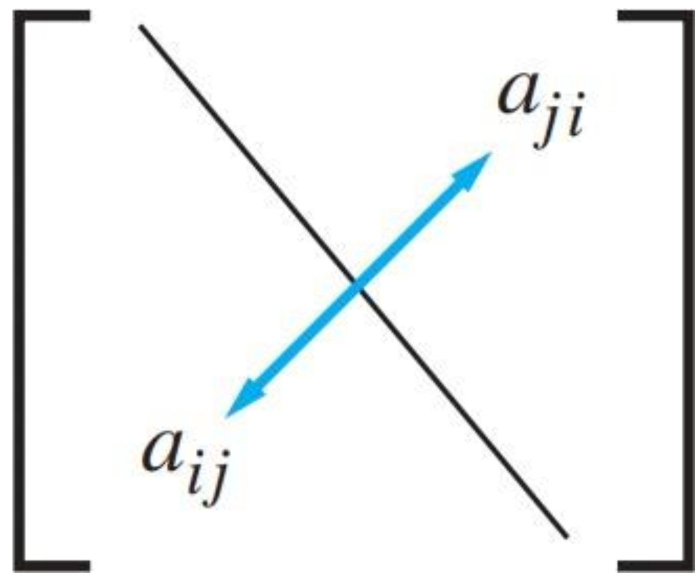
$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\mathbf{A}$ $\qquad$ $\mathbf{A}^t$

Matrices (11/14)

Symmetric

A square matrix $\mathbf{A}$ is called *symmetric* if $\mathbf{A} = \mathbf{A}^t$



Zero–One Matrices

A matrix all of whose entries are either **0** or **1**

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$$

join and *meet* (Zero–One Matrices)

meet

$$b_1 \wedge b_2 = \begin{cases} 1 & \text{if } b_1 = b_2 = 1 \\ 0 & \text{otherwise,} \end{cases}$$

$$b_1 \vee b_2 = \begin{cases} 1 & \text{if } b_1 = 1 \text{ or } b_2 = 1 \\ 0 & \text{otherwise.} \end{cases}$$

join

Example (1/3)

Find the join and meet of the zero–one matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$$

Matrices (14/14)

Example (2/3)

Find the join and meet of the zero–one matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$$

Solution: We find that the join of **A** and **B** is

$$\mathbf{A} \vee \mathbf{B} = \begin{bmatrix} 1 \vee 0 & 0 \vee 1 & 1 \vee 0 \\ 0 \vee 1 & 1 \vee 1 & 0 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Matrices (14/14)

Example (3/3)

Find the join and meet of the zero–one matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$$

Solution:

The meet of **A** and **B** is

$$\mathbf{A} \wedge \mathbf{B} = \begin{bmatrix} 1 \wedge 0 & 0 \wedge 1 & 1 \wedge 0 \\ 0 \wedge 1 & 1 \wedge 1 & 0 \wedge 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

شكرًا لحسن استماعكم

Thank you