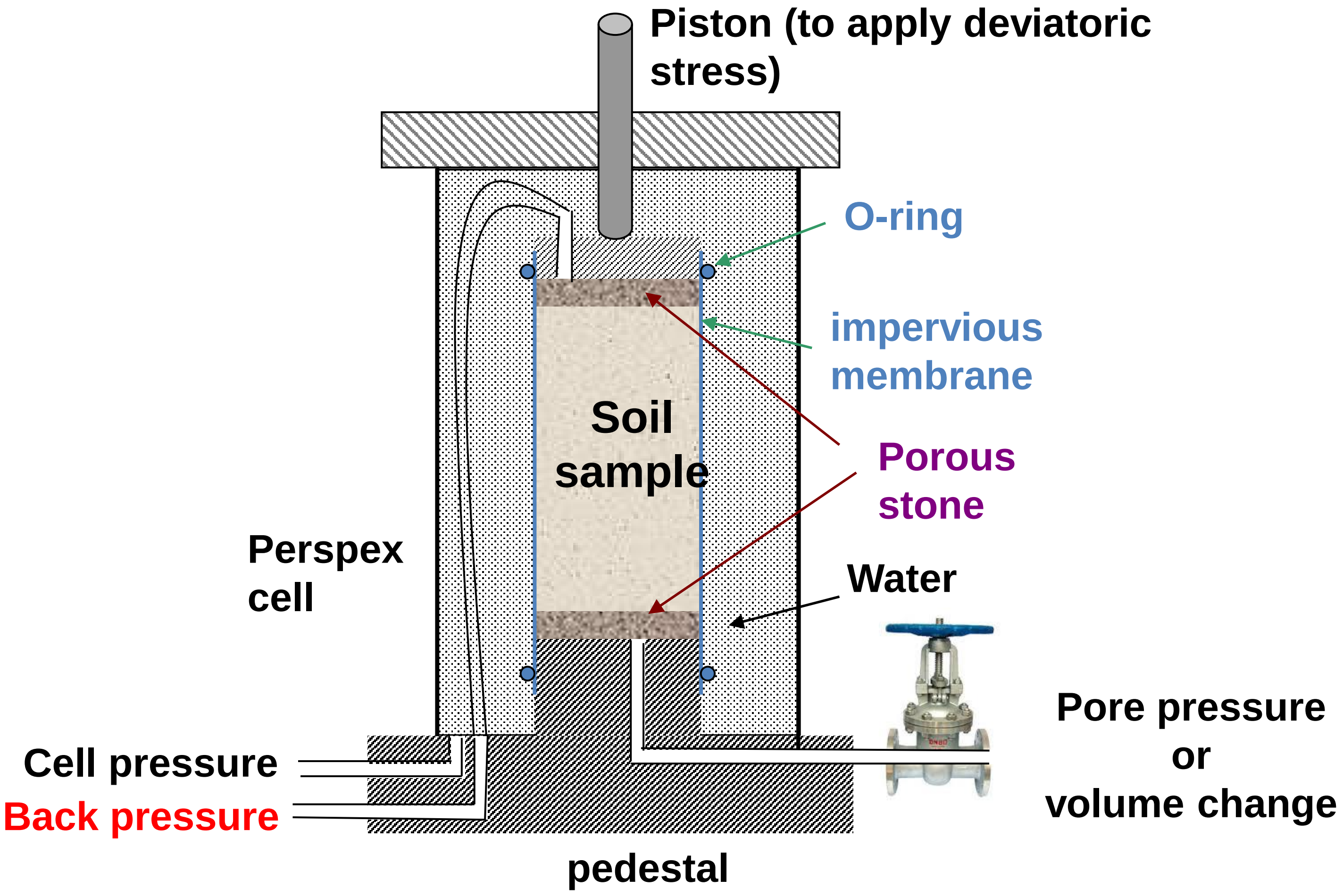
Shear Strength of Soil
Chapter 12

Triaxial Shear Test Device



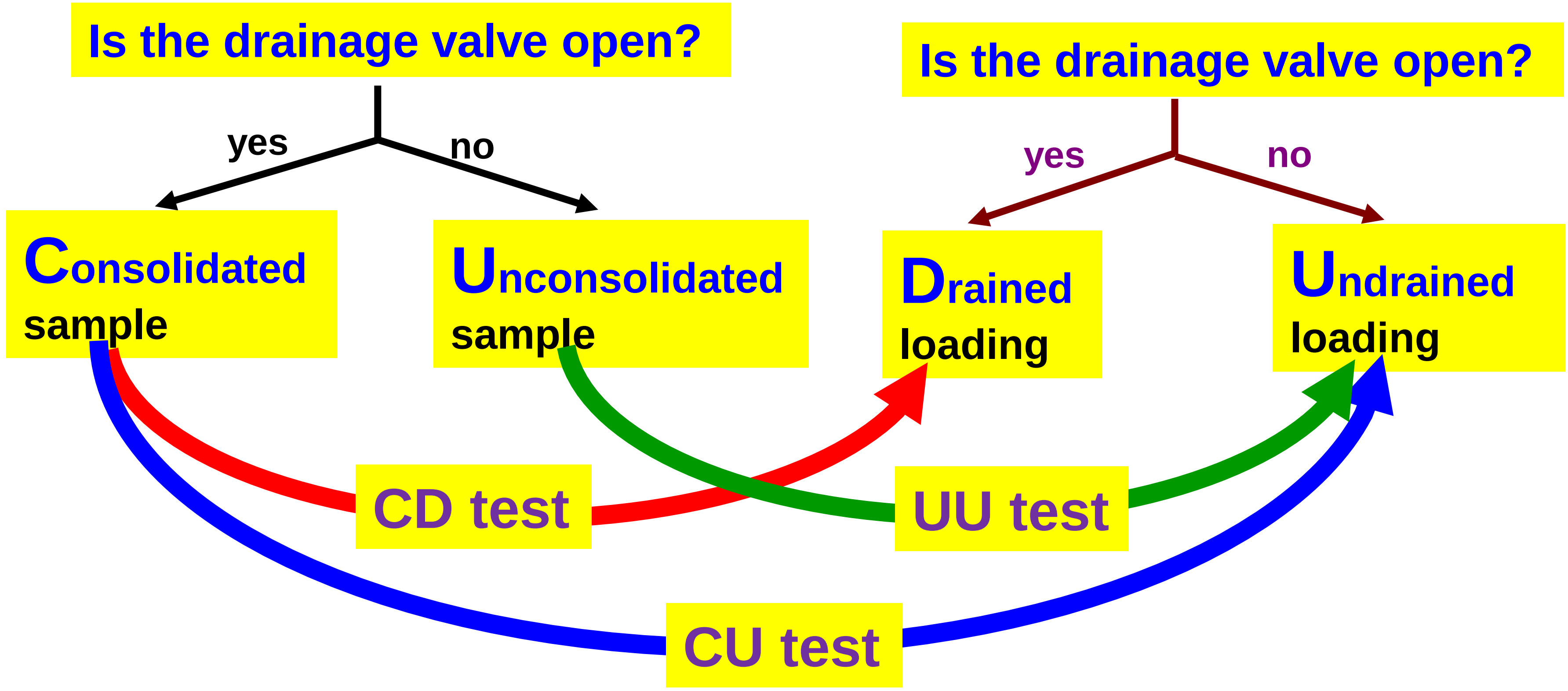
Types of Triaxial Test

Step 1

Under Confining (cell) pressure σ_3

Step 2

Shearing (loading)



Is the drainage valve open?

yes

no

Consolidated sample

Unconsolidated sample

Is the drainage valve open?

yes

no

Drained loading

Undrained loading

CD test

UU test

CU test

II. Consolidated Unrained Test (CU Test)

- As the name implies, the test specimen is first consolidated (drainage valves open) under the desired consolidation stresses.

- After consolidation is complete, the drainage valves are closed, and the specimen is loaded to failure in undrained shear.

- Often, the pore water pressures developed during shear are measured, and both the total and effective stresses may be calculated during shear and at failure. Thus this test can either be a total or an effective stress test.

- This test is sometimes called the *R-test*.

- The CU test is the most common type of triaxial test.

II. Consolidated Unrained Test (CU Test)

- CD tests on clay soils take considerable time. For this reason, CU tests can be conducted on such soils with pore pressure measurements to obtain drained shear strength parameters.

- Because drainage is not allowed in these tests during the application of deviator stress, they can be performed quickly.

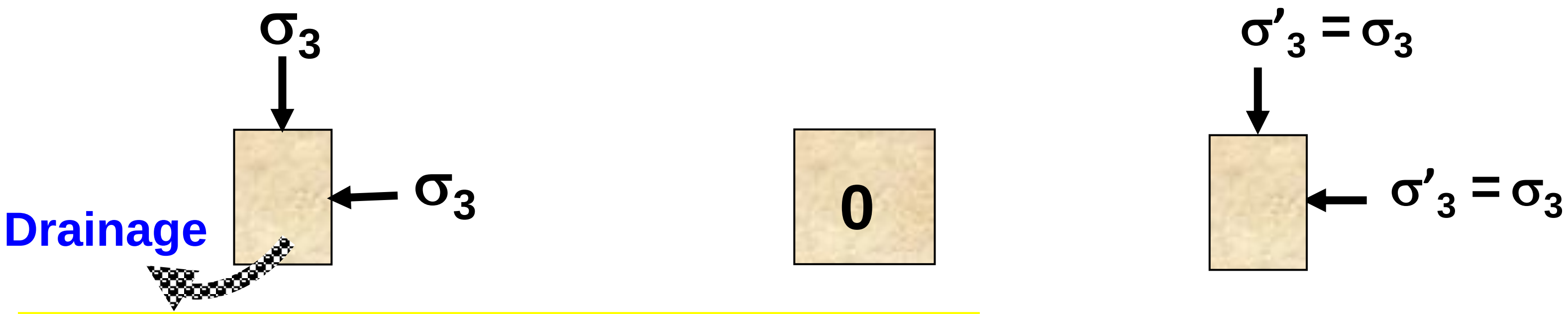
- Like the CD test, the axial stress can be increased incrementally or at a constant rate of strain..

- Positive pore pressures occur in normally consolidated clays and negative pore pressures occur in overconsolidated clays.

Stress conditions for the consolidated undrained test

Total, σ = **Neutral, u** + **Effective, σ'**

Step 1: At the end of consolidation



σ_3

Drainage

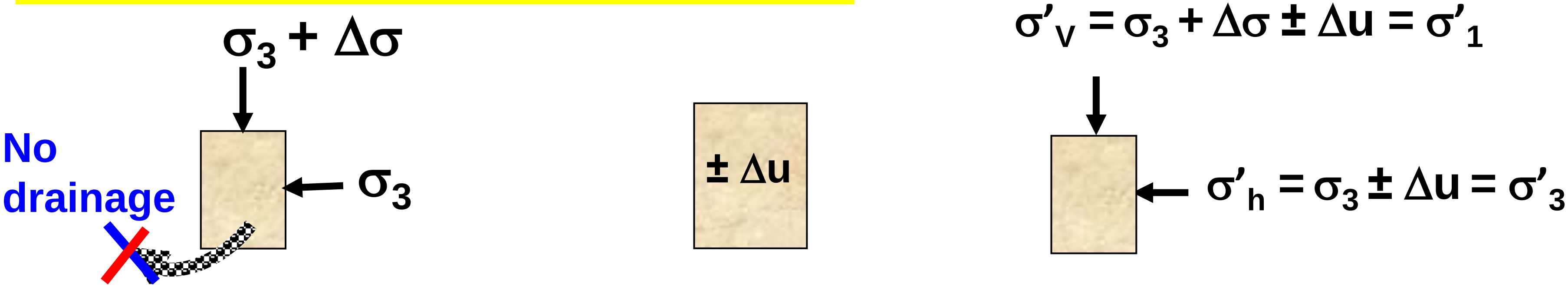
σ_3

0

$\sigma'_3 = \sigma_3$

$\sigma'_3 = \sigma_3$

Step 2: During axial stress increase



$\sigma_3 + \Delta\sigma$

No drainage

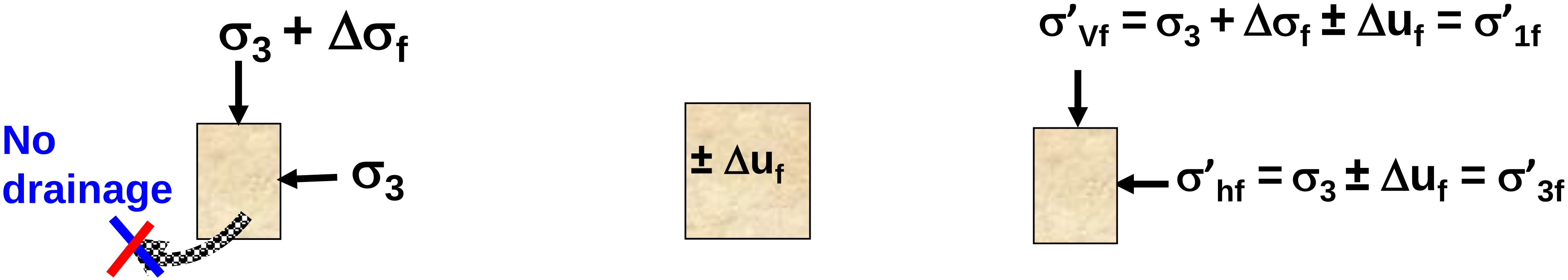
σ_3

$\pm \Delta u$

$\sigma'_V = \sigma_3 + \Delta\sigma \pm \Delta u = \sigma'_1$

$\sigma'_h = \sigma_3 \pm \Delta u = \sigma'_3$

Step 3: At failure



$\sigma_3 + \Delta\sigma_f$

No drainage

σ_3

$\pm \Delta u_f$

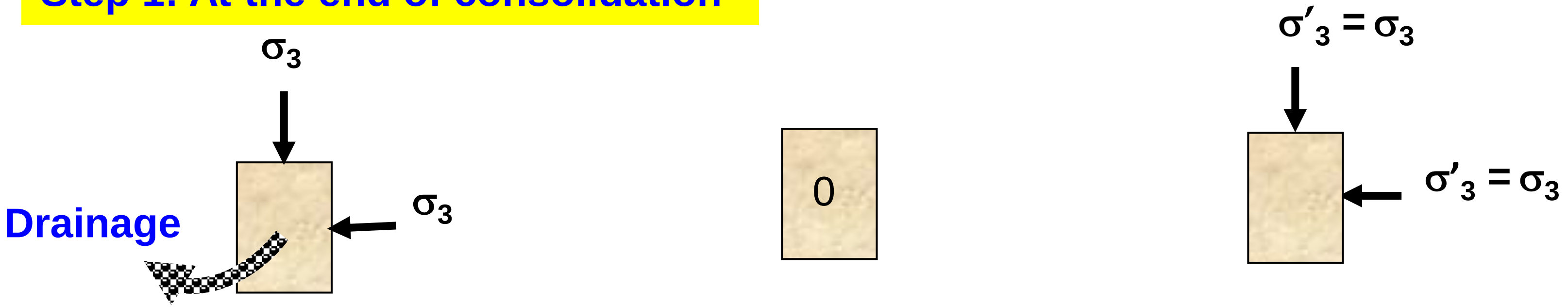
$\sigma'_{Vf} = \sigma_3 + \Delta\sigma_f \pm \Delta u_f = \sigma'_{1f}$

$\sigma'_{hf} = \sigma_3 \pm \Delta u_f = \sigma'_{3f}$

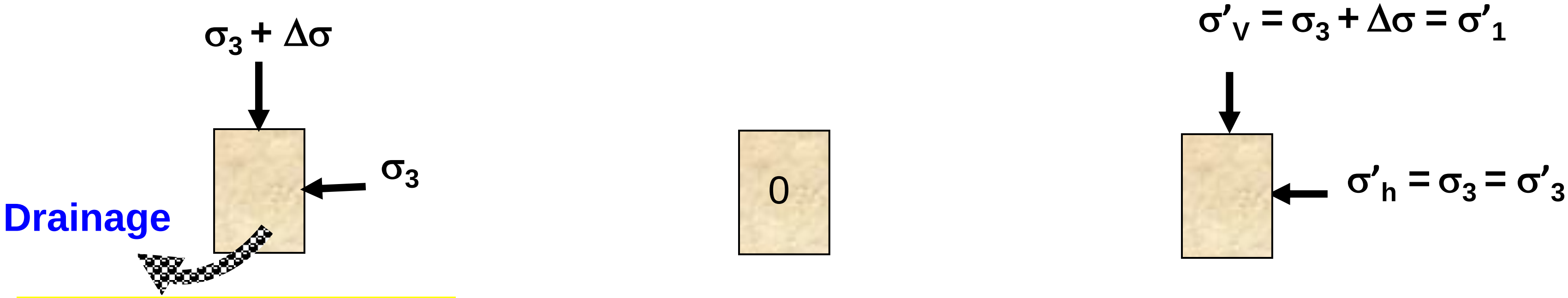
Stress conditions for the consolidated drained test

Total, σ = **Neutral, u** + **Effective, σ'**

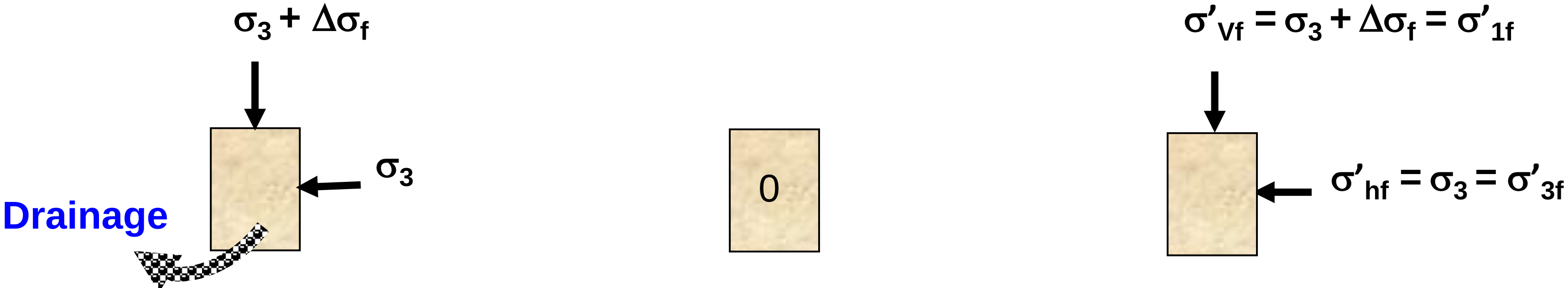
Step 1: At the end of consolidation



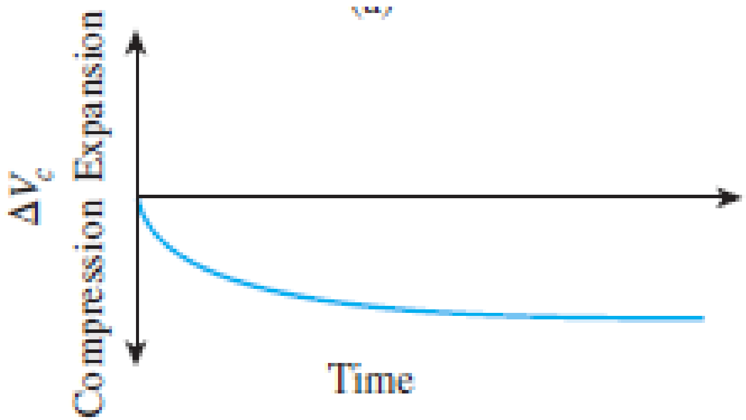
Step 2: During axial stress increase



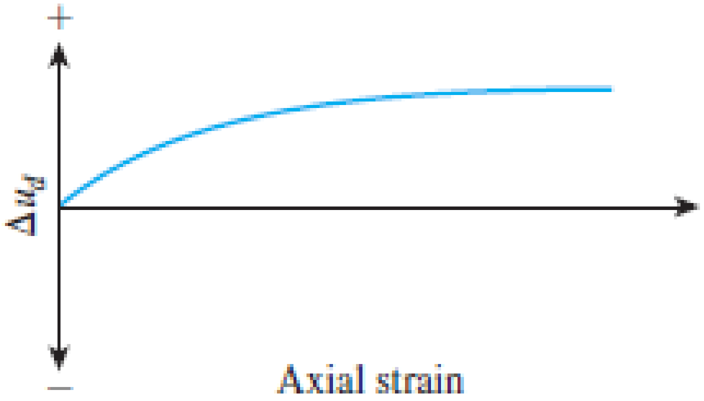
Step 3: At failure



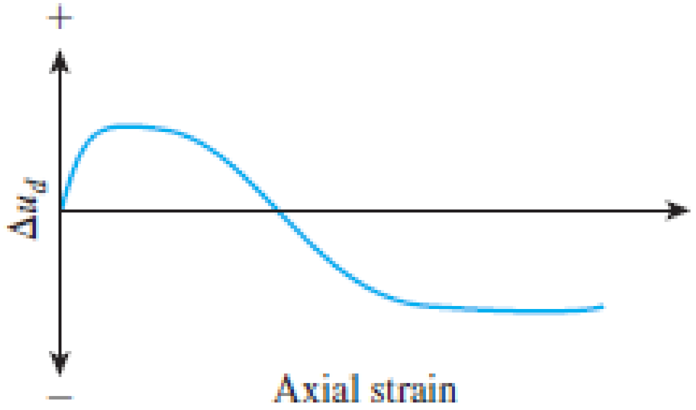
Consolidated Unrained Test (CU Test)



volume change in specimen caused by confining pressure

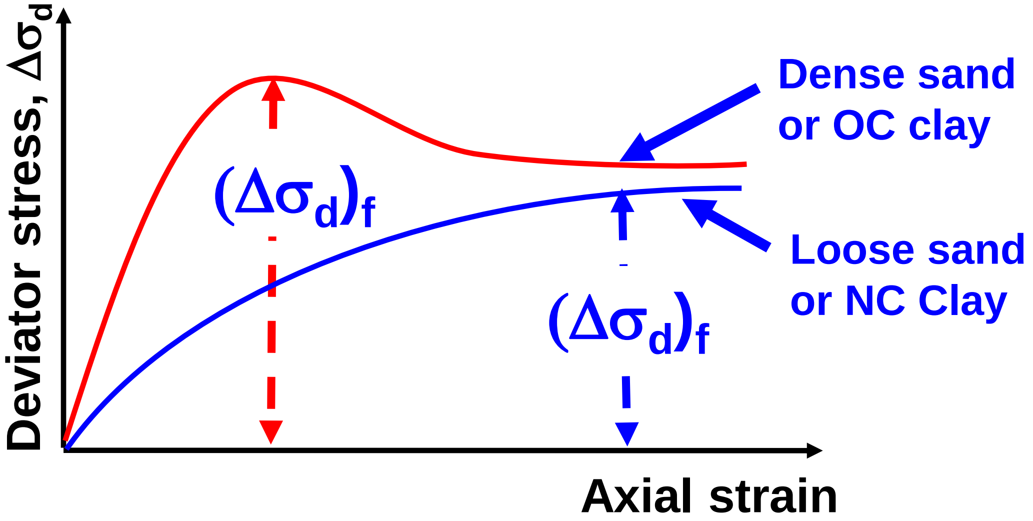


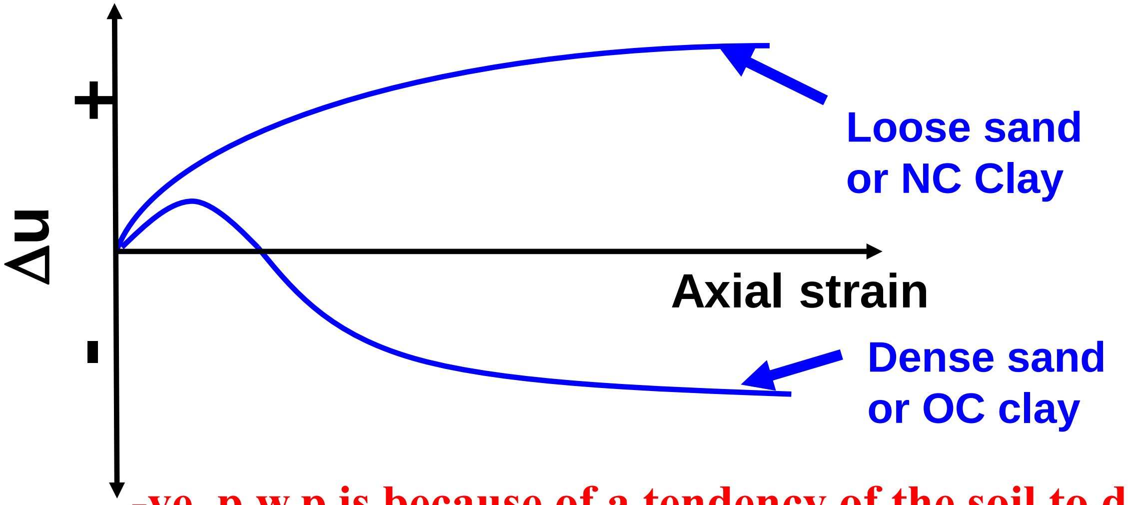
Variation of pore water pressure with axial strain for loose sand and normally consolidated clay



Variation of pore water pressure with axial strain for dense sand and overconsolidated clay.

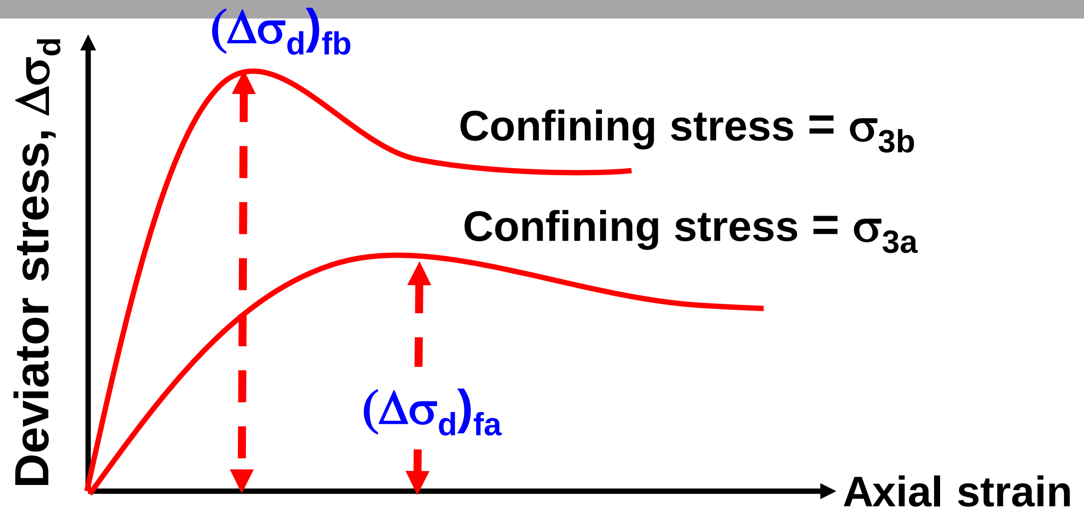
Stress-strain relationship during shearing (CU Test)



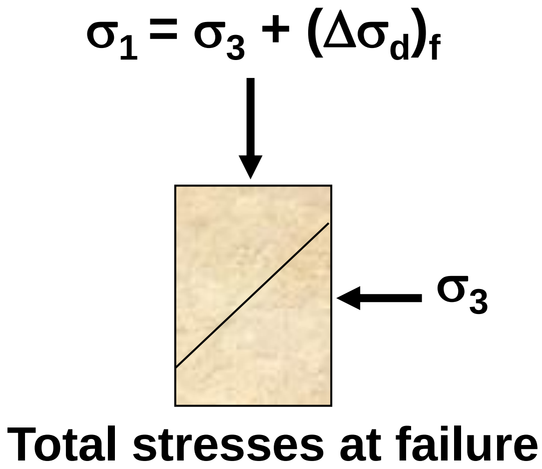


-ve p.w.p is because of a tendency of the soil to dilate.

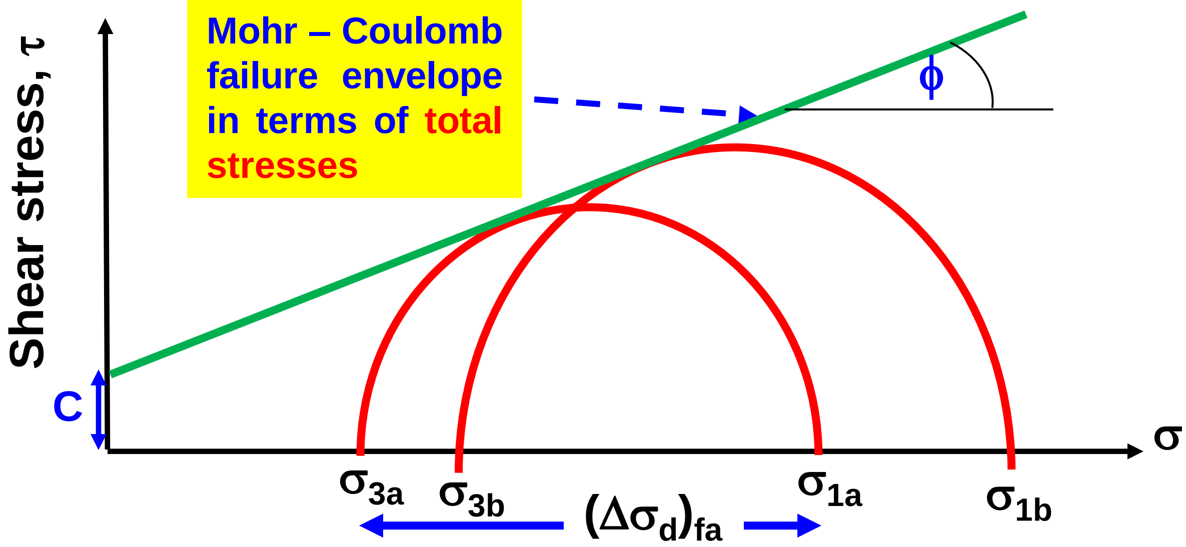
Shear Strength Parameters C and ϕ



$\sigma_1 = \sigma_3 + (\Delta\sigma_d)_f$



Total stresses at failure

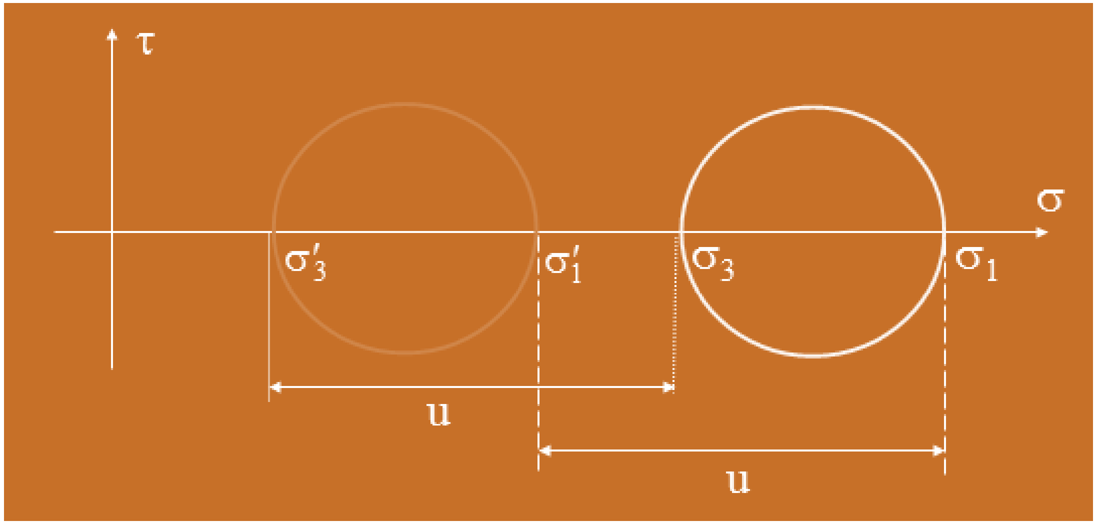


Mohr – Coulomb failure envelope in terms of total stresses

C and ϕ are total strength parameters (Sometimes called C_{cu} and ϕ_{cu} which are consolidated-undrained cohesion and angle of shearing resistance, respectively).

Effective and total stress Mohr circles

- Unlike the consolidated-drained test, the total and effective principal stresses are not the same in the consolidated-undrained test.
- However, since we can get both the total and effective stress circles at failure for a CU test when we measure the induced pore water pressures, it is possible to define the Mohr-Coulomb failure envelopes in terms of both total and effective stresses.



- For any point in the soil a total and an effective stress Mohr circle can be drawn. These are the same size with

$$\sigma_1' - \sigma_3' = \sigma_1 - \sigma_3$$

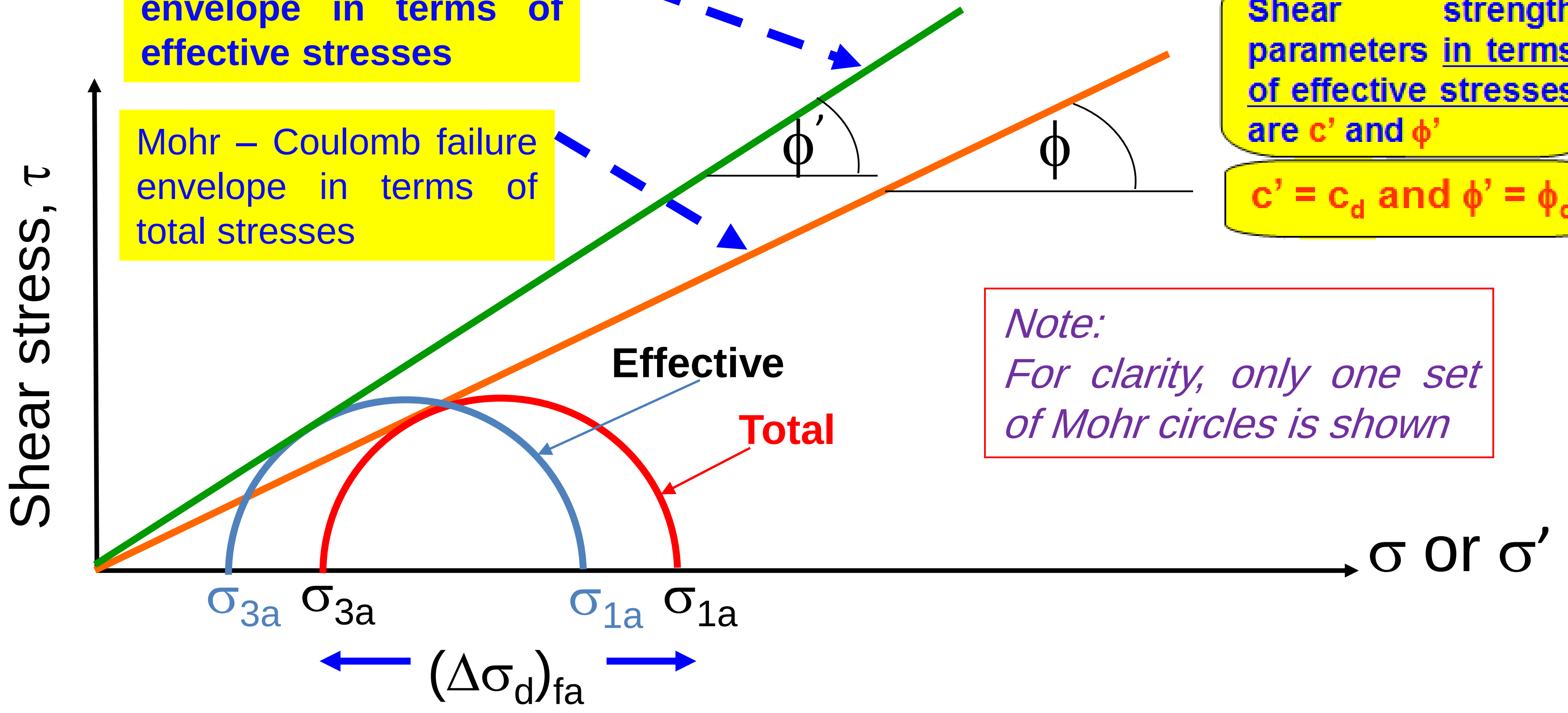
- The two circles are displaced horizontally by the pore pressure, u.

Failure envelopes for sand and NC Clay, C_{cu} and C' = 0

One CU test would be sufficient to determine ϕ ($=\phi_{cu}$) and ϕ' ($= \phi_d$) of sand or NC clay

Mohr – Coulomb failure envelope in terms of effective stresses

Mohr – Coulomb failure envelope in terms of total stresses



Shear strength parameters in terms of total stresses are c_{cu} and ϕ_{cu}

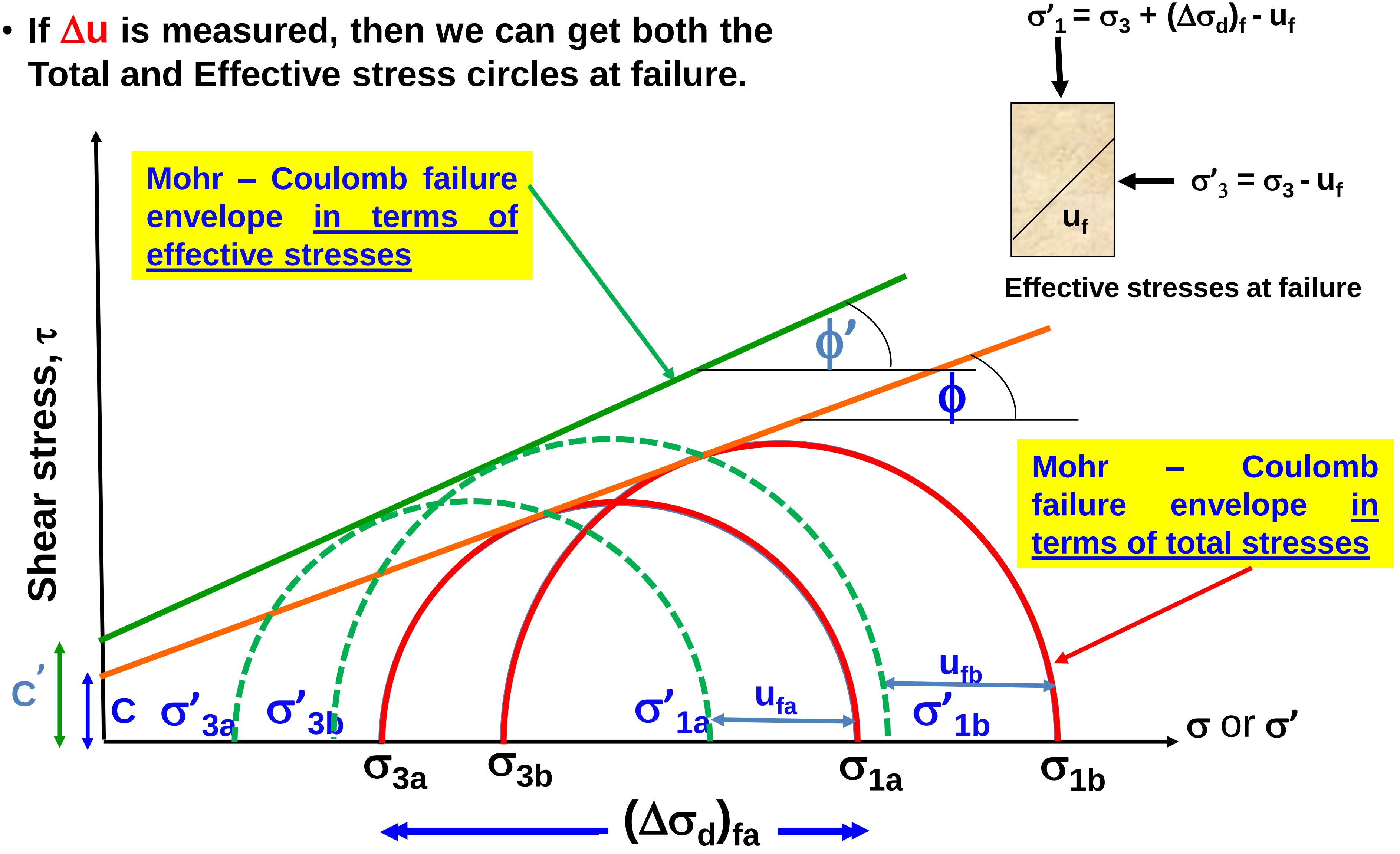
Shear strength parameters in terms of effective stresses are c' and ϕ'

$c' = c_d$ and $\phi' = \phi_d$

Note:
For clarity, only one set of Mohr circles is shown

Shear Strength Parameters C and ϕ

- If Δu is measured, then we can get both the Total and Effective stress circles at failure.

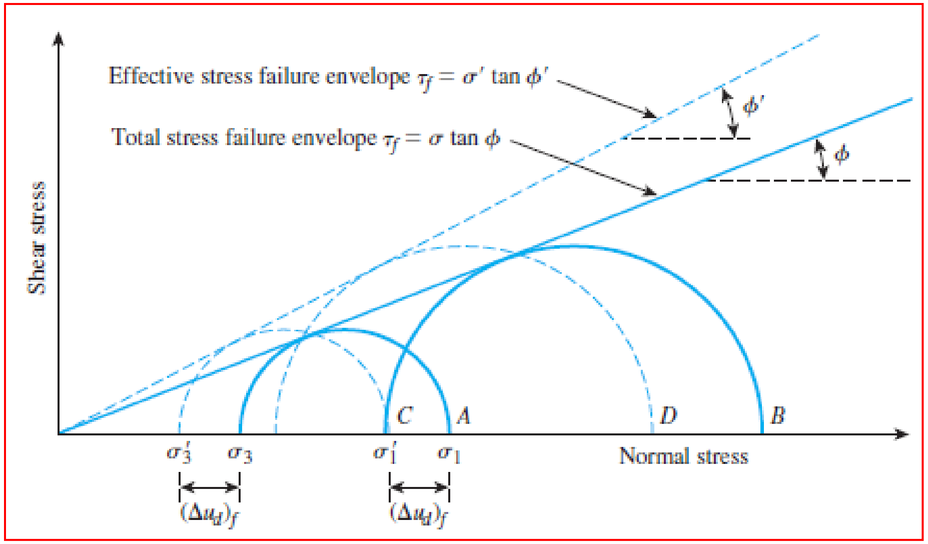


Mohr – Coulomb failure envelope in terms of effective stresses

Mohr – Coulomb failure envelope in terms of total stresses

Effective stresses at failure

Failure envelopes for NC and OC Clays

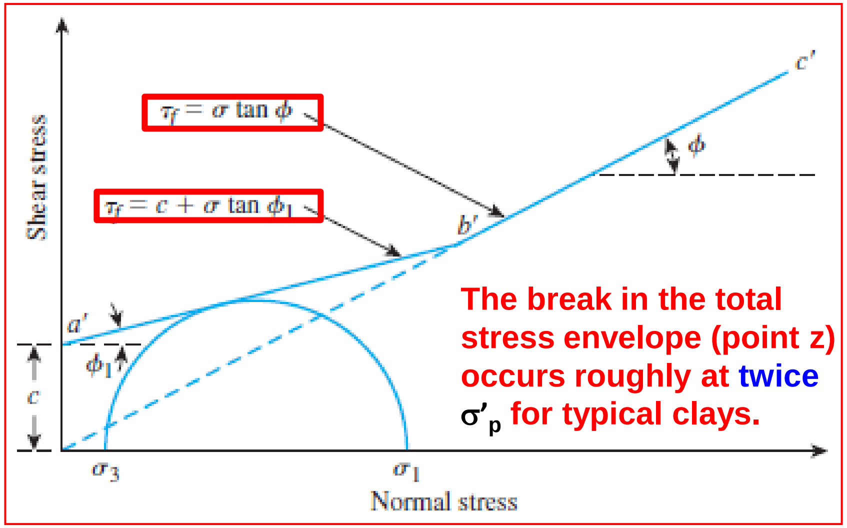


NC

(Same as that obtained from CD test)

OC

Typically, the complete Mohr failure envelopes are determined by tests on several specimens consolidated over the working stress range of the field problem.



The break in the total stress envelope (point z) occurs roughly at twice σ'_p for typical clays.

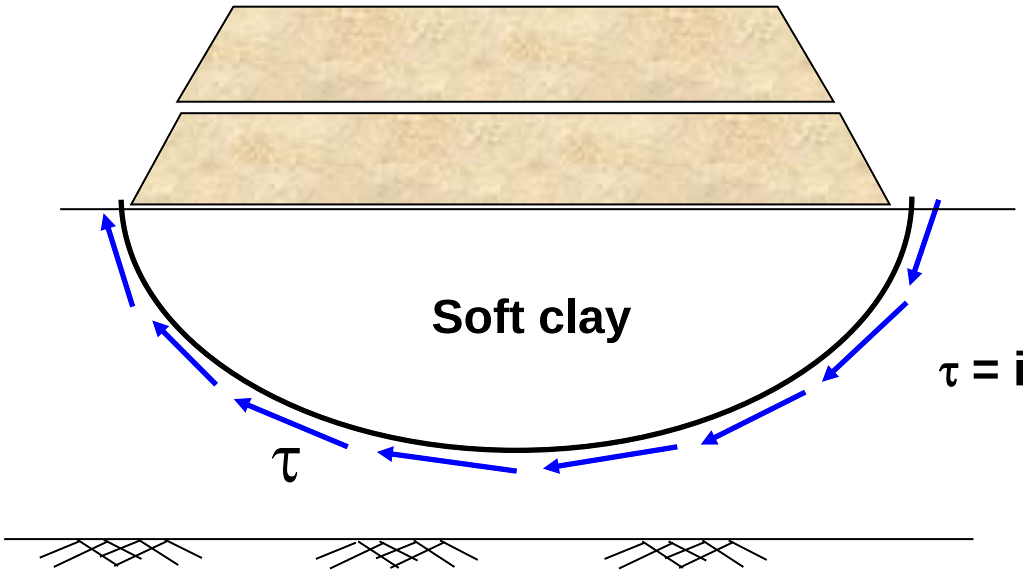
Note on CU test

- Shear Strength parameters in terms of total stresses are C and ϕ or C_{cu} and ϕ_{cu}
- Shear Strength parameters in terms of effective stresses are C' and ϕ'.
- If the specimen tends to contract or consolidate during shear, then the induced p.w.p. is +ve. This is in loose sand and N.C. clay.
- If the specimen tends to EXPAND or swell during shear, the Δu decreases and may be –ve. This occurs in Dense sand and OC clay.
- Since the shear strength is controlled by the effective stress in the specimen at failure, the Mohr failure hypothesis is valid in terms of *effective stresses only*. Hence, the point of tangency of effective M-C failure envelope to the Mohr circle of effective stress is used to define θ_f .
- It is tacitly assumed that the Mohr-Coulomb strength parameters in terms of effective stresses determined by CU tests with pore pressure measurements would be the same as those determined by CD tests. We used the same symbols, C' and ϕ' for the parameters determined both ways.

Some practical applications of CU analysis for clays

- ○ **CU strengths are used for stability problems where the soils have first become fully consolidated and are at equilibrium with the existing stress system**
- ○ **Then, for some reasons, additional stresses are applied quickly with no drainage occurring.**
- ○ **Practical examples include**

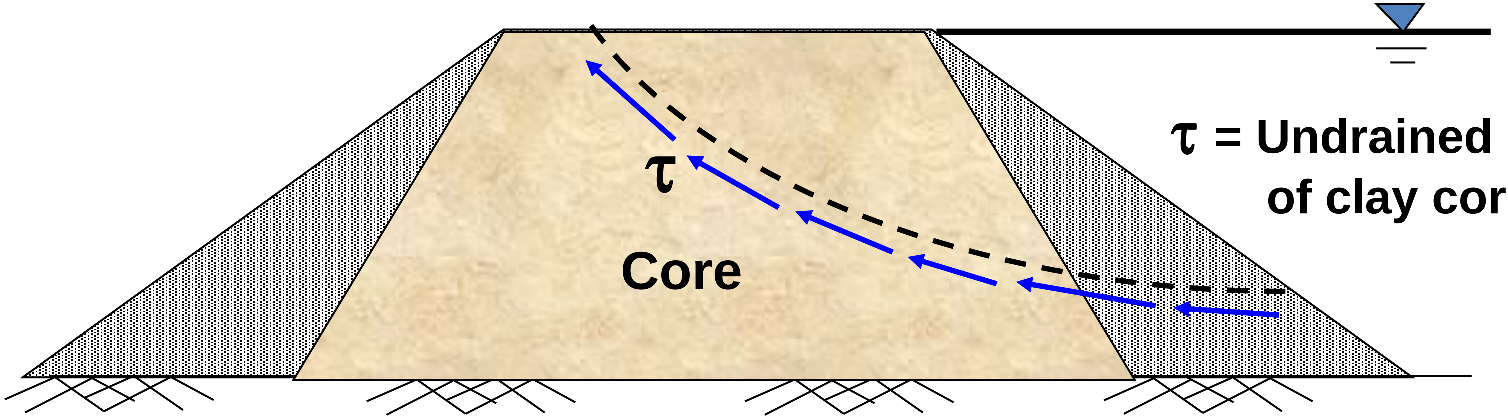
1. Embankment constructed rapidly over a soft clay deposit



τ = in situ undrained shear strength

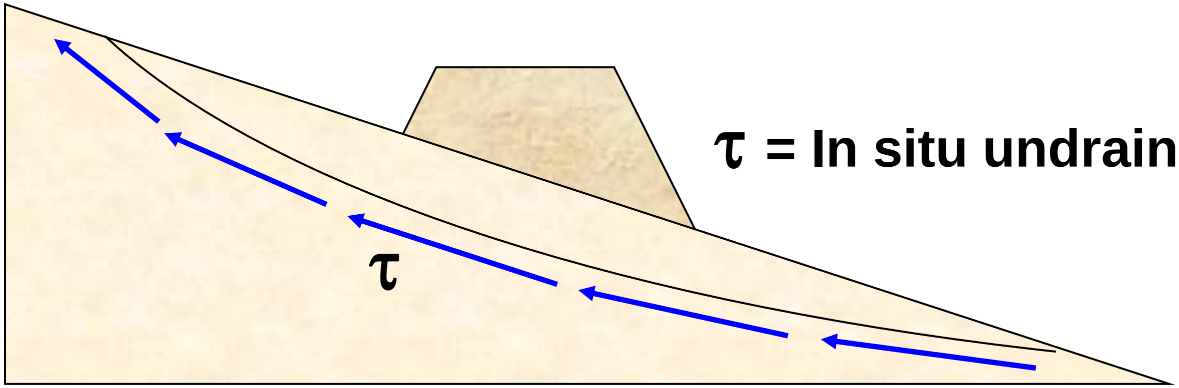
Some practical applications of CU analysis for clays

2. Rapid drawdown behind an earth dam. No drainage of the core



τ = Undrained shear strength of clay core

3. Rapid construction of an embankment on a natural slope

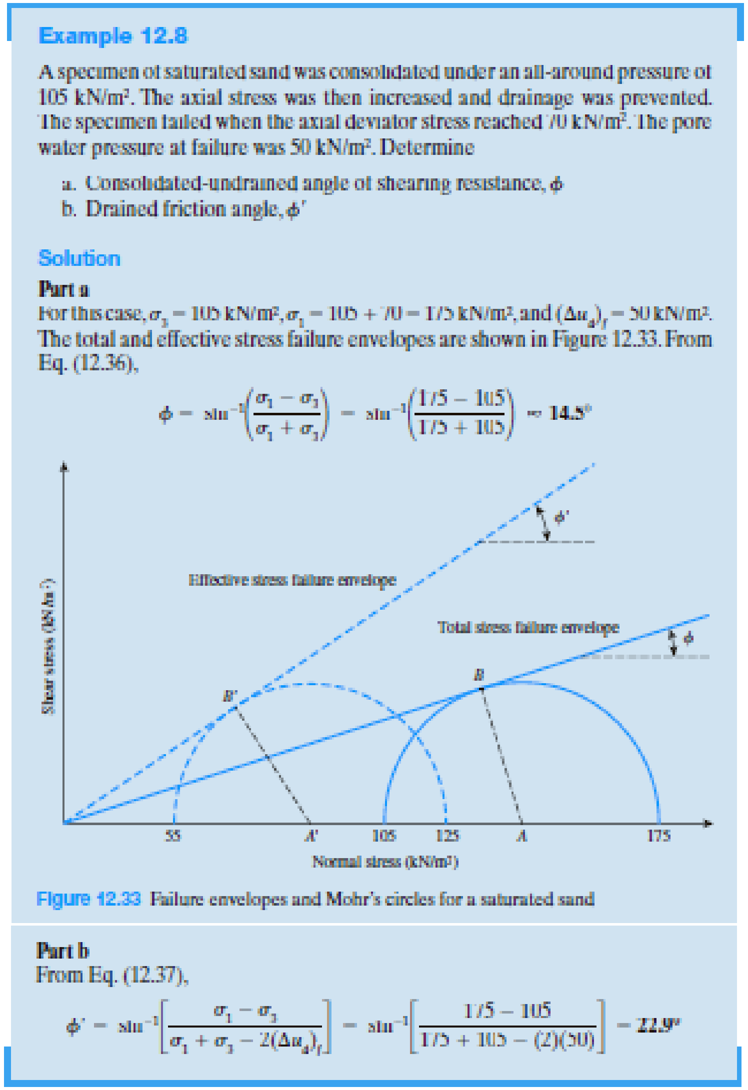


τ = In situ undrained shear strength

Disadvantages of CU Test

- It requires the measurement of Δu which is not an easy task and requires a great deal of care.
- The sample cannot be assured to be fully saturated.
- Effect of rate of loading. The stress-deformation and strength response of clay soils is rate-dependent; that is, usually the faster you load a clay, the stronger it becomes.
- There are two objectives that are incompatible.
- The rate of loading in one hand shall be slow that the proper pressures measured at the ends of the specimen are the same as those occurring in the vicinity of the failure plane.
- On the other hand, the rate of loading in the field may be quite rapid, and therefore for correct modeling of the field situation, the rate of loading in the laboratory sample should be comparable.

Example 12.8



Example 12.8

A specimen of saturated sand was consolidated under an all-around pressure of 105 kN/m². The axial stress was then increased and drainage was prevented. The specimen failed when the axial deviator stress reached 70 kN/m². The pore water pressure at failure was 50 kN/m². Determine

a. Consolidated-undrained angle of shearing resistance, ϕ
b. Drained friction angle, ϕ'

Solution

Part a

For this case, $\sigma_3 = 105$ kN/m², $\sigma_1 = 105 + 70 = 175$ kN/m², and $(\Delta u_d)_f = 50$ kN/m². The total and effective stress failure envelopes are shown in Figure 12.33. From Eq. (12.36),

$$\phi = \sin^{-1}\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right) = \sin^{-1}\left(\frac{175 - 105}{175 + 105}\right) \approx \mathbf{14.5^\circ}$$

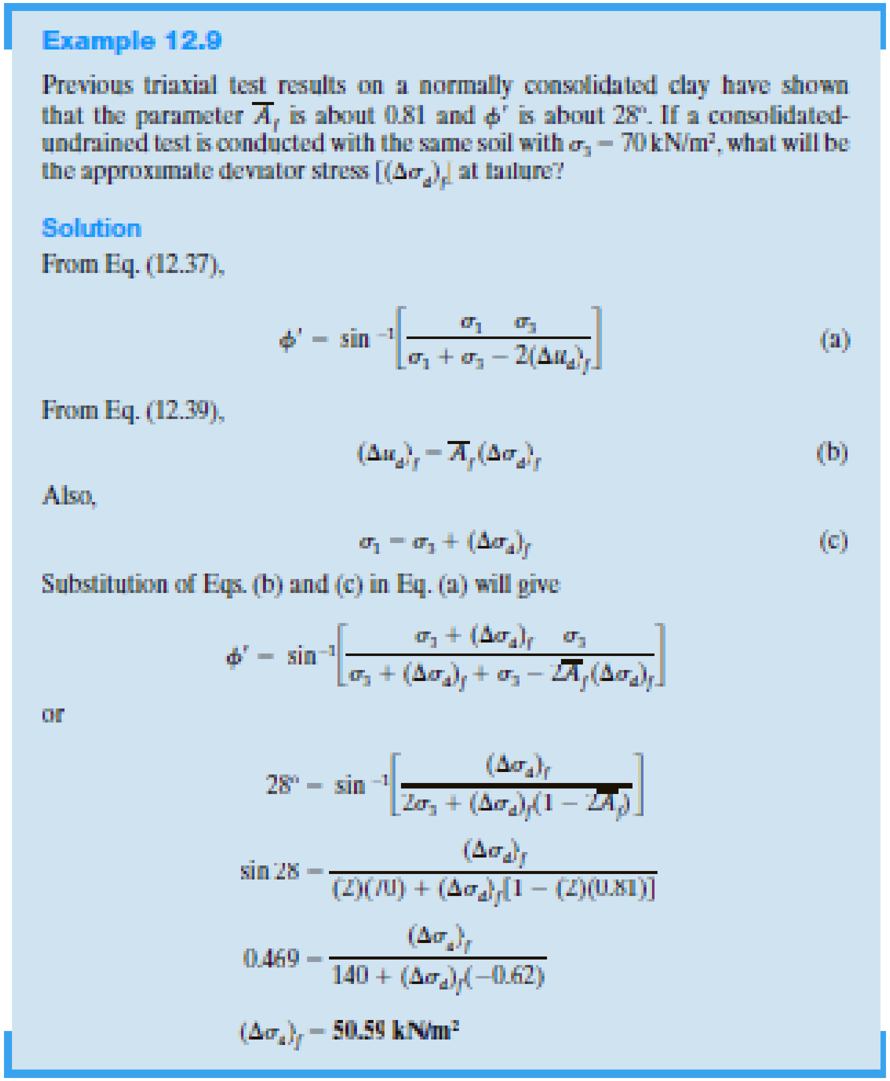
Figure 12.33 Failure envelopes and Mohr's circles for a saturated sand

Part b

From Eq. (12.37),

$$\phi' = \sin^{-1}\left[\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2(\Delta u_d)_f}\right] = \sin^{-1}\left[\frac{175 - 105}{175 + 105 - (2)(50)}\right] = \mathbf{22.9^\circ}$$

Example 12.9



Example 12.9

Previous triaxial test results on a normally consolidated clay have shown that the parameter $\overline{A}_f$ is about 0.81 and ϕ' is about 28°. If a consolidated-undrained test is conducted with the same soil with $\sigma_3 = 70\ \text{kN/m}^2$, what will be the approximate deviator stress $[(\Delta\sigma_d)_f]$ at failure?

Solution

From Eq. (12.37),

$$\phi' = \sin^{-1}\left[\frac{\sigma_1 \quad \sigma_3}{\sigma_1 + \sigma_3 - 2(\Delta u_d)_f}\right] \tag{a}$$

From Eq. (12.39),

$$(\Delta u_d)_f = \overline{A}_f(\Delta\sigma_d)_f \tag{b}$$

Also,

$$\sigma_1 = \sigma_3 + (\Delta\sigma_d)_f \tag{c}$$

Substitution of Eqs. (b) and (c) in Eq. (a) will give

$$\phi' = \sin^{-1}\left[\frac{\sigma_3 + (\Delta\sigma_d)_f \quad \sigma_3}{\sigma_3 + (\Delta\sigma_d)_f + \sigma_3 - 2\overline{A}_f(\Delta\sigma_d)_f}\right]$$

or

$$28° = \sin^{-1}\left[\frac{(\Delta\sigma_d)_f}{2\sigma_3 + (\Delta\sigma_d)_f(1 - 2\overline{A}_f)}\right]$$

$$\sin 28 = \frac{(\Delta\sigma_d)_f}{(2)(70) + (\Delta\sigma_d)_f[1 - (2)(0.81)]}$$

$$0.469 = \frac{(\Delta\sigma_d)_f}{140 + (\Delta\sigma_d)_f(-0.62)}$$

$$(\Delta\sigma_d)_f = \mathbf{50.59\ kN/m^2}$$

Example

A normally consolidated clay was consolidated under a stress of 150 kPa, then sheared undrained in axial compression. The principal stress difference at failure was 100 kPa, and the induced pore water pressure at failure was 88 kPa.

Determine:-

a. The Mohr-Coulomb strength parameters in terms of both total and effective stresses analytically and graphically.

b. The theoretical angle of the failure plane in the specimen.

σ_3 =150 kPa

σ_1 =150 +100 =250 kPa

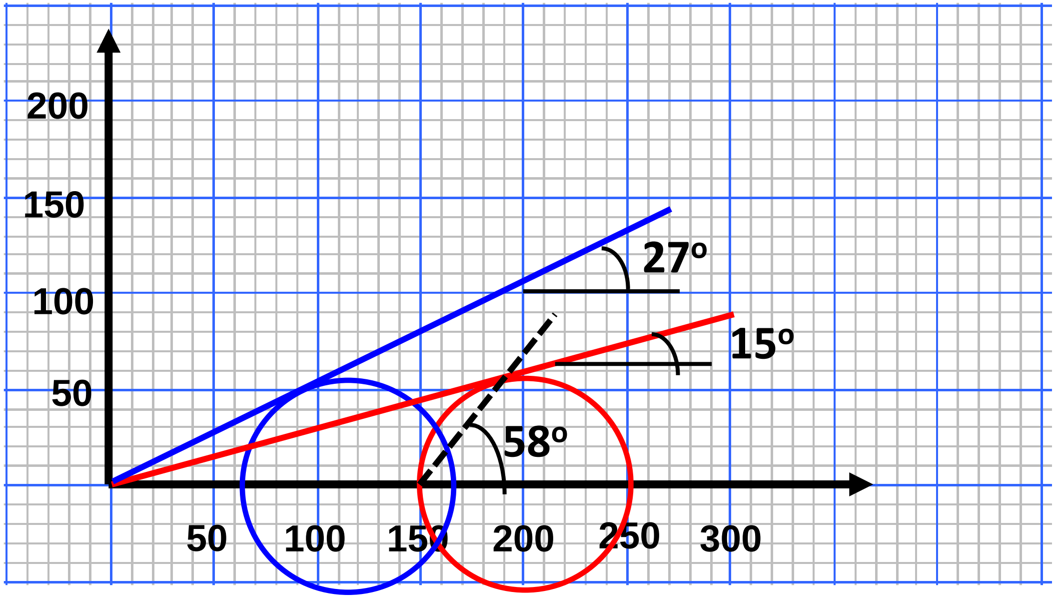
σ'_3 = 150-88 =62 kPa

σ'_1 = 250-88 =162 kPa

$\text{Sin } \phi = 100/(250+150)... > \phi = 14.5^\circ$

$\text{Sin } \phi' = 100/(162+62)... > \phi = 26.5^\circ$

$\theta_f = 45 + \phi'/2 = 58^\circ$



Note:

- Failure plane is obtained from effective Mohr circle
- Measuring p.w.p. makes it possible to obtained effective strength parameters

Example

The shear strength of a normally consolidated clay can be given by the equation $\tau_f = \sigma \tan 27°$. Following are the results of a consolidated-undrained test on the clay.

- Chamber-confining pressure = 150 kN/m^2
- Deviator stress at failure = 120 kN/m^2

a. Determine the consolidated-undrained friction angle

b. Pore water pressure developed in the specimen at failure

σ_3 =150 kN/m^2

σ_1 =150 +120 =270 kN/m^2

Sin ϕ = (270-150)/(270+150)…. > ϕ =16.6^o

SIN ϕ' = $(\sigma'_1 - \sigma'_{3)} / (\sigma'_1 + \sigma'_3)$

But deviatoric stress of total and effective are always equal, hence

$\sigma'_1 - \sigma'_3 = 120$

Sin 27 = 120/$(\sigma'_1 + \sigma'_3)$

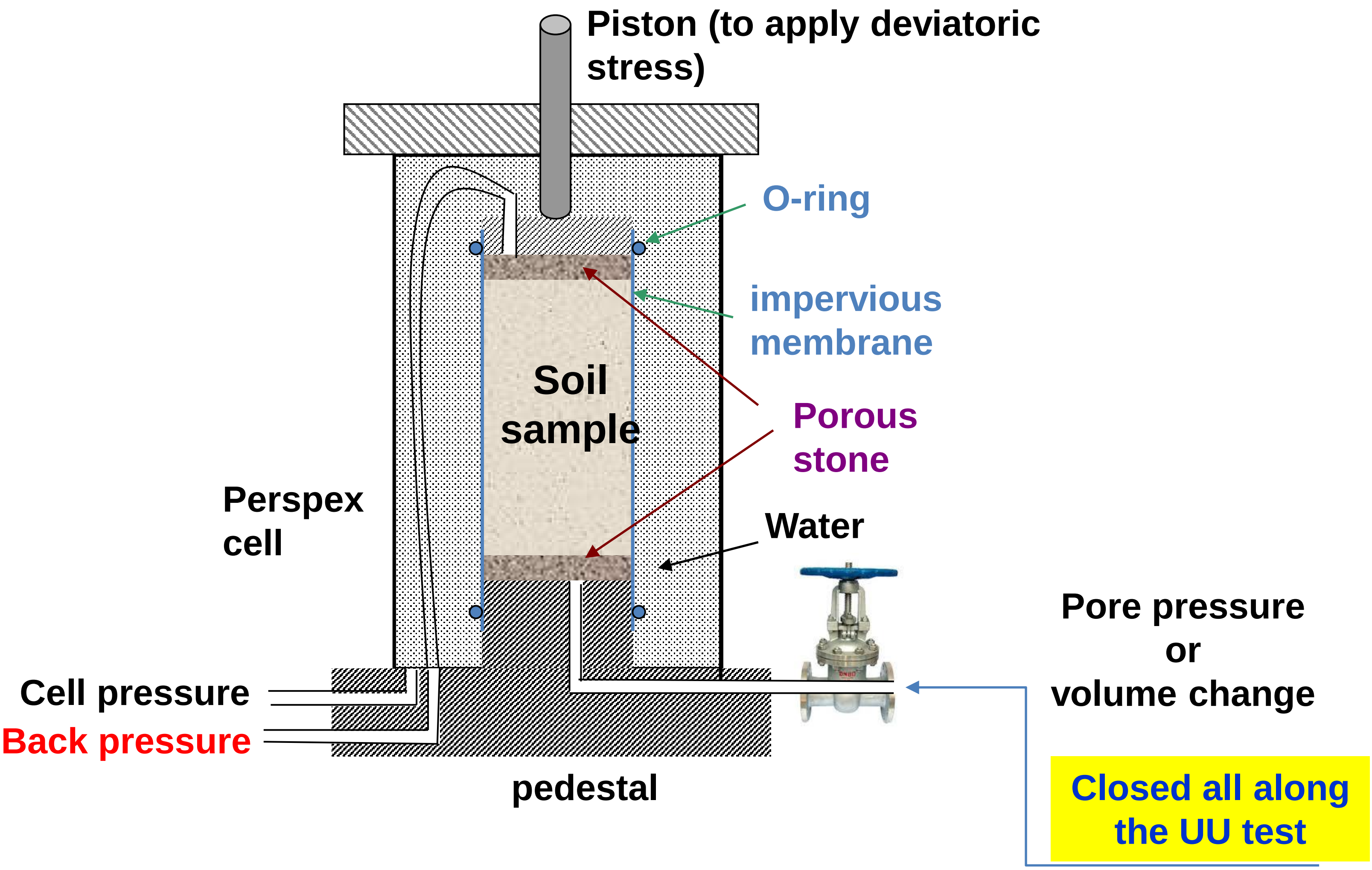
hence $(\sigma'_1 + \sigma'_3)$ = 120/sin 27 = **264.3 kPa**

The difference between the center of total and effective Mohr circles is equal to u, or

u = $(\sigma_1 + \sigma_3)/2 - (\sigma'_1 - \sigma'_3)/2$ = (270+150)/2 –(264.3)/2

Hence **u = 77.85 kPa**

III. Unonsolidated Unrained Test (UU Test)



Closed all along the UU test

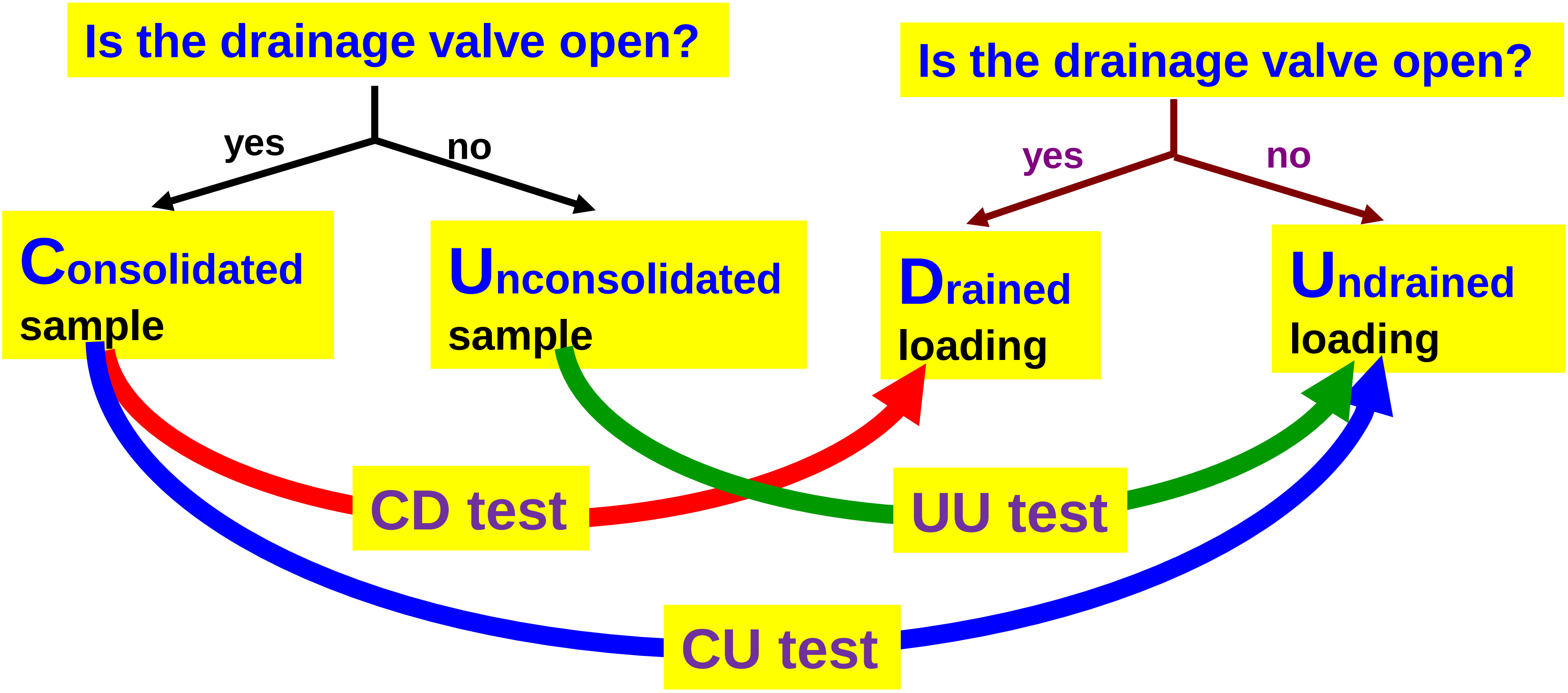
Types of Triaxial Test

Step 1

Under Confining (cell) pressure σ_3

Step 2

Shearing (loading)



Is the drainage valve open?

yes

no

Consolidated sample

Unconsolidated sample

Is the drainage valve open?

yes

no

Drained loading

Undrained loading

CD test

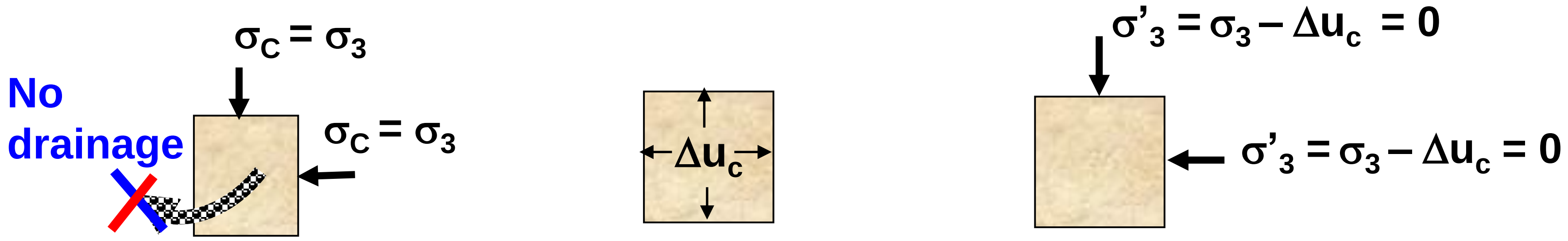
UU test

CU test

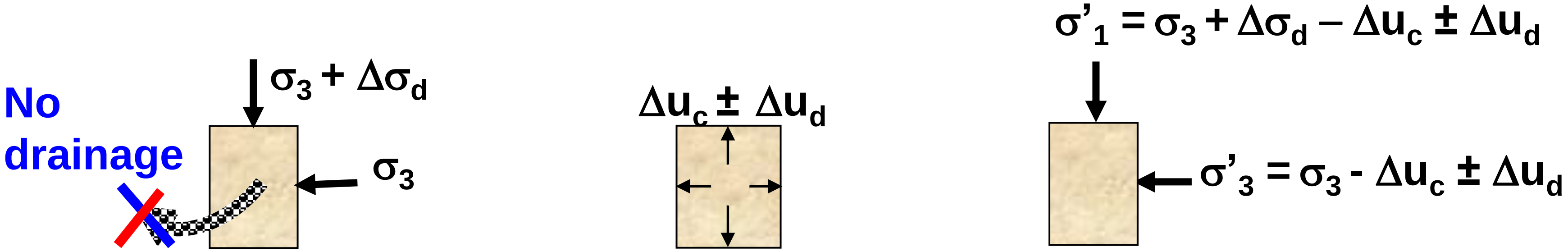
Stress conditions for the unconsolidated undrained test

Total, σ = **Neutral, u** + **Effective, σ’**

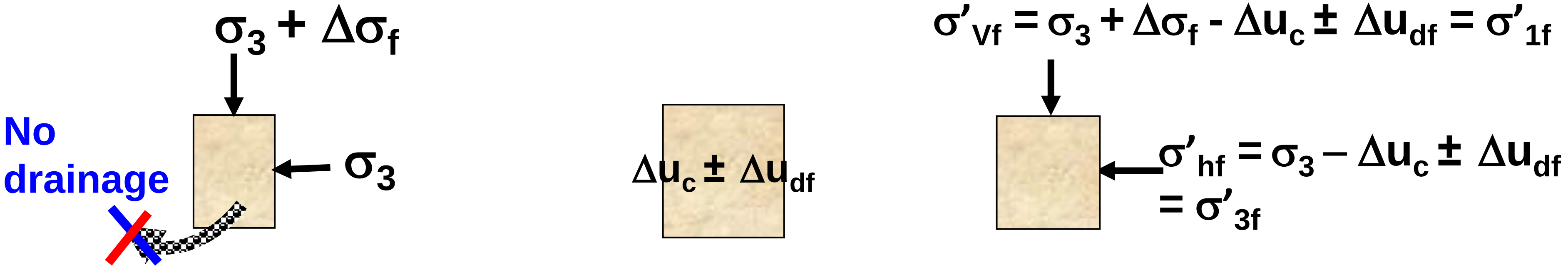
Step 1: After application of cell pressure



Step 2: During application of axial load

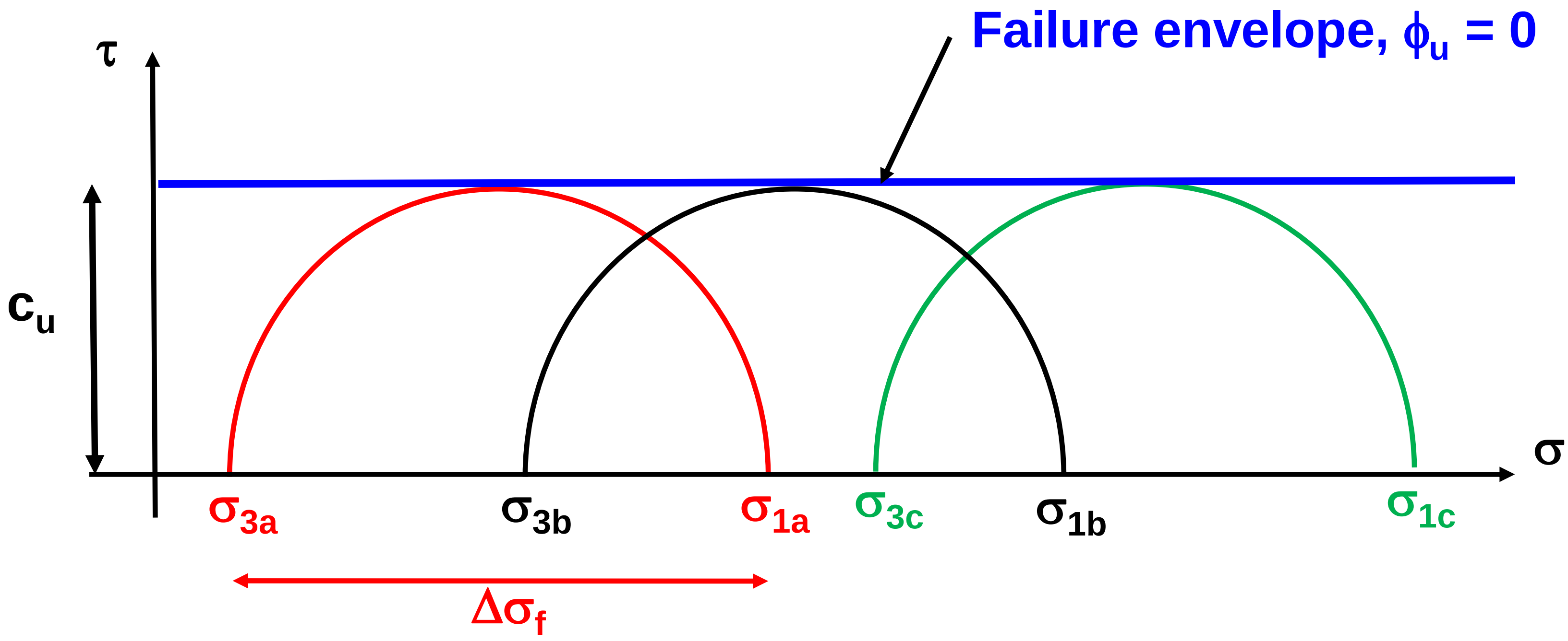


Step 3: At failure



Shear Strength Parameters C and ϕ

- Three identical saturated soil samples are sheared to failure in UU triaxial tests. Each sample is subjected to a different cell pressure. No water can drain at any stage. At failure the Mohr circles are found to be as shown.



- All tests for fully saturated clays, which are assumed to be at the same void ratio (density) and water content, and consequently they will have the same shear strength since there is no CONSOLIDATION allowed.

Notes on UU TEST

- Drainage is not allowed both during the application of the confining pressure σ_3 and during shearing.
- The test specimen is sheared to failure by the application of deviator stress, $\Delta\sigma_d$, and drainage is prevented.
- Because of the application of chamber confining pressure σ_3, the pore water pressure in the soil specimen will increase by u_c.
- A further increase in the pore water pressure (u_d) will occur because of the deviator stress application.
- Usually Δu is not measured in this test. This test is total stress test. Analysis is in terms of σ gives C_u and ϕ_u.
- The added axial stress at failure $(\Delta\sigma_d)_f$ is practically the same regardless of the chamber confining pressure.

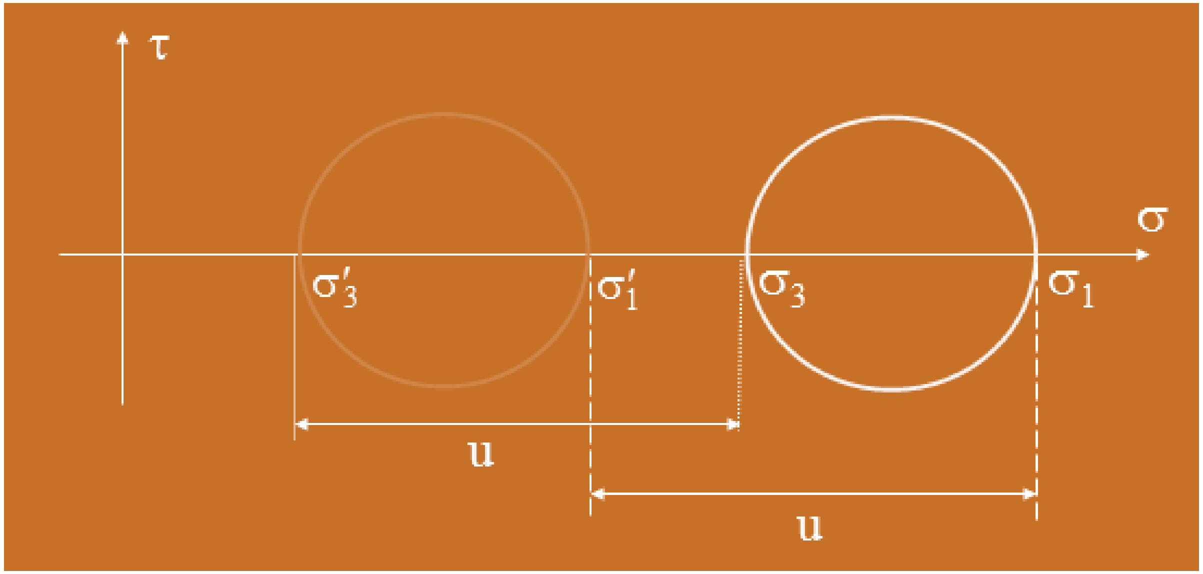
Notes on UU TEST

- All Mohr circles at failure will have the same diameter and the Mohr failure envelope will be a horizontal straight line and hence is called a $\phi = 0$ condition with $\tau = s_u = c_u$ = constant.
- $\tau_f = c = c_u = s_u$ is called undrained shear strength and is equal to the radius of the Mohr's circle.
- The ϕ =0 concept is applicable to only saturated clays and silts.
- Since drainage is not allowed at any stage the test can be performed very quickly. So it is called Quick test or just Q-test. (10-20 mins.)
- Typically, stress-strain curves for UU test are not different from CU and CD stress-strain curves for the same soils.
- Intact specimens are required for this test, so it is conducted usually on clay samples.

Effective and total stress Mohr circles

- If Δu is measured, although it is not measured in this test, then the effective stresses can be estimated and Mohr circle for that is drawn.

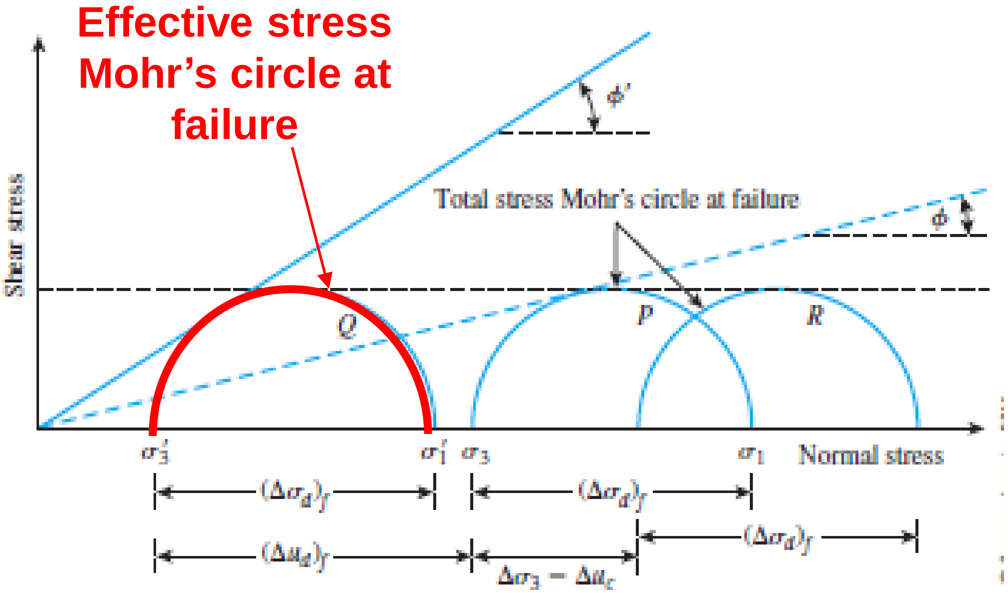
$$\sigma'_1 - \sigma'_3 = \sigma_1 - \sigma_3$$



- The deviator stress $(\sigma_d)_f$ to cause failure is the same as long as the soil is fully saturated and fully undrained during both stages of the test.

- The total and effective Mohr circles are displaced horizontally by the pore pressure, u.

Effective and total stress Mohr circles


Effective stress Mohr's circle at failure

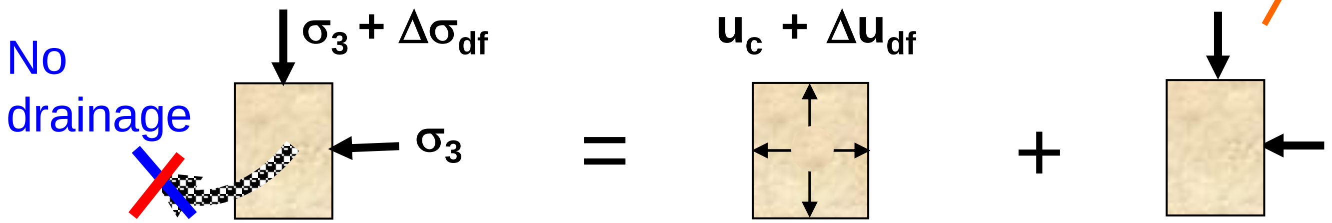
- The different total stress Mohr circles with a single effective stress Mohr circle indicate that the pore pressure is different for each sample.
- As discussed previously increasing the cell pressure without allowing drainage has the effect of increasing the pore pressure by the same amount ($\Delta u = \Delta\sigma_c$) with no change in effective stress.
- The change in pore pressure during shearing is a function of the initial effective stress and the moisture content. As these are identical for the three samples an identical strength is obtained.

Why does $\Delta\sigma_f$ is the same for all specimens?

- There is only ONE **UU** effective stress Mohr Circle at failure, no matter what the confining pressure. But why this is the case?

Specimen I: at Failure

$$\sigma'_1 = \cancel{\sigma_3} + \Delta\sigma_d - \cancel{u_c} - \Delta u_{df}$$

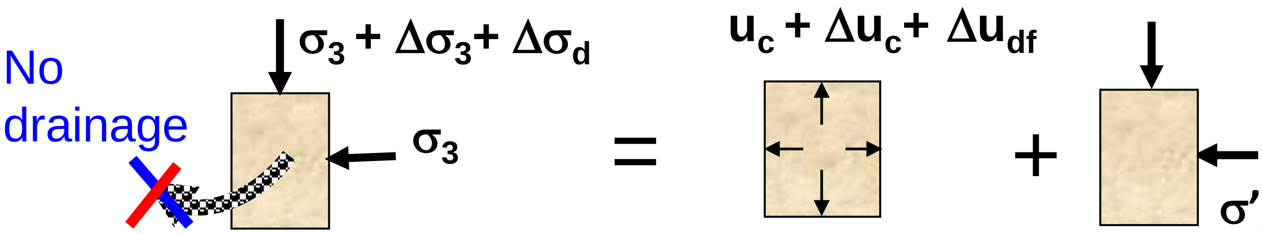


$$\sigma'_3 = \cancel{\sigma_3} - \cancel{u_c} - \Delta u_{df}$$

For fully saturated clays, B = 1 (hence, $u_c = \sigma_3$)

Specimen II: at Failure

$$\sigma'_1 = \cancel{\sigma_3} + \cancel{\Delta\sigma_3} + \Delta\sigma_d - \cancel{u_c} - \cancel{\Delta u_c} - \Delta u_{df}$$



$$\sigma'_3 = \cancel{\sigma_3} + \cancel{\Delta\sigma_3} - \cancel{u_c} - \cancel{\Delta u_c} - \Delta u_{df}$$

Because the effective confining pressure is the same for Specimen I and II.

Example

In an unconsolidated undrained triaxial test the undrained strength is measured as 17.5 kPa. Determine the cell pressure used in the test if the effective strength parameters are c’ = 0, Φ’ = 26º and the pore pressure at failure is 43 kPa.

Analytical solution

Undrained strength = 17.5 = $\frac{(\sigma_1 - \sigma_3)}{2} = \frac{(\sigma_1' - \sigma_3')}{2}$

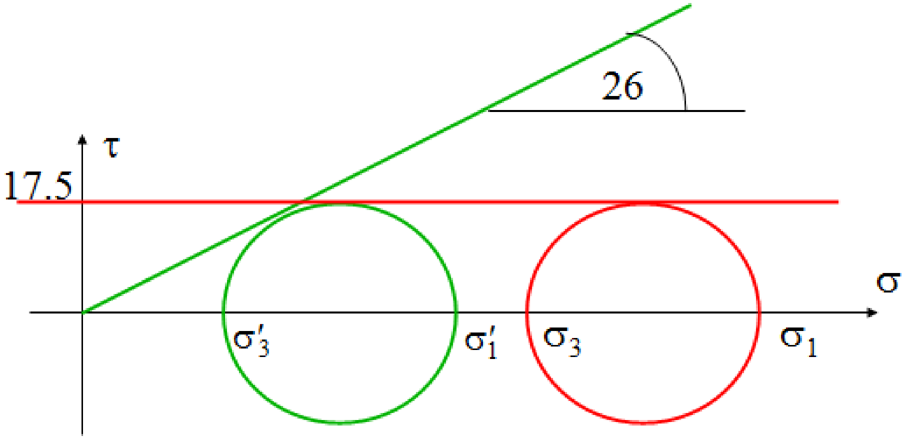
Failure criterion $\sigma_1 = \sigma_3 \tan^2(45 + \phi/2) + 2c\tan(45 + \phi/2)$

2 Eqs. With 2 unknowns

Hence $\sigma_1' =$ 57.4 kPa, $\sigma_3' =$ 22.4 kPa

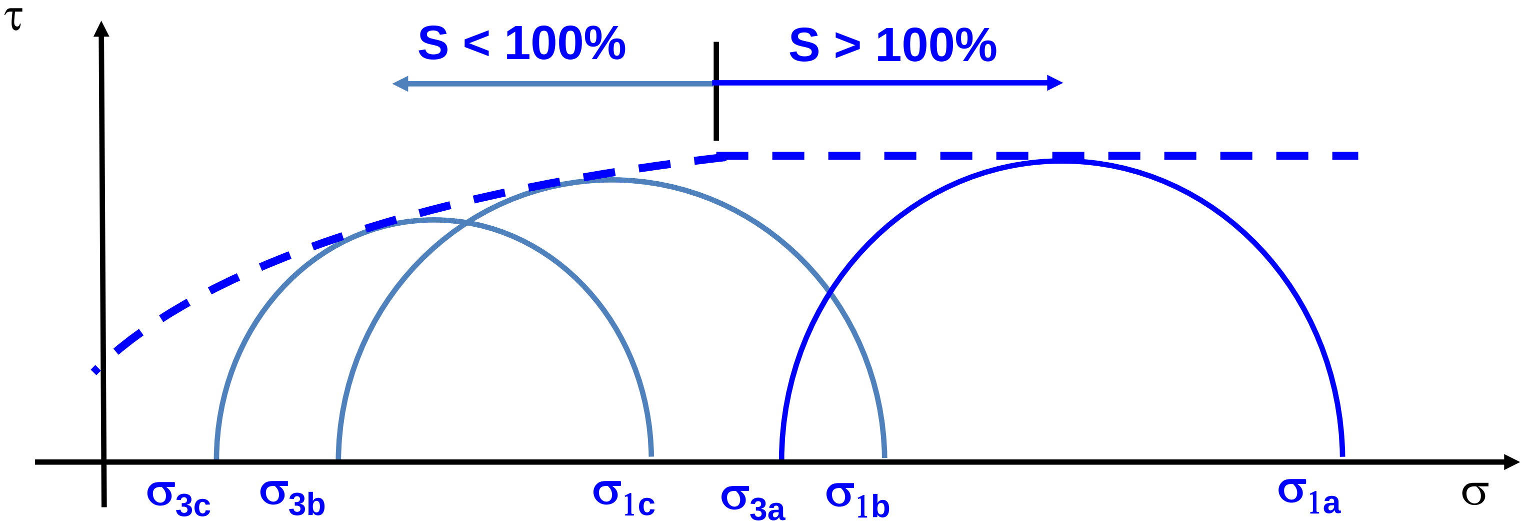
and cell pressure (total stress) = $\sigma_3' + u =$ 65.4 k

Graphical solution



Remarks

- It is often found that a series of undrained tests from a particular site give a value of ϕ_u that is not zero (C_u not constant). If this happens either:
  - The samples are not saturated, or
  - The samples have different moisture contents
- If the samples are not saturated analyses based on undrained behavior will not be correct.
- The undrained strength C_u is not a fundamental soil property. If the moisture content changes so will the undrained strength.

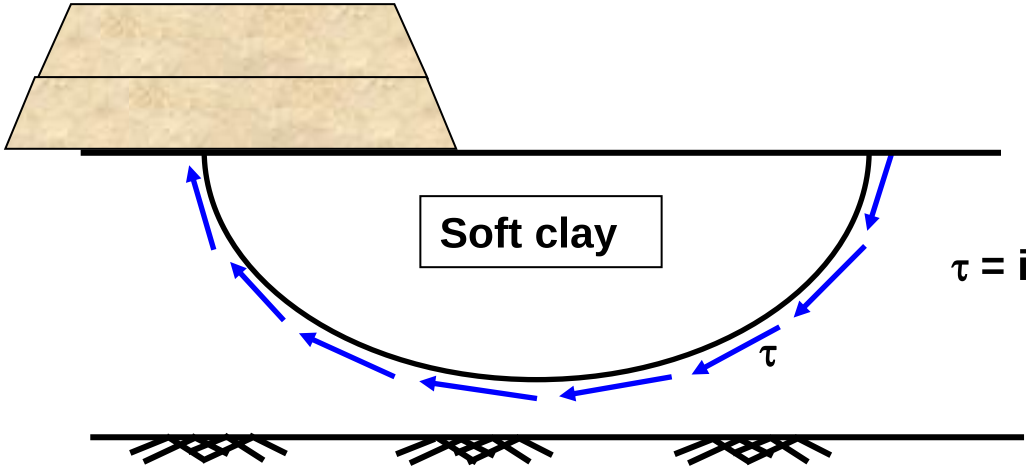


Effect of degree of saturation on failure envelope

Some practical applications of UU analysis for clays

- Like the CD and CU tests, the UU strength is applicable to certain critical design situations in engineering practice.
- These situations are where the engineering loading is assumed to take place so rapidly that there is no time for the induced pore water pressure to dissipate or for consolidation to occur during the loading period.
- UU test simulates the short term condition in the field. Thus, C_u can be used to analyze the short term behavior of soils

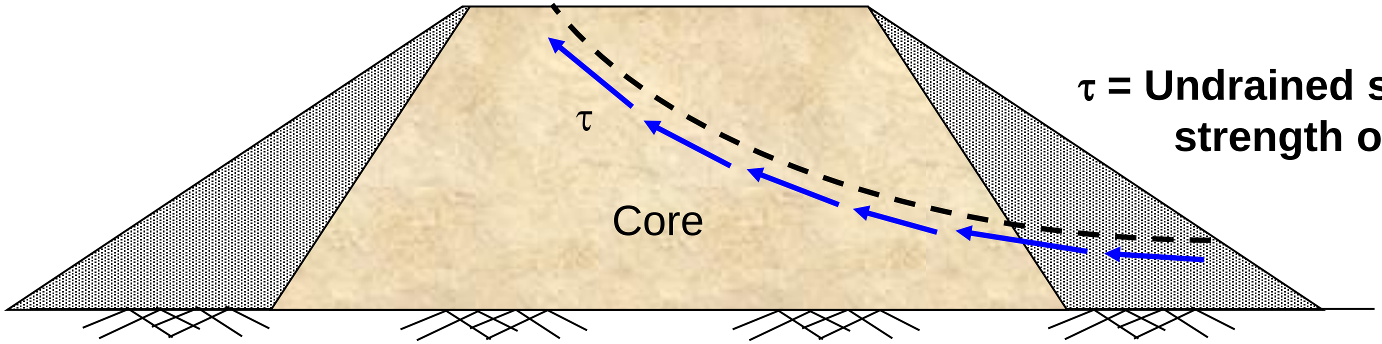
1. Embankment constructed rapidly over a soft clay deposit



τ = in situ undrained shear strength

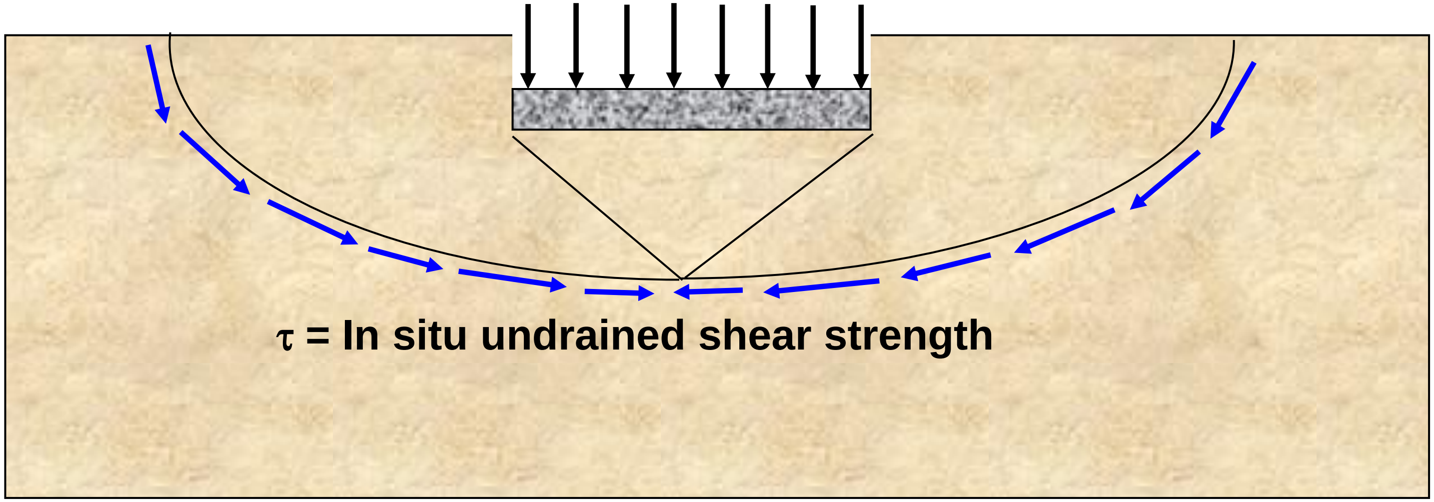
Some practical applications of UU analysis for clays

2. Large earth dam constructed rapidly with no change in water content of soft clay



τ = Undrained shear strength of clay core

3. Footing placed rapidly on clay deposit

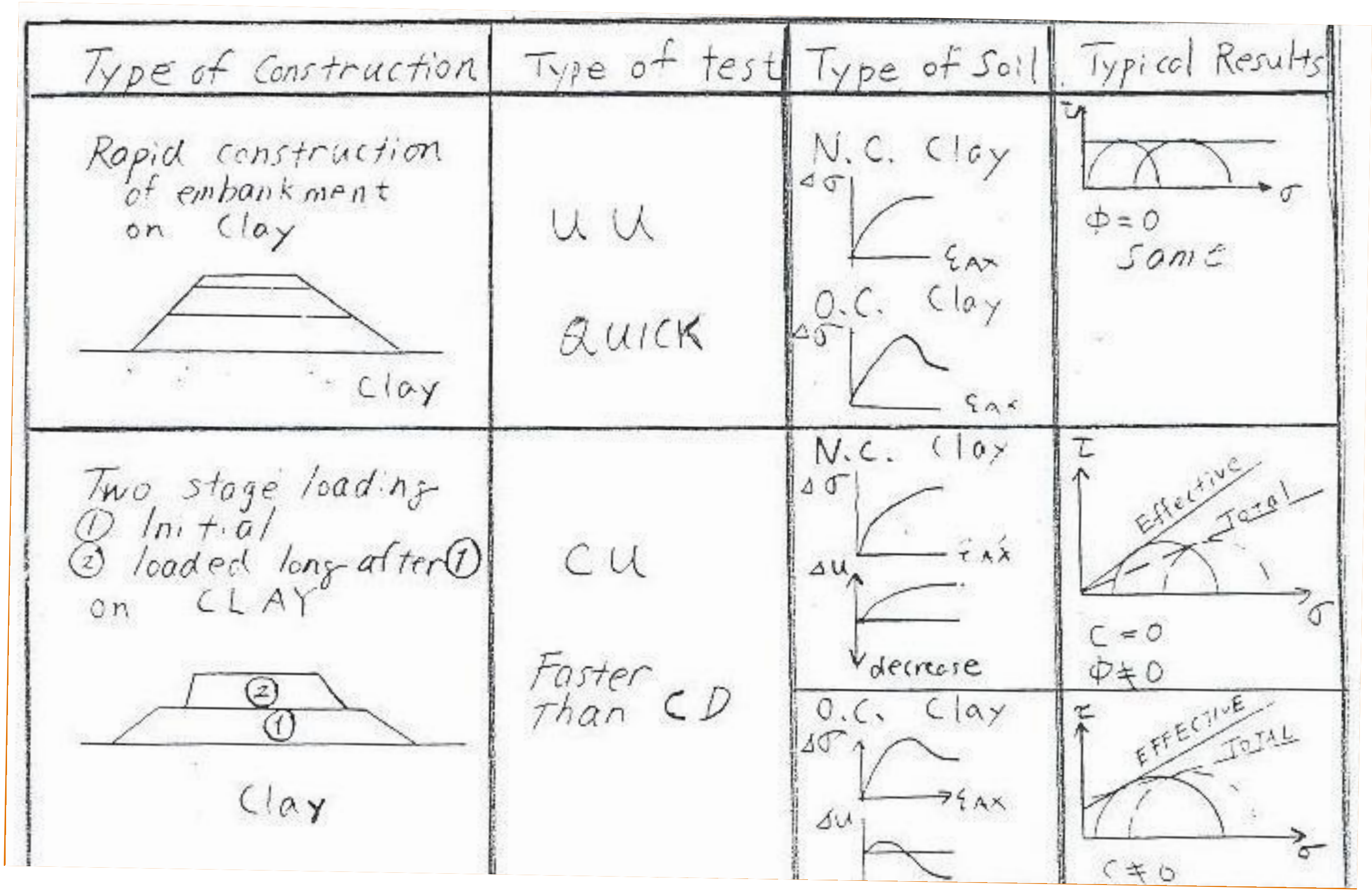


τ = In situ undrained shear strength

Types of Triaxial Test

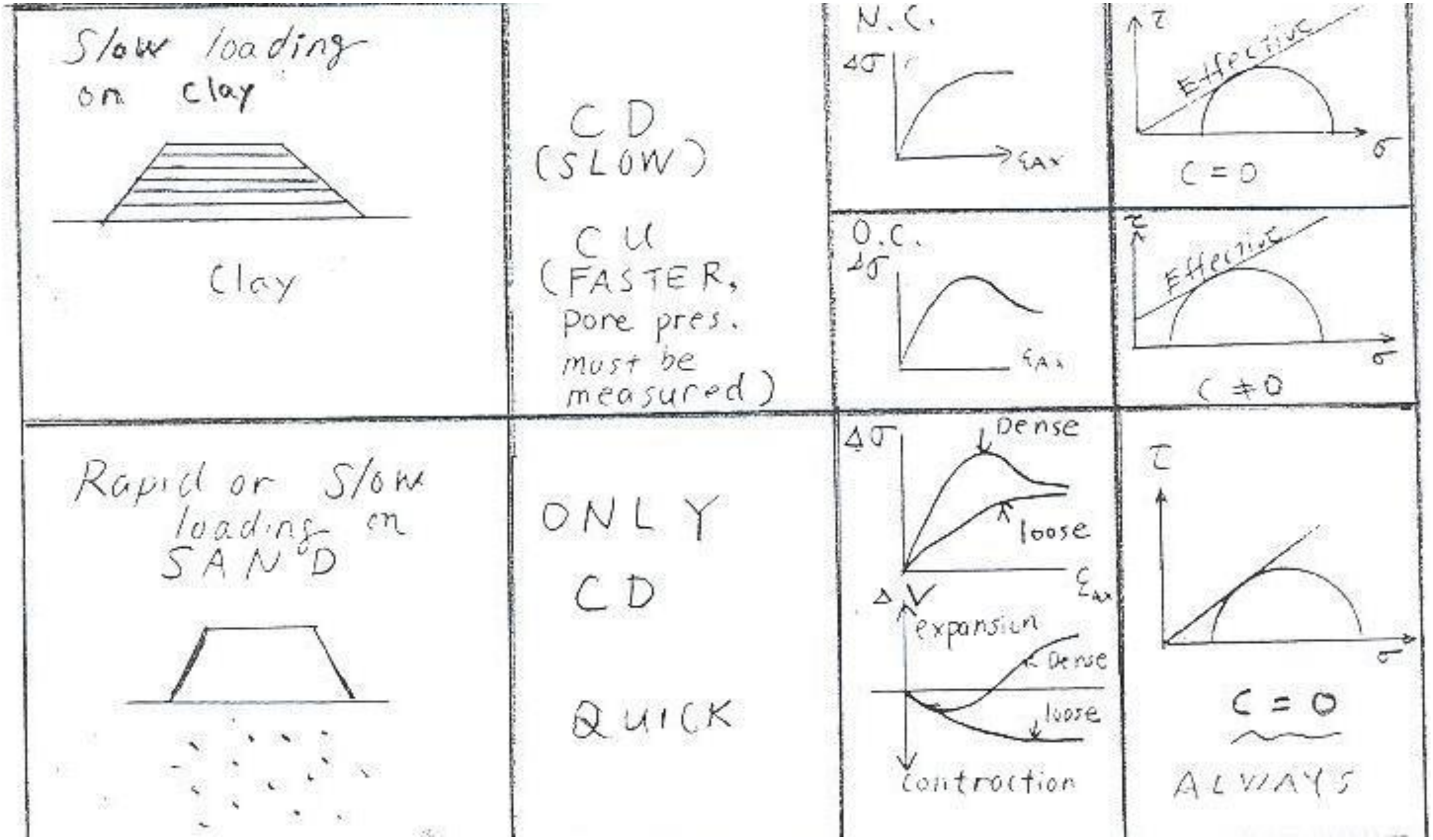
Test	Drainage during confinement	Drainage during shear	Pore water pressure build up?	Total or Effective	Type of test “duration”
CD	Open	Open	No if the test is slow	Effective	Slow for clay S- test
CU	Open	Closed	Yes	Total Effective if p.w.p is measured	
UU	Closed	Closed	Yes	Total	Fast Q-test

Applications



Type of Construction	Type of test	Type of Soil	Typical Results
Rapid construction of embankment on Clay Clay	UU QUICK	N.C. Clay $\Delta\sigma$ ε_{AX} O.C. Clay $\Delta\sigma$ ε_{AX}	τ σ $\Phi = 0$ Same
Two stage loading ① Initial ② loaded long after ① on CLAY ② ① Clay	CU Faster than CD	N.C. Clay $\Delta\sigma$ ε_{AX} Δu decrease	τ Effective Total σ $c = 0$ $\Phi \neq 0$
		O.C. Clay $\Delta\sigma$ ε_{AX} Δu	τ EFFECTIVE TOTAL σ $c \neq 0$

Applications



Slow loading on clay Clay	CD (SLOW) CU (FASTER, pore pres. must be measured)	N.C. O.C.	C = 0 C ≠ 0
Rapid or Slow loading on SAND	ONLY CD QUICK		C = 0 ALWAYS

Unconfined Compression Test (UC Test)

- This a special class or type of UU test. In this test the confining pressure $\sigma_3 = 0$.
- Axial load is rapidly applied and at failure $\sigma_3 = 0$ and the value of σ_1 necessary to cause failure is called the Unconfined Compression Strength q_u.

Failure by shear

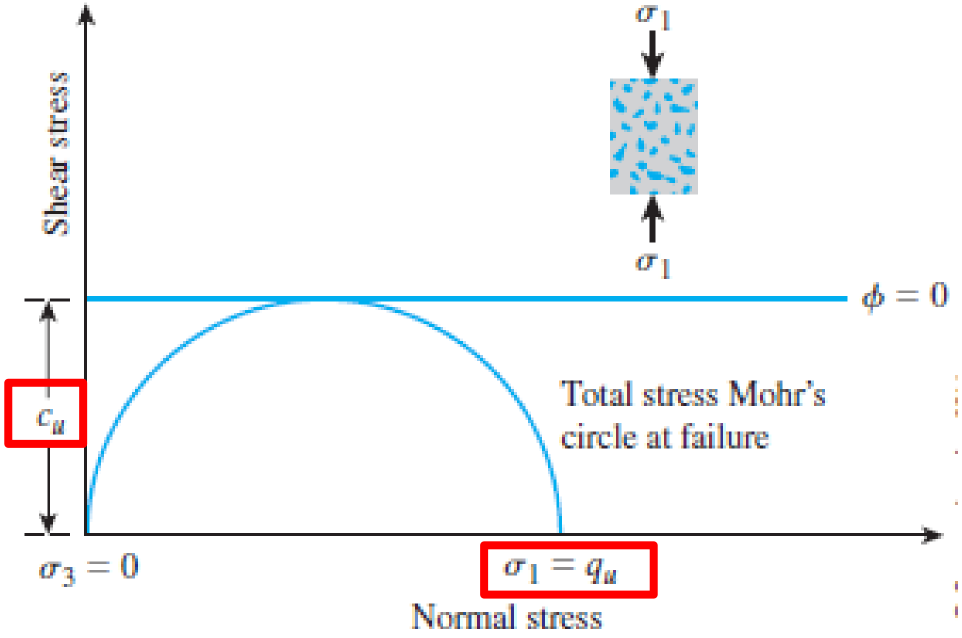
Failure by Bulging

Unconfined Compression Test (UC Test)

- Because the undrained shear strength is independent of the confining pressure as long as the soil is fully saturated and fully undrained, we have

$$\boxed{\tau = \frac{\sigma_1}{2} = \frac{q_u}{2} = C_u}$$

- The difference in q_u between different tests will depend on the level of compaction for each sample.



- Since we said that in UU test strength is independent of σ_3, theoretically the value of C_u obtained from unconfined compression test or UU test must be the same. In practice, however C_u from UC is slightly lower than that from UU test.

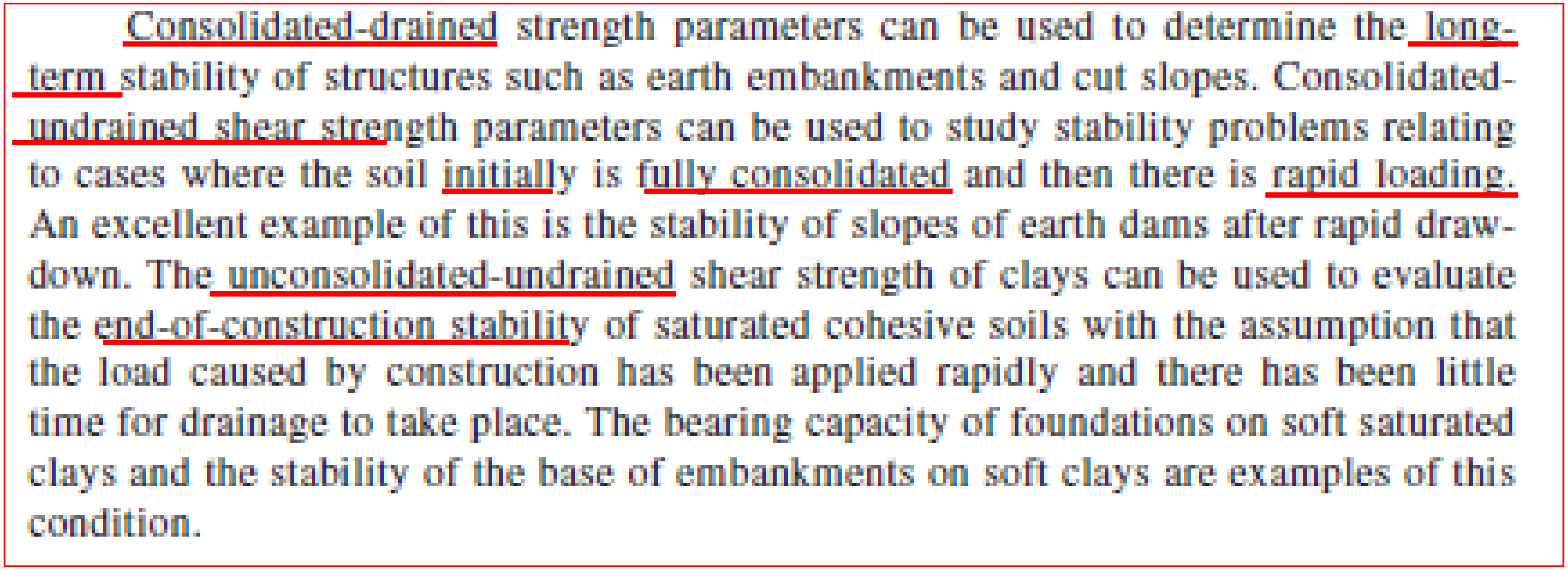
Notes

- The effective stress conditions at failure are identical for both UU and UC tests. And if the effective stress conditions are the same in both tests, then the strengths will be the same. For this to be true the following assumptions must be satisfied.

  1. The specimen must be 100% saturated.
  2. The specimen must be intact and contains no defects.
  3. The soil must be very fine (clays)
  4. The specimen must be sheared rapidly to failure.

Comments on Triaxial Tests

Three types of strength parameters (Consolidated-drained, consolidated-undrained, and unconsolidated undrained) were introduced. There use depends on drainage conditions.



Consolidated-drained strength parameters can be used to determine the long-term stability of structures such as earth embankments and cut slopes. Consolidated-undrained shear strength parameters can be used to study stability problems relating to cases where the soil initially is fully consolidated and then there is rapid loading. An excellent example of this is the stability of slopes of earth dams after rapid draw-down. The unconsolidated-undrained shear strength of clays can be used to evaluate the end-of-construction stability of saturated cohesive soils with the assumption that the load caused by construction has been applied rapidly and there has been little time for drainage to take place. The bearing capacity of foundations on soft saturated clays and the stability of the base of embankments on soft clays are examples of this condition.

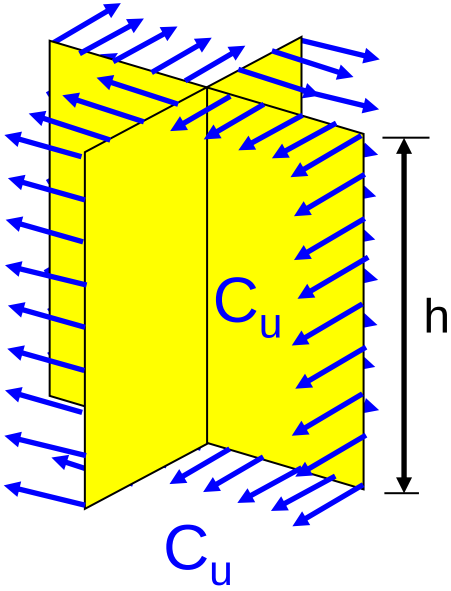
Vane Shear Test

- Since the test is very fast, Unconsolidated Undrained (UU) can be expected.
- If T is the maximum torque applied at the head of the torque rod to cause failure, it should be equal to the sum of the resisting moment of the shear force along the side surface of the soil cylinder (M_s) and the resisting moment of the shear force at each end (M_e).

$$T = M_s + M_e + M_e = M_s + 2M_e$$

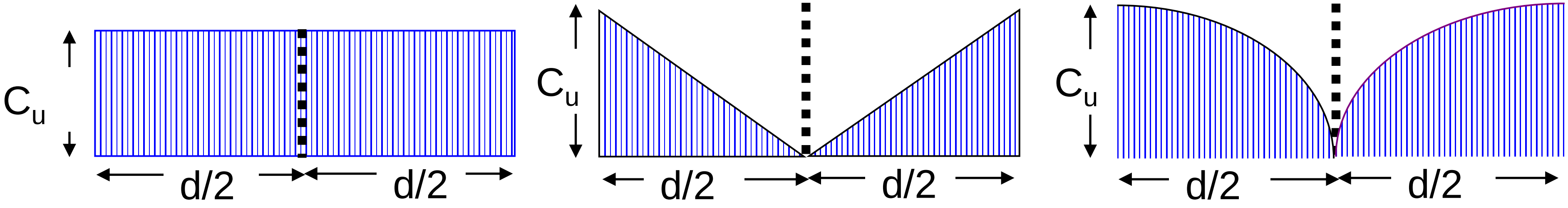
M_s – Shaft shear resistance along the circumference

$$M_s = \pi d h C_u \frac{d}{2} = \pi C_u \frac{d^2 h}{2}$$



Vane Shear Test

M_e depends on the assumed distribution of shear strength mobilization at the ends of the soil cylinder.



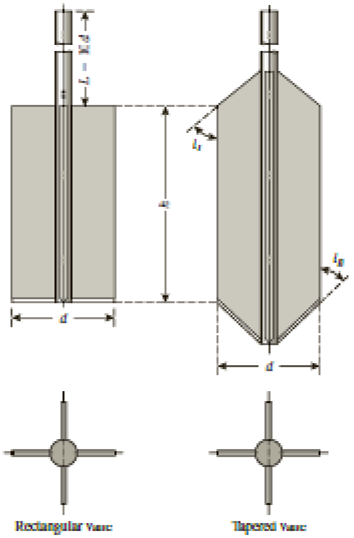
- In general, the torque, T, at failure can be expressed as

$$C_u = \frac{T}{\pi\left(\frac{d^2h}{2} + \beta\frac{d^3}{4}\right)}$$

β = 1/2 for triangular distribution
β = 2/3 for uniform distribution
β = 3/5 for parabolic distribution

Remarks

Figure 12.47 Laboratory vane shear test device (*Courtesy of ELE International*)



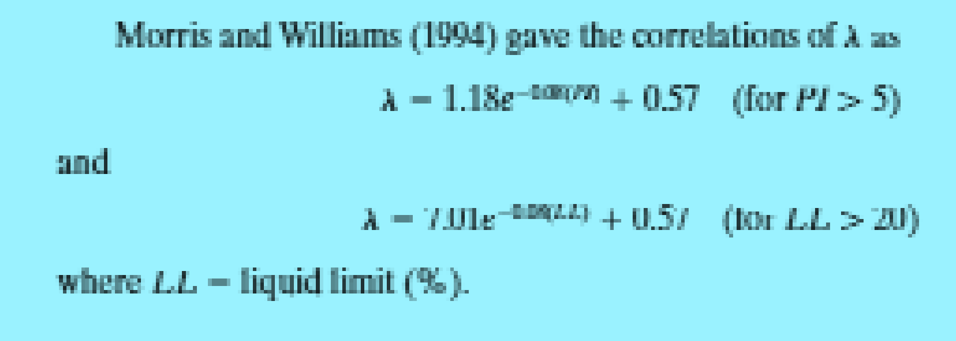
Remarks

- The undrained shear strength obtained from a vane shear test also depends on the rate of application of torque T.

- Bjerrum (1974) has shown that as the plasticity of soils increases, C_u obtained by vane shear tests may give unsafe results for foundation design. Therefore, he proposed the following correction.

$$C_{u(design)} = \lambda C_{u(vane\ shear)}$$

Where, λ = correction factor = 1.7 – 0.54 log (PI)
PI = Plasticity Index



Morris and Williams (1994) gave the correlations of λ as

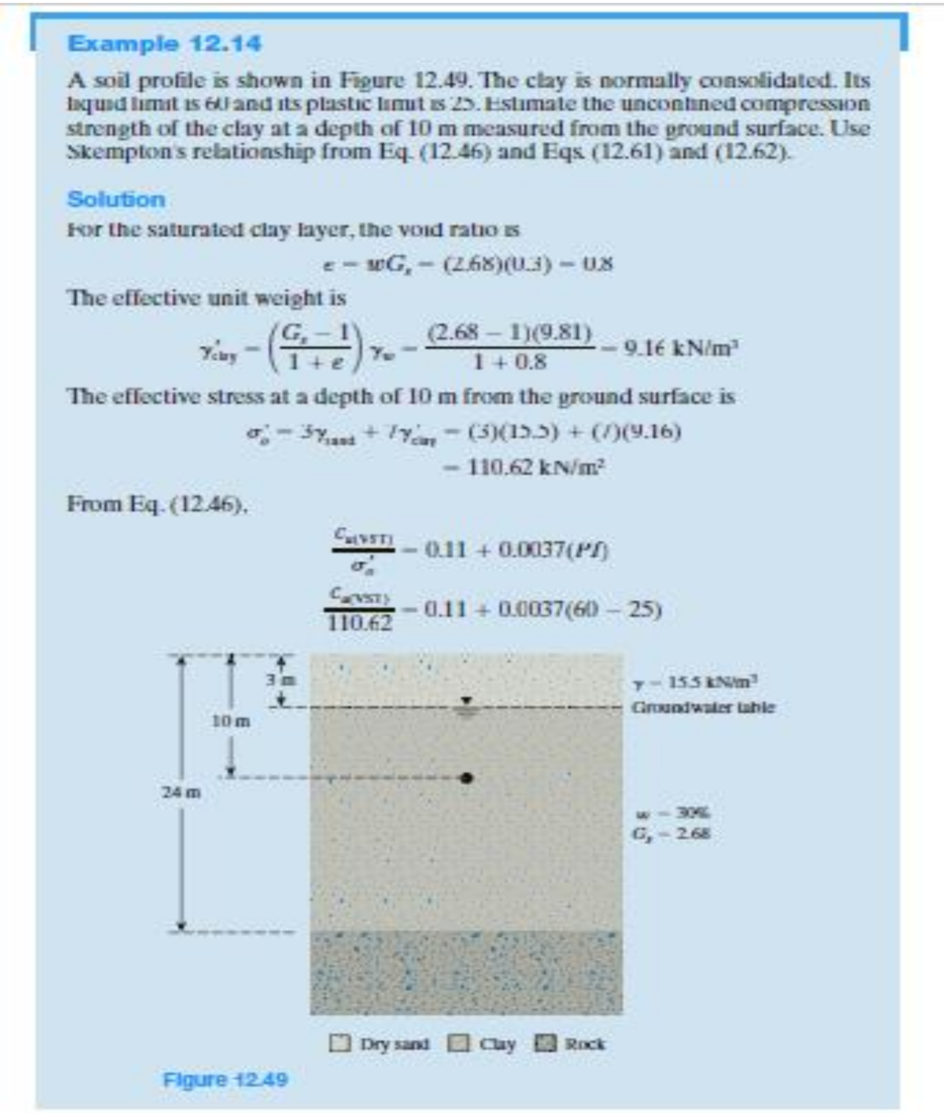
$$\lambda = 1.18e^{-0.08(PI)} + 0.57 \quad (\text{for } PI > 5)$$

and

$$\lambda = 7.01e^{-0.08(LL)} + 0.57 \quad (\text{for } LL > 20)$$

where LL = liquid limit (%).

Example 12.14



Example 12.14

A soil profile is shown in Figure 12.49. The clay is normally consolidated. Its liquid limit is 60 and its plastic limit is 25. Estimate the unconfined compression strength of the clay at a depth of 10 m measured from the ground surface. Use Skempton's relationship from Eq. (12.46) and Eqs. (12.61) and (12.62).

Solution

For the saturated clay layer, the void ratio is

$$e = wG_s = (2.68)(0.3) = 0.8$$

The effective unit weight is

$$\gamma'_{\text{clay}} = \left(\frac{G_s - 1}{1 + e}\right)\gamma_w = \frac{(2.68 - 1)(9.81)}{1 + 0.8} = 9.16 \text{ kN/m}^3$$

The effective stress at a depth of 10 m from the ground surface is

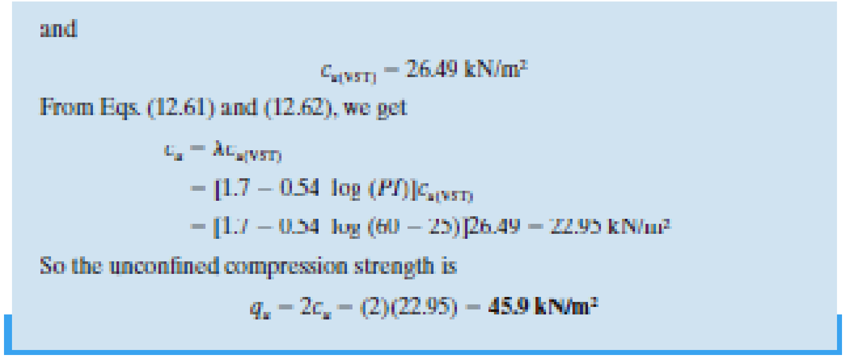
$$\sigma'_o = 3\gamma_{\text{sand}} + 7\gamma'_{\text{clay}} = (3)(15.5) + (7)(9.16)$$
$$= 110.62 \text{ kN/m}^2$$

From Eq. (12.46),

$$\frac{c_{u(\text{VST})}}{\sigma'_o} = 0.11 + 0.0037(PI)$$

$$\frac{c_{u(\text{VST})}}{110.62} = 0.11 + 0.0037(60 - 25)$$

Figure 12.49



and

$$c_{u(\text{VST})} = 26.49 \text{ kN/m}^2$$

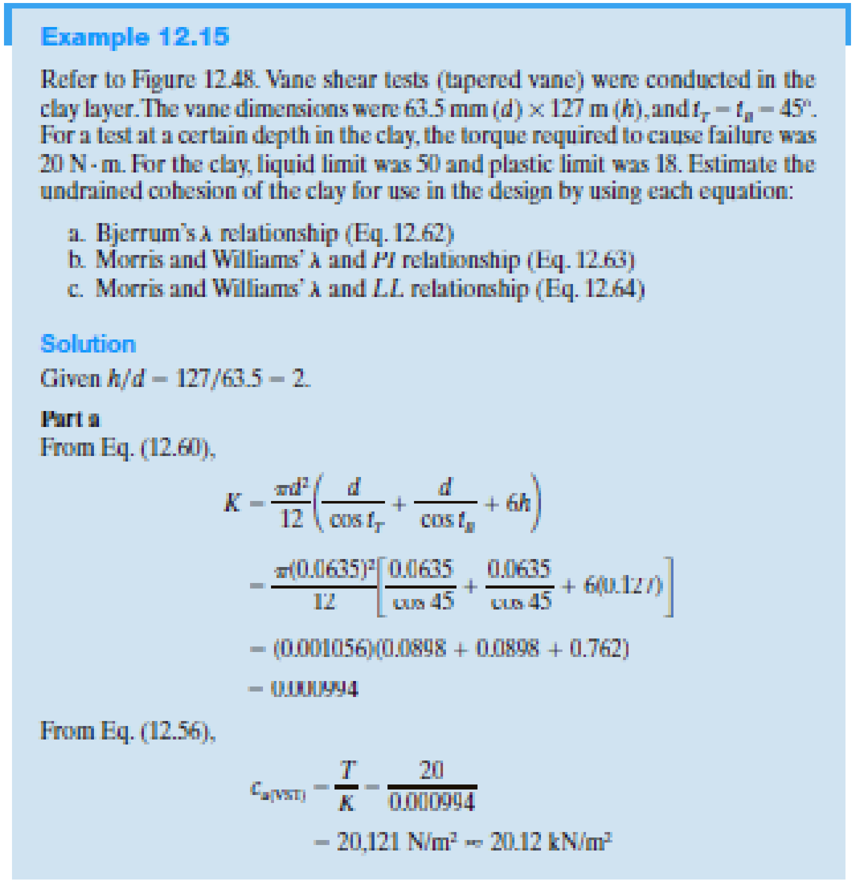
From Eqs. (12.61) and (12.62), we get

$$c_u = \lambda c_{u(\text{VST})}$$
$$= [1.7 - 0.54 \log (PI)]c_{u(\text{VST})}$$
$$= [1.7 - 0.54 \log (60 - 25)]26.49 = 22.95 \text{ kN/m}^2$$

So the unconfined compression strength is

$$q_u = 2c_u = (2)(22.95) = \mathbf{45.9 \text{ kN/m}^2}$$

Example 12.15



Example 12.15

Refer to Figure 12.48. Vane shear tests (tapered vane) were conducted in the clay layer. The vane dimensions were 63.5 mm (d) × 127 m (h), and $i_T = i_B = 45°$. For a test at a certain depth in the clay, the torque required to cause failure was 20 N · m. For the clay, liquid limit was 50 and plastic limit was 18. Estimate the undrained cohesion of the clay for use in the design by using each equation:

a. Bjerrum's λ relationship (Eq. 12.62)
b. Morris and Williams' λ and PI relationship (Eq. 12.63)
c. Morris and Williams' λ and LL relationship (Eq. 12.64)

Solution

Given $h/d = 127/63.5 = 2$.

Part a

From Eq. (12.60),

$$K = \frac{\pi d^2}{12}\left(\frac{d}{\cos i_T} + \frac{d}{\cos i_B} + 6h\right)$$

$$= \frac{\pi(0.0635)^2}{12}\left[\frac{0.0635}{\cos 45} + \frac{0.0635}{\cos 45} + 6(0.127)\right]$$

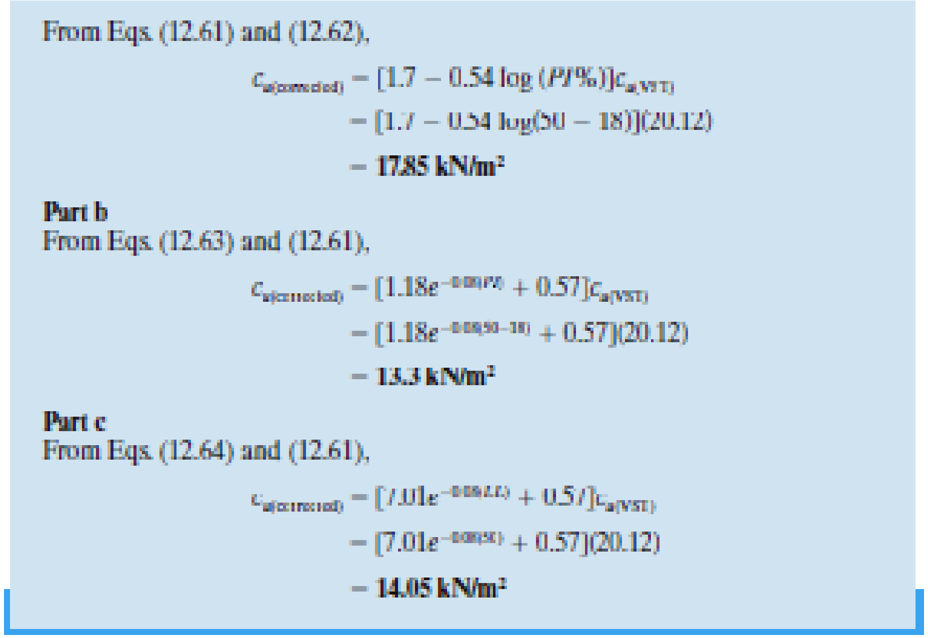
$$= (0.001056)(0.0898 + 0.0898 + 0.762)$$

$$= 0.000994$$

From Eq. (12.56),

$$c_{u(\text{VST})} = \frac{T}{K} = \frac{20}{0.000994}$$

$$= 20{,}121 \text{ N/m}^2 \approx 20.12 \text{ kN/m}^2$$



From Eqs. (12.61) and (12.62),

$$c_{u(\text{corrected})} = [1.7 - 0.54 \log (PI\%)]c_{u(\text{VST})}$$

$$= [1.7 - 0.54 \log(50 - 18)](20.12)$$

$$= \mathbf{17.85 \text{ kN/m}^2}$$

Part b

From Eqs. (12.63) and (12.61),

$$c_{u(\text{corrected})} = [1.18e^{-0.08(PI)} + 0.57]c_{u(\text{VST})}$$

$$= [1.18e^{-0.08(50-18)} + 0.57](20.12)$$

$$= \mathbf{13.3 \text{ kN/m}^2}$$

Part c

From Eqs. (12.64) and (12.61),

$$c_{u(\text{corrected})} = [7.01e^{-0.08(LL)} + 0.57]c_{u(\text{VST})}$$

$$= [7.01e^{-0.08(50)} + 0.57](20.12)$$

$$= \mathbf{14.05 \text{ kN/m}^2}$$

Remarks

- Vane shear tests can be conducted in the laboratory and in the field during soil exploration.

- In the field, where considerable variation in the undrained shear strength can be found with depth, vane shear tests are extremely useful.