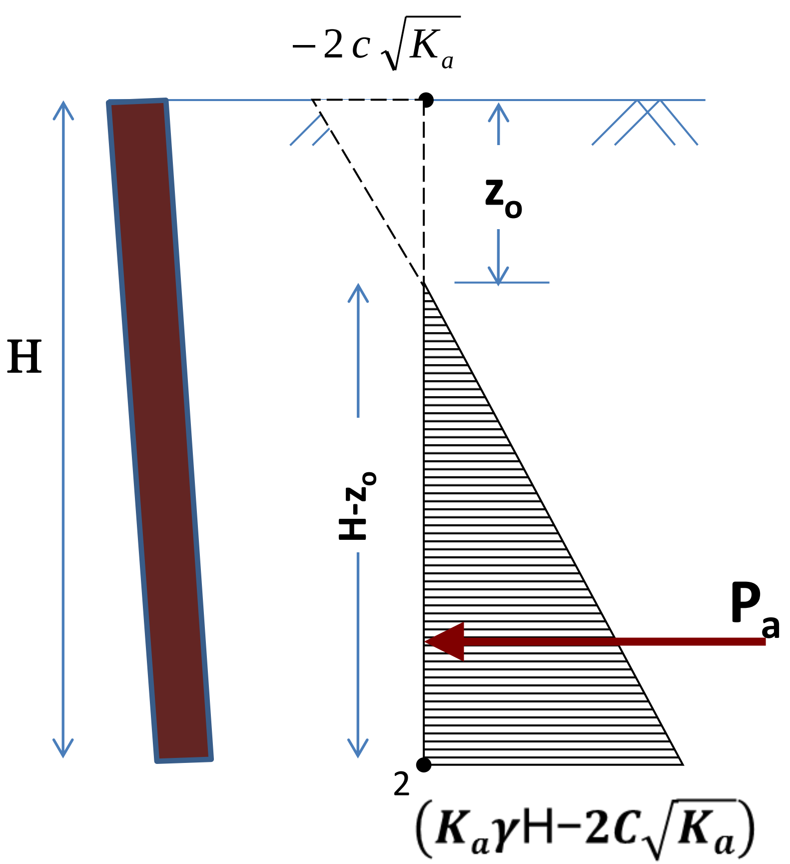
TOPICS

- Introduction
- Coefficient of Lateral Earth Pressure
- Types and Conditions of Lateral Earth Pressures
- Lateral Earth pressure Theories
- Rankine’s Lateral Earth Pressure Theory
- Lateral Earth Pressure Distribution – Cohesionless Soils
- **Lateral Earth Pressure Distribution – C – ϕ Soils**
- Coulomb’s Lateral Earth Pressure Theory

Earth Pressure Distribution

ii. C- ϕ Soils

1. Horizontal Ground Surface



Active Case:

z_o = depth of tension crack

= it is the depth at which active lateral earth pressure is zero

$$0 = K_a \gamma z_o - 2c\sqrt{K_a}$$

$$z_o = \frac{2c}{\gamma\sqrt{K_a}}$$

Earth Pressure force (P_a)

= Area of Earth pressure diagram

$$P_a = \frac{1}{2}(K_a\gamma H - 2C\sqrt{K_a})(H - z_0)$$

For $\phi = 0 \quad K_a = 1$

$$P_a = \frac{1}{2}(\gamma H - 2C)(H - z_0)$$

Point of application of P_a

$(H-z_o)/3$ from the base

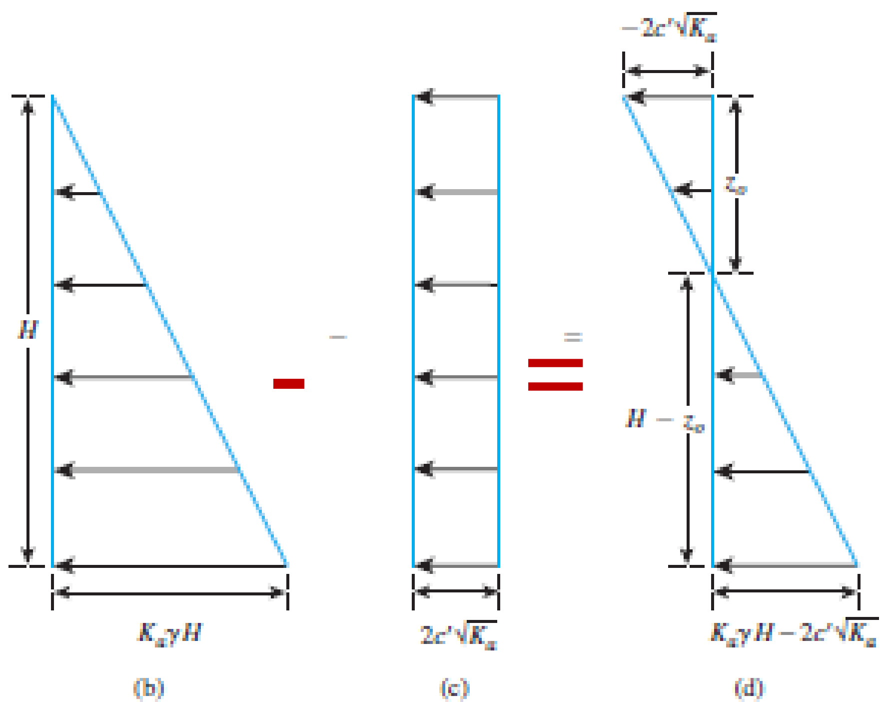
Earth Pressure Distribution

- For calculation of the total active force, common practice is to take the tensile cracks into account. However, if it is not taken then:

$$P_a = \frac{1}{2}K_a\gamma H^2 - 2\sqrt{K_a}c'\,H$$

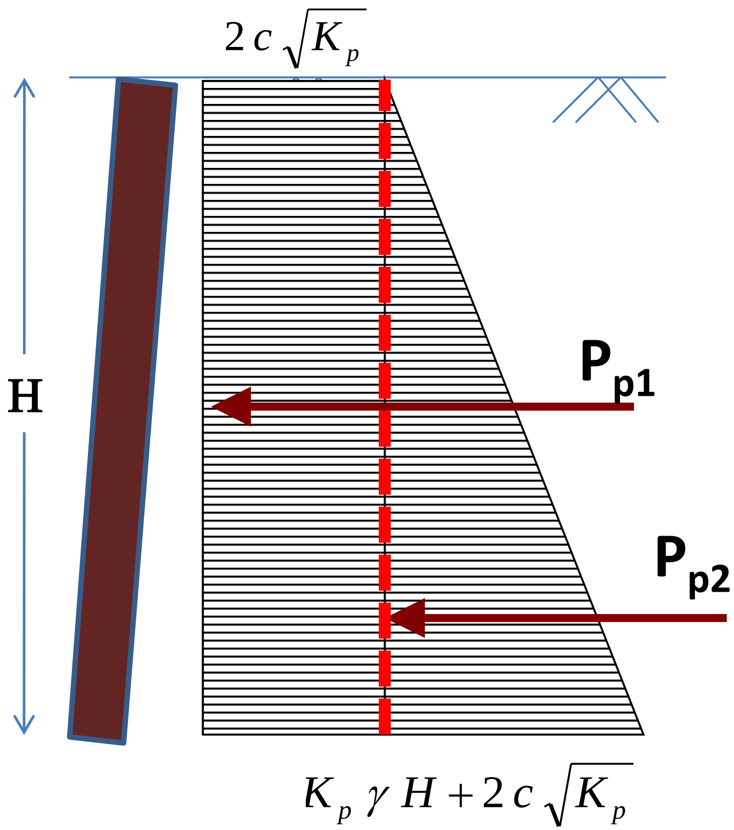
For the $\phi = 0$ condition,

$$P_a = \frac{1}{2}\gamma H^2 - 2c_u H$$



Earth Pressure Distribution

1. Horizontal Ground Surface



For $\phi = 0$, $K_p = 1$, $c = c_u$

$$P_p = \frac{1}{2}\gamma H^2 + 2c_u H$$

Passive Case:

- No tension cracks

$$P_{p1} = 2\sqrt{K_P}\, c\, H$$

$$P_{p2} = \frac{1}{2} K_p \gamma H^2$$

- Earth Pressure force (P_p)

=Area of Earth pressure diagram

$$P_p = \frac{1}{2} K_p \gamma H^2 + 2\sqrt{K_P}\, c\, H$$

- Point of application of P_P

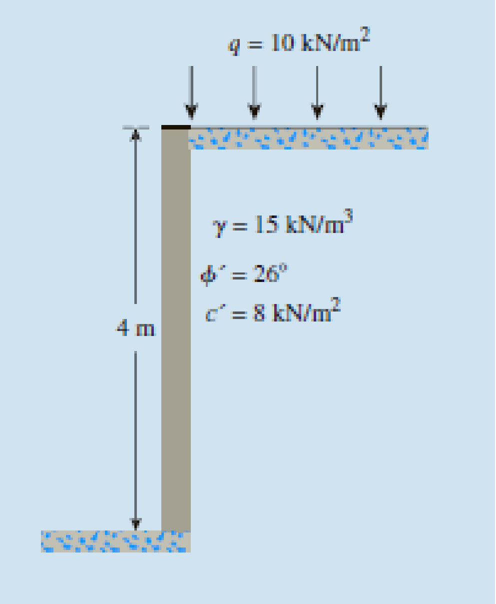
As done before take moment at the base

EXAMPLE 13.8

Example 13.8

A frictionless retaining wall is shown in Figure 13.23a. Determine:

a. The active force P_a after the tensile crack occurs
b. The passive force P_p



Solution

Part a

Given $\phi' = 26°$, we have

$$K_a = \frac{1 - \sin\phi'}{1 + \sin\phi'} = \frac{1 - \sin 26°}{1 + \sin 26°} = 0.39$$

From Eq. (13.31),

$$\sigma'_a = K_a \sigma'_o - 2c'\sqrt{K_a}$$

At $z = 0$,

$$\sigma'_a = (0.39)(10) - (2)(8)\sqrt{0.39} = 3.9 - 9.99 = -6.09 \text{ kN/m}^2$$

At $z = 4$ m,

$$\sigma'_a = (0.39)[10 + (4)(15)] - (2)(8)\sqrt{0.39} = 27.3 - 9.99$$
$$= 17.31 \text{ kN/m}^2$$

EXAMPLE 13.8

The pressure distribution is shown in Figure 13.23b. From this diagram,

$$\frac{6.09}{z_o} = \frac{17.31}{4 \quad z_o}$$

or

$$z_o = 1.04 \text{ m}$$

After the tensile crack occurs,

$$P_a = \frac{1}{2}(4 \quad z_o)(17.31) = \left(\frac{1}{2}\right)(2.96)(17.31) = \mathbf{25.62\ kN/m}$$

Part b

Given $\phi' = 26°$, we have

$$K_p = \frac{1 + \sin\phi'}{1 - \sin\phi'} = \frac{1 + \sin 26°}{1 - \sin 26°} = \frac{1.4384}{0.5616} = 2.56$$

From Eq. (13.35),

$$\sigma'_p = K_p\sigma'_o + 2\sqrt{K_p}c'$$

At $z = 0, \sigma'_o = 10 \text{ kN/m}^2$ and

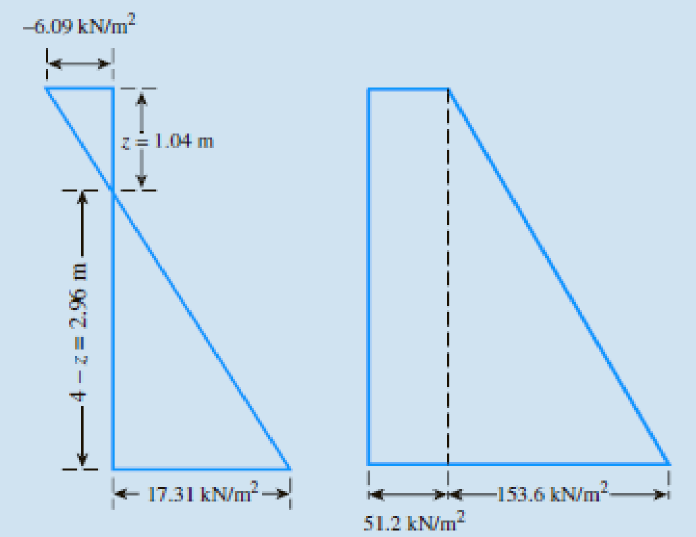
$$\sigma_p = (2.56)(10) + 2\sqrt{2.56}(8)$$
$$= 25.6 + 25.6 = 51.2 \text{ kN/m}^2$$

Again, at $z = 4 \text{ m}, \sigma'_o = (10 + 4 \times 15) = 70 \text{ kN/m}^2$ and

$$\sigma'_p = (2.56)(70) + 2\sqrt{2.56}(8)$$
$$= 204.8 \text{ kN/m}^2$$

The pressure distribution is shown in Figure 13.23c. The passive resistance per unit length of the wall is

$$P_p = (51.2)(4) + \frac{1}{2}(4)(153.6)$$
$$= 204.8 + 307.2 = \mathbf{512\ kN/m}$$



EXAMPLE 13.9

Example 13.9

A retaining wall is shown in Figure 13.24a. Determine P_a after the occurrence of the tensile crack.

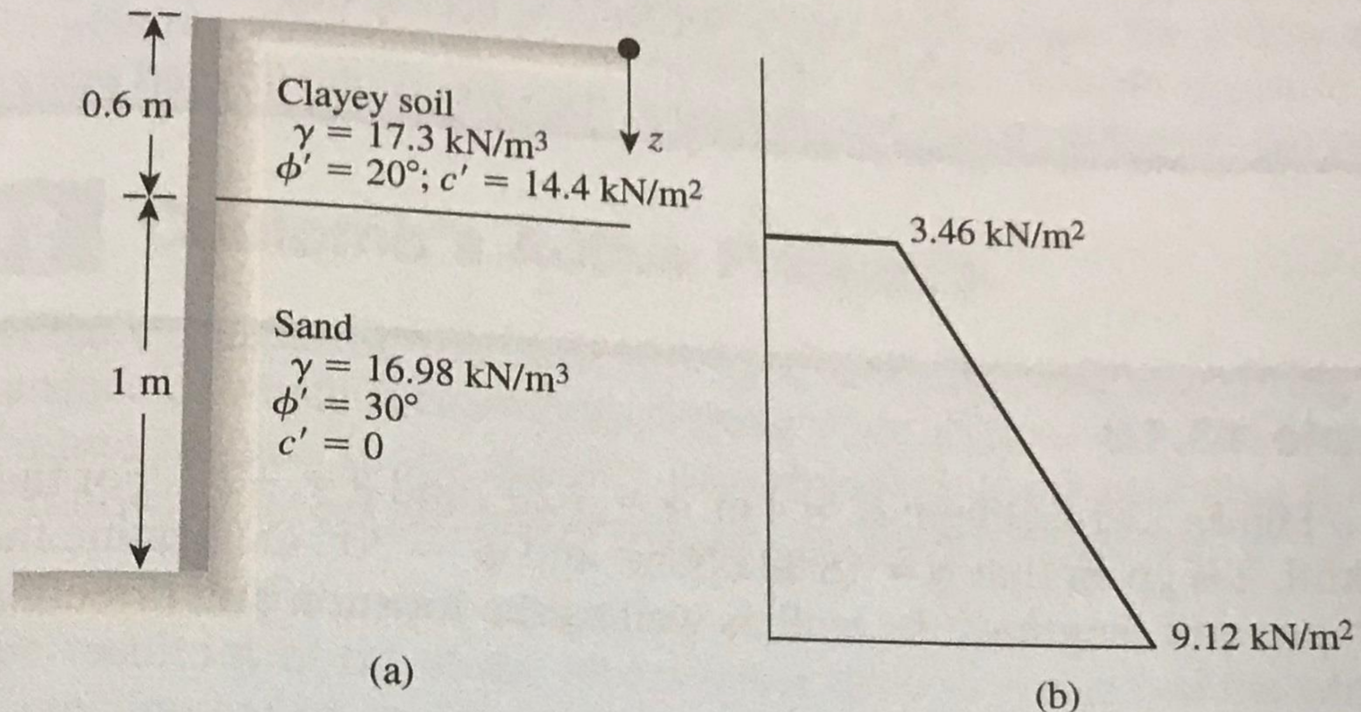


Figure 13.24

EXAMPLE 13.9

Solution

For the upper layer,

$$K_a = K_{a(1)} = \tan^2\left(45 - \frac{20}{2}\right) = 0.49$$

From Eq (13.64),

$$z_o = \frac{2c'}{\gamma\sqrt{K_a}} = \frac{(2)(14.4)}{(17.3)\sqrt{0.49}} = 2.38 \text{ m}$$

Since the depth of the clayey soil layer is 0.6 m (which is less than z_o), the tensile crack will develop up to $z = 0.6$ m. Now

$$K_a = K_{a(2)} = \tan^2\left(45 - \frac{30}{2}\right) = \frac{1}{3}$$

At $z = 0.6$ m,

$$\sigma_o = \sigma'_o = (0.6)(17.3) = 10.38 \text{ kN/m}^2$$

So,

$$\sigma'_a = \sigma'_o K_{a(2)} = (10.38)\left(\frac{1}{3}\right) = 3.46 \text{ kN/m}^2$$

At $z = 1.6$ m,

$$\sigma'_o = (0.6)(17.3) + (1)(16.98) = 10.38 + 16.98 = 27.36 \text{ kN/m}^2$$

$$\sigma'_a = \sigma'_o K_{a(2)} = (27.36)\left(\frac{1}{3}\right) = 9.12 \text{ kN/m}^2$$

The pressure distribution diagram after the occurrence of the tensile crack is shown in Figure 13.24b. From this

$$P_a = \left(\frac{1}{2}\right)(3.46 + 9.12)(1) = \mathbf{6.29\ kN/m}$$

2nd Midterm Exam-Fall 36-37 QUESTION #3

The soil conditions adjacent to a sheet pile wall are given in Fig. 1 below. A surcharge pressure of 50 kN/m² being carried on the surface behind the wall. For soil 1, a sand above the water table, $c' = 0$ kN/m² and $\phi' = 38^o$ and $\gamma = 18$ KN/m³. For soil 2, a saturated clay, $c' = 10$ kN/m² and $\phi' = 28^o$ and $\gamma_{sat} = 20$ KN/m³.

- **Calculate K_a and K_p for each of soils (1) and (2).**
- **Complete the given table for the Rankine active pressure at 6 and 9 m depth behind the behind the wall shown in Fig.1.**
- **Complete the given table for the Rankine passive pressure at 1.5 and 4.5 m depth in front of the wall shown in Fig.1.**

SOLUTION

Soil 1: $K_a = (1-\sin 38)/(1+\sin 38) = 0.24$

$K_p = 1/K_a = 4.17$

Soil 2: $K_a = (1-\sin 28)/1+\sin 28) = 0.36$

$K_p = 1/K_a = 2.78$

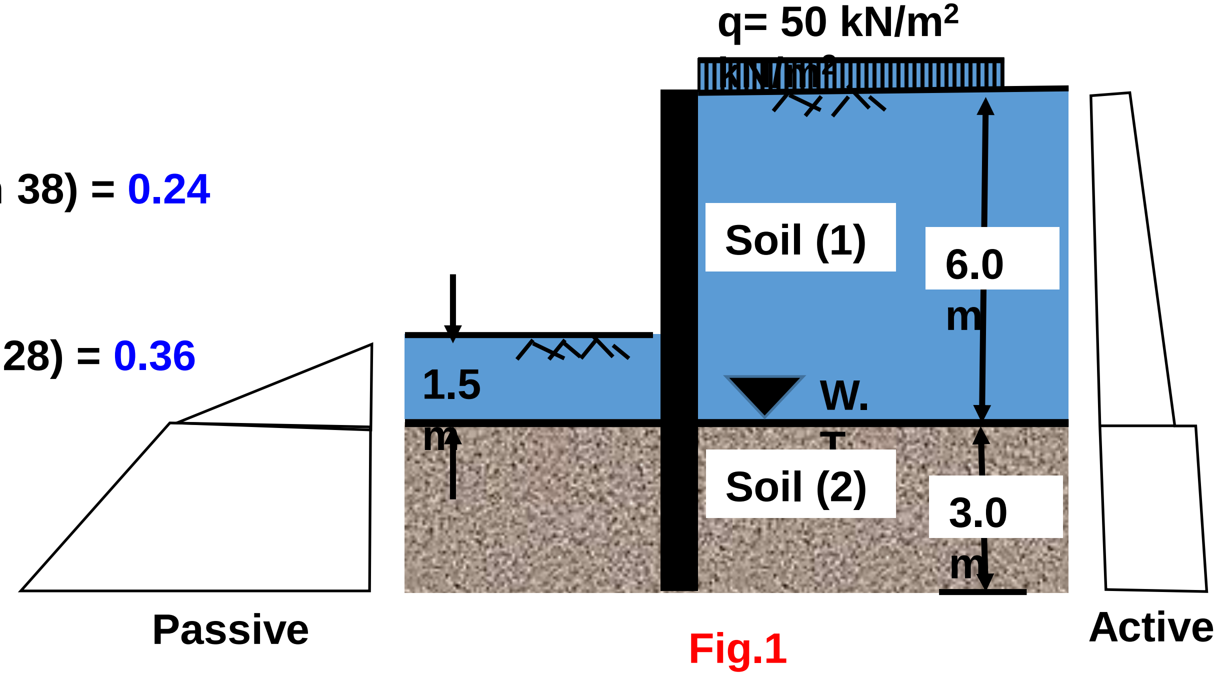


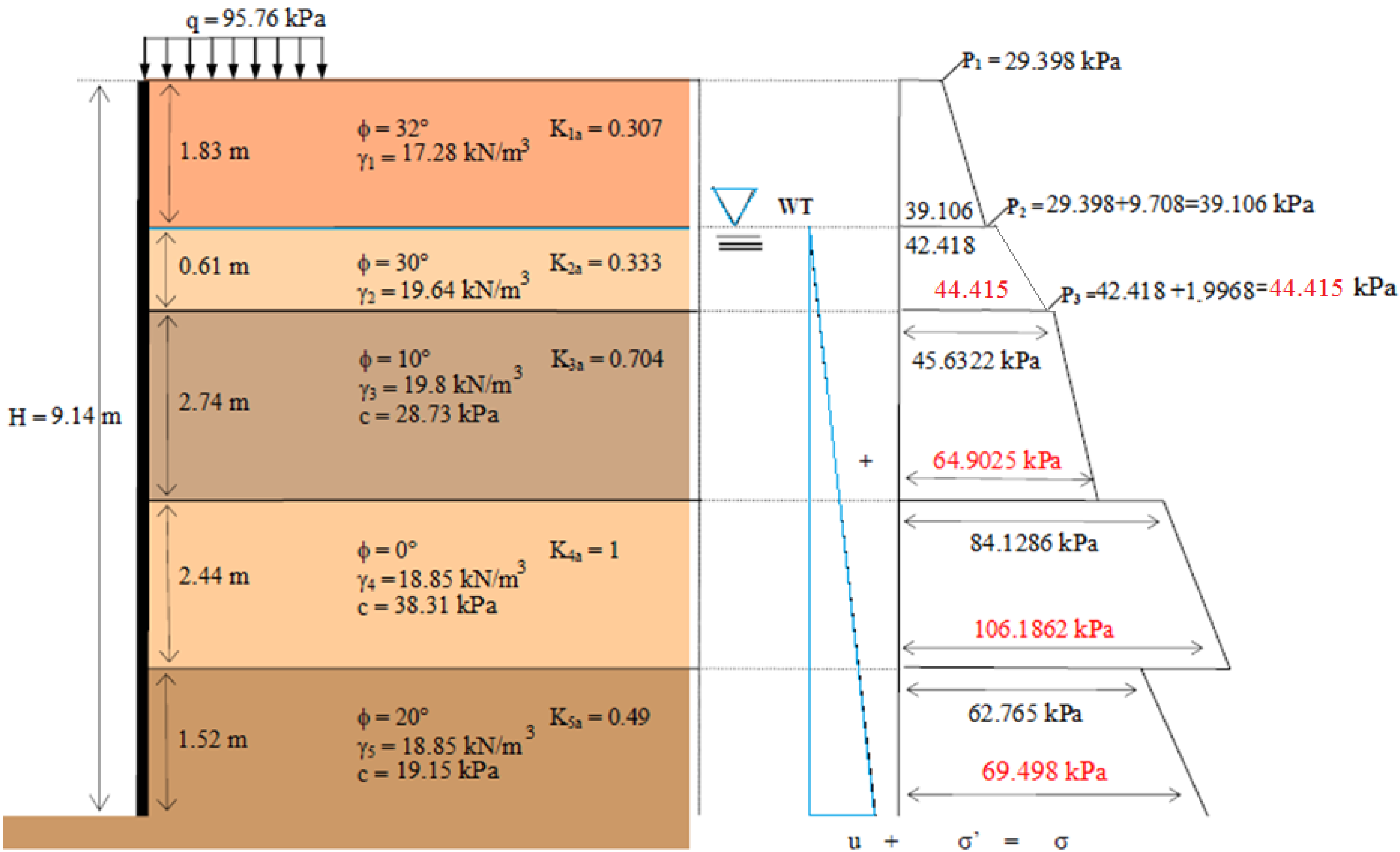
Fig.1

Table 1. Active and passive earth pressures on sheet pile wall shown in Fig. 1.

Depth (meter)	Soil		
		Active Pressure (kN/m^2)	
0	1	0.24 x 50	= 12
6	1	0.24 x (50 + 18 x 6)	= 37.9
6	2	0.36 x (50 + 18 x 6) - 2 x $\sqrt{0.36}$ x 10	= 44.9
9	2	0.36 x (50 + 18 x 6 + 10.2 x 3) - 2 x $\sqrt{0.36}$ x 10 + 9.81 x 3	= 85.33
		Passive Pressure (kN/m^2)	
0	1		= 0
1.5	1	4.17 x 18 x 1.5	= 112.6
1.5	2	2.78 x 18 x 1.5 + 2 x $\sqrt{2.78}$ x 10	= 108.4
4.5	2	2.78 x (18 x 1.5 + 10.2 x 3)+ 2 x $\sqrt{2.78}$ x 10 + 9.81 x 3	= 222.93

EXAMPLE

Plot the Rankine pressure diagram and find the resultant force F and its location under an active pressure condition.



SOLUTION

At h=0' $p_1 = q\, K_{1a} = (95.76)(0.307) = 29.398$ kPa

At h = -1.83 $\Delta p_2 = \gamma_1 h\, K_{1a} = (17.28)(1.83)(0.307) = 9.708$ kPa

At h = -(1.83+dh) = $[q + (\gamma_1)\, 1.83]\, K_{2a} = [95.76 + (17.28)(1.83)]\,(0.333) = 42.418$ kPa

At h = -2.44 $\Delta p_3 = (\gamma_2 - \gamma_w)\, h\, K_{2a} = (19.64 - 9.81)(0.61)(0.333) = 1.9968$ kPa → 42.418 + 1.9968 = 44.415 kPa

At h = -(2.44+dh) = $[q + (\gamma_1)\, 1.83 + (\gamma_2 - \gamma_w)\, 0.61]\, K_{3a} - 2c\sqrt{(K_{3a}}$ from $p = \gamma h\, K_a - 2c\sqrt{K_a}$

$= [95.76 + (17.28)1.83 + (19.64 - 9.81)0.61]\,(0.704) - 2(28.73)(0.84) = 45.6322$ kPa

At h= -5.18 $\Delta p_4 = (\gamma_3 - \gamma_w)\, h\, K_{3a} = (19.8\text{-}9.81)(2.74)(0.704) = 19.27$ kPa ∴ 45.632+19.27 = 64.9025 kPa

At h = - (5.18 + dh) = [95.76 + 31.62 + 5.996 + 27.3726] (1) – 2(38.31)(1) = 84.1286 kPa

At h = -7.62 $\Delta p_5 = (\gamma_4 - \gamma_w)\, h\, K_{4a} = (18.85 - 9.81)(2.44)(1) = 22.0576$ kPa

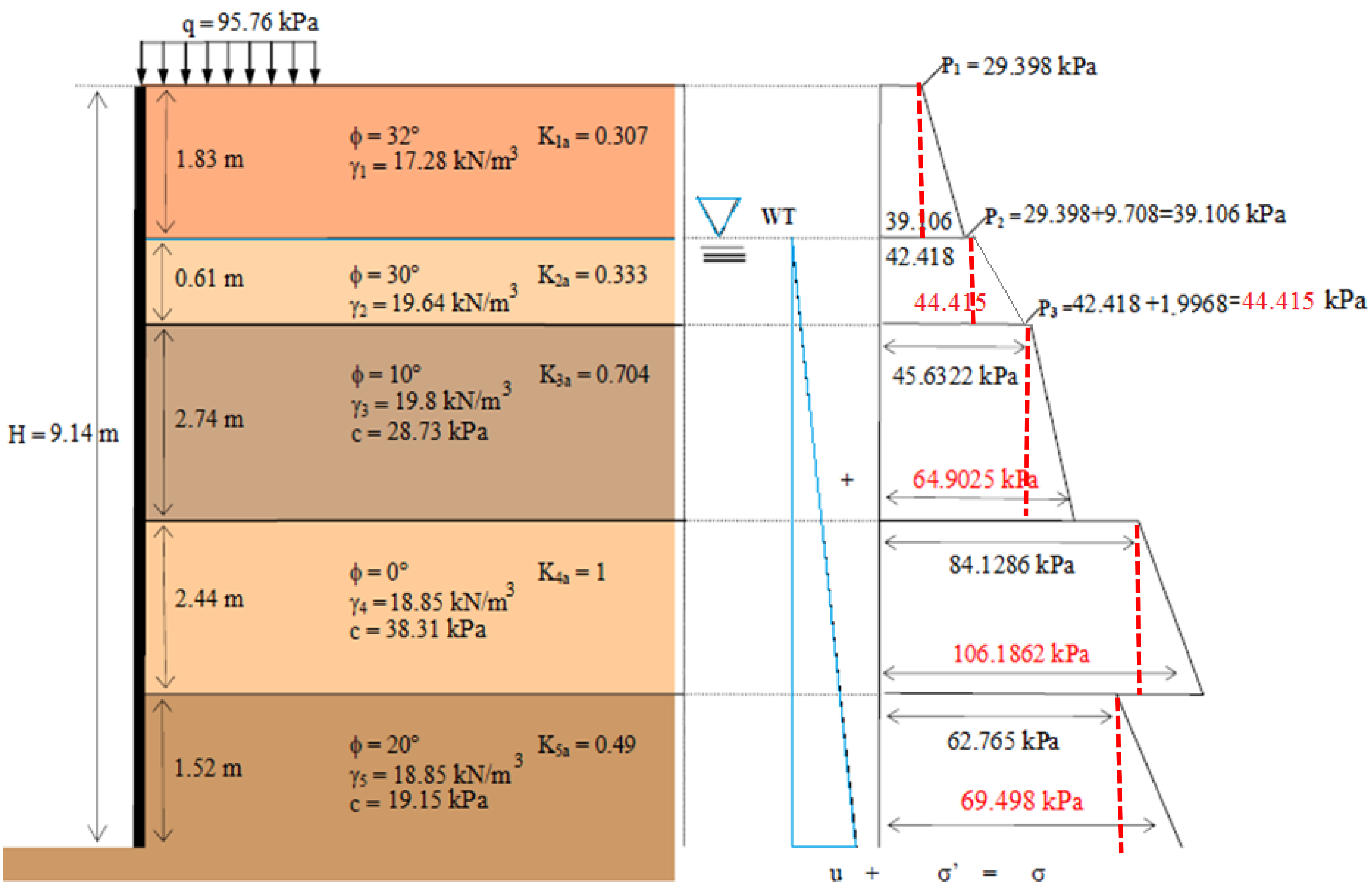
∴ 84.1268+22.0576= 106.1862 kPa

At h = - (7.62 + dh) = [95.76+31.62+ 5.996+27.3726+22.0576](0.49) – 2(19.15)(0.7) = 62.765 kPa

At h = -9.14 $\Delta p_6 = (\gamma_5 - \gamma_w)\, h\, K_{5a} = (18.85\text{-}9.81)(1.52)(0.49) = 6.733$ kPa

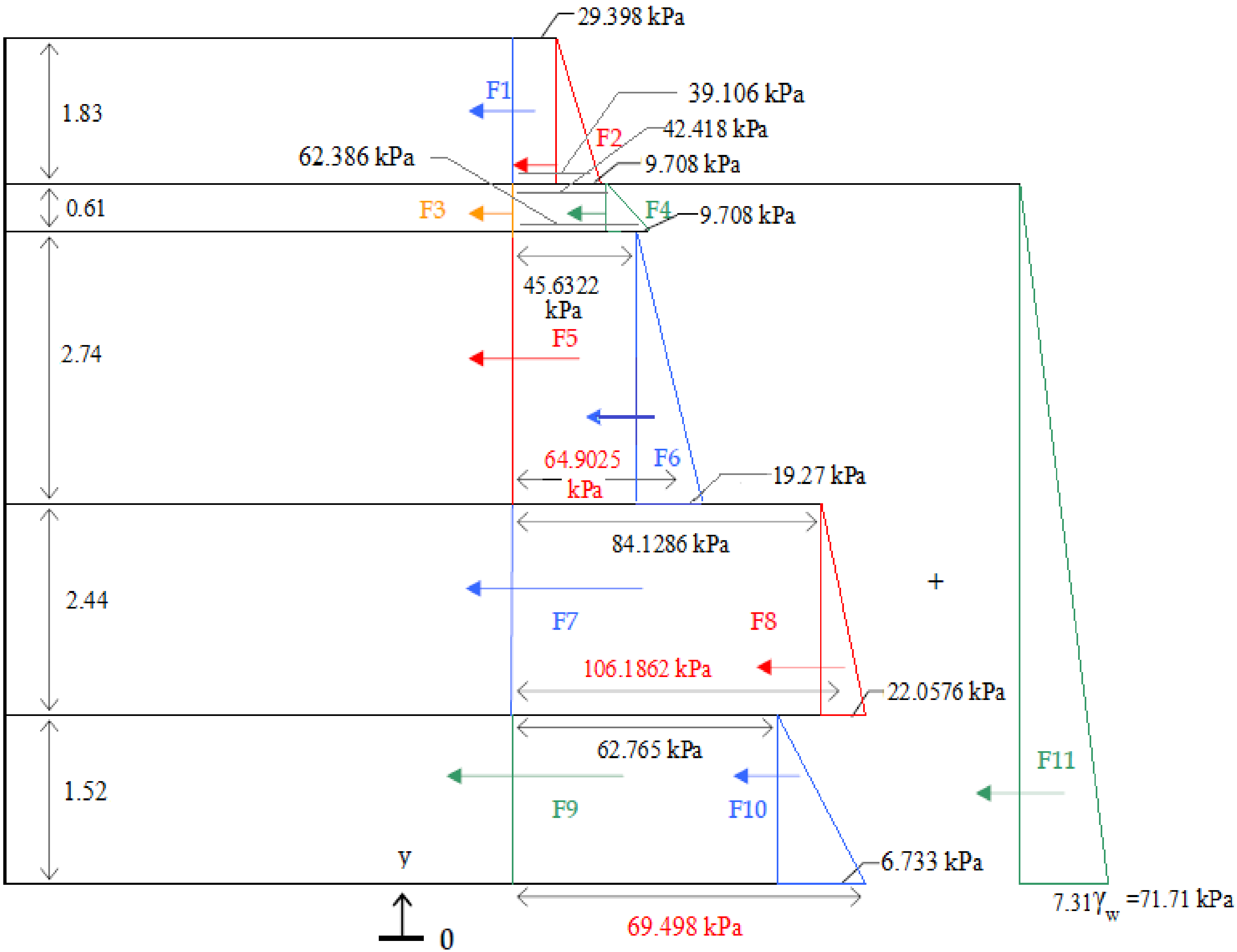
∴ 62.765+6.33 =69.498 kPa

SOLUTION



q = 95.76 kPa
H = 9.14 m
1.83 m
ϕ = 32°
γ1 = 17.28 kN/m3
K1a = 0.307
0.61 m
ϕ = 30°
γ2 = 19.64 kN/m3
K2a = 0.333
2.74 m
ϕ = 10°
γ3 = 19.8 kN/m3
c = 28.73 kPa
K3a = 0.704
2.44 m
ϕ = 0°
γ4 = 18.85 kN/m3
c = 38.31 kPa
K4a = 1
1.52 m
ϕ = 20°
γ5 = 18.85 kN/m3
c = 19.15 kPa
K5a = 0.49
WT
P1 = 29.398 kPa
39.106
P2 = 29.398+9.708=39.106 kPa
42.418
44.415
P3 = 42.418 + 1.9968=44.415 kPa
45.6322 kPa
64.9025 kPa
84.1286 kPa
106.1862 kPa
62.765 kPa
69.498 kPa
+
u + σ' = σ

SOLUTION



29.398 kPa
1.83
F1
39.106 kPa
42.418 kPa
F2
62.386 kPa
9.708 kPa
0.61
F3
F4
9.708 kPa
45.6322
kPa
2.74
F5
64.9025
kPa
F6
19.27 kPa
84.1286 kPa
2.44
F7
F8
106.1862 kPa
22.0576 kPa
62.765 kPa
1.52
F9
F10
F11
6.733 kPa
y
0
69.498 kPa
7.31γw =71.71 kPa

SOLUTION

$F1 = (29.398\ kPa)(1.83) = 53.798\ kN$

$F2 = 0.5(9.708\ kPa)(1.83) = 8.8828\ kN$

$F3 = (42.418)(0.61) = 25.875\ kN$

$F4 = 0.5(19.968\ kPa)(0.61) = 7.8919\ kN$

$F5 = (45.6322\ kPa)(2.74) = 125.032\ kN$

$F6 = 0.5(19.27\ kPa)(2.74) = 26.3999\ kN$

$F7 = (84.1286\ kPa)(2.44) = 205.2738\ kN$

$F8 = 0.5(22.0576\ kPa)(2.44) = 26.91\ kN$

$F9 = (62.765\ kPa)(1.52) = 95.4\ kN$

$F10 = 0.5(6.733\ kPa)(1.52) = 5.117\ kN$

$F11 = 0.5(71.71)(7.31) = 262.104\ kN$

The resultant R is, $\mathbf{R = \Sigma\ F}i = 842.68448\ kN$

The location of R is……$\Sigma M_0 = 0$ (about 0)

$842.68448(y) = (53.798)(8.225) + (8.8828)(7.92) + (25.875)(7.005) + (7.8919)(6.903) + (125.032)(5.33) + (26.3999)(4.873) + (205.273)(2.74) + (26.91)(2.33) + (95.4)(0.76) + (5.117)(0.506) + (262.104)(2.4366)$

$\therefore$ 57.1 y = 2882.53945→→→→→ y = 3.4206 m **<u>above “0”</u>**

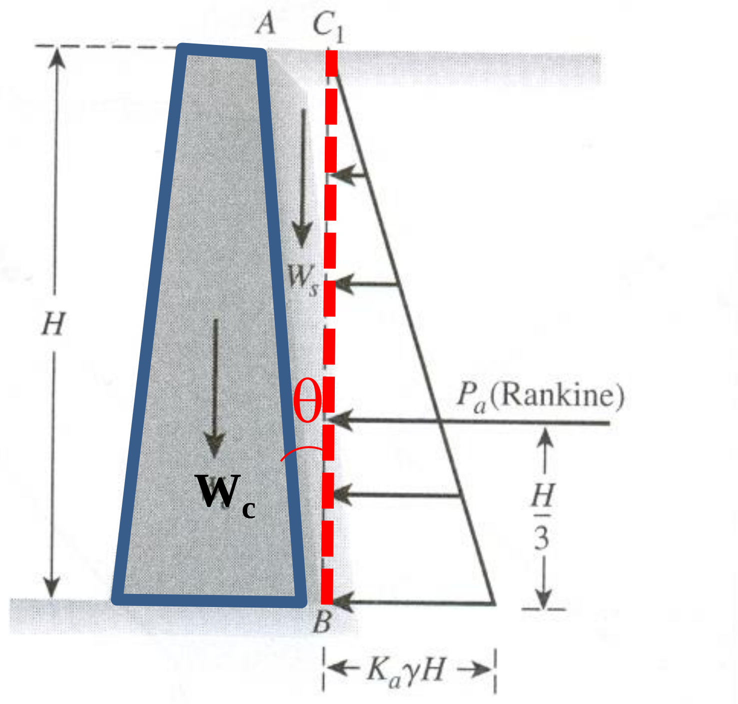
RECOMMENDED PROCEDURE

1. **Calculate the appropriate k for each soil**
2. **Calculate σ_V at a specified depth**
3. **Add q if any**
4. **Multiply the sum of $\sigma_V + q$ by the appropriate k (for upper and lower soil) and subtract (or add for passive) cohesion part if exists.**
5. **Calculate water pressure**
6. **Divide each trapezoidal area into a rectangle and a triangle**
7. **Calculate areas and that give the lateral forces**
8. **Locate point of application for each force**
9. **Find the resultant force**
10. **Take moments about the base of the wall and find location of the resultant**

Rankine's Earth Pressure Theory- Special Cases

I. Horizontal Ground & Inclined Wall Back

- No lower bound (Mohr's Circle) solution is available for this case.
- Assume an imaginary vertical wall BC_1
- The weight of the wedge of soil (W_s) is added vectorally to the earth pressure force for stability analysis.



- **Notes**
  - Same as vertical wall only we consider W_s in addition to P_a when analysing the stability of the wall.
  - This is only approximate solution.
  - Only active case is provided (It is more practical).

$$W_s = 1/2.\gamma.H^2.\tan\theta$$

II. Inclined Ground & Vertical Wall Back

- In this case, the direction of Rankine's active or passive pressures are **no longer horizontal**. Rather, they are inclined at an angle α with the horizontal.

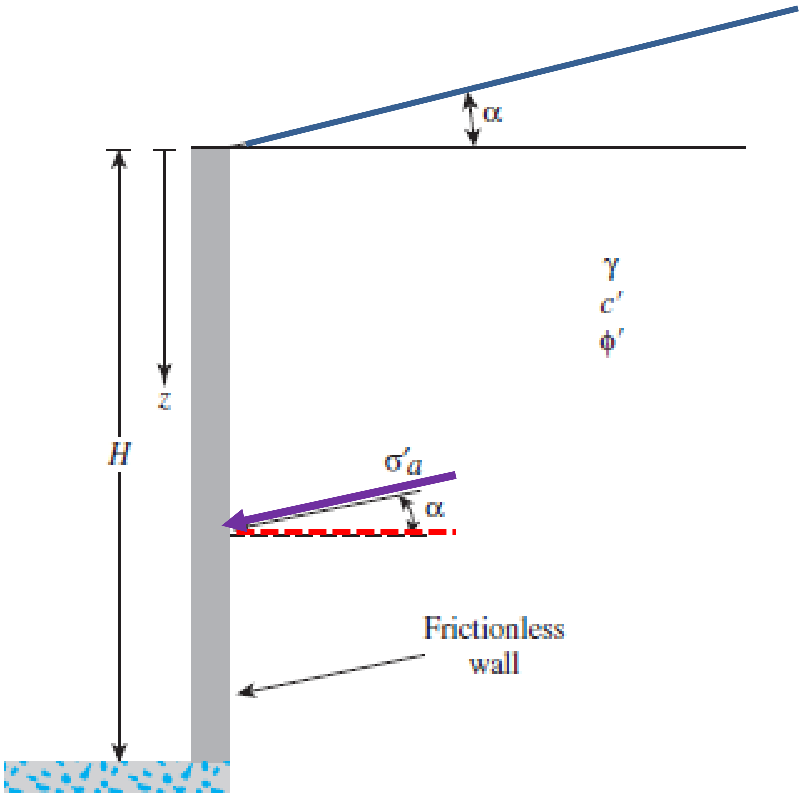
- If the backfill is a **granular** soil with a friction angle ϕ, and $C = 0$,

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$$

$$K_p = \cos\alpha \frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}$$

$$P_a = \frac{1}{2} K_a \gamma H^2$$

$$P_p = \frac{1}{2} \gamma H^2 K_p$$



For horizontal ground surface $\alpha = 0$

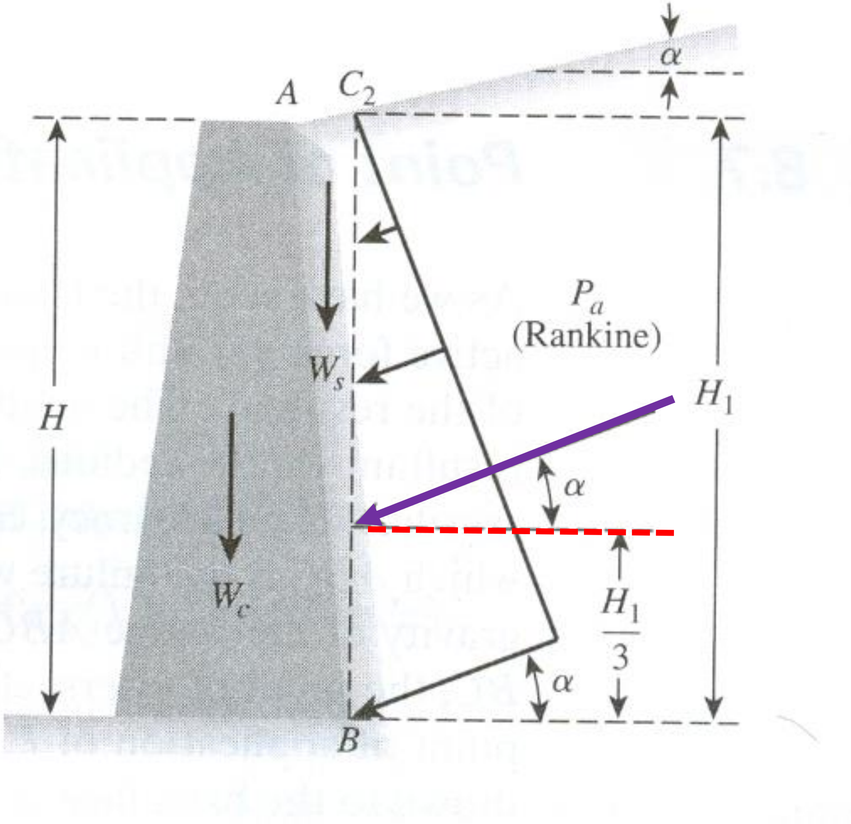
$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi}$$

The line of action of the resultant acts at a distance of $H/3$ measured from the bottom of the wall.

III. Inclined Ground & Inclined Wall Back – Approximate Solution

- Assume an imaginary vertical wall BC_2
- The weight of the wedge of soil (W_s) is added vectorally to the earth pressure force for stability analysis.



$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$$

$$K_p = \cos\alpha \frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}$$

For horizontal ground surface α = 0

$$K_a = \frac{1-\sin\phi}{1+\sin\phi}$$

$$K_P = \frac{1+\sin\phi}{1-\sin\phi}$$

REMARKS

- P_a acts parallel to the ground surface
- For stability analysis W_s is vectorally added to P_a
- Plane BC_2 is not the minor principal plane.
- This is only an approximate solution. No available lower bound (Mohr Circle) solution for this case.
- Upper bound solution (kinematic) for this case is given by Coulomb.
- Rankine kinematic upper bound solutions are special cases or approximation to Coulomb solution and Coulomb solution is a generalization of Rankine solution. (Rankine 1857, Coulomb 1776).
- Wall inclination affects the value of H_1 and W_s. For vertical wall, $H_1 = H$, $W_s = 0$.

I. ACTIVE CASE - Granular Backfill

W = weight of soil wedge.

F = reaction from supporting soil.

P_a = **maximum** reaction from wall required for equilibrium.

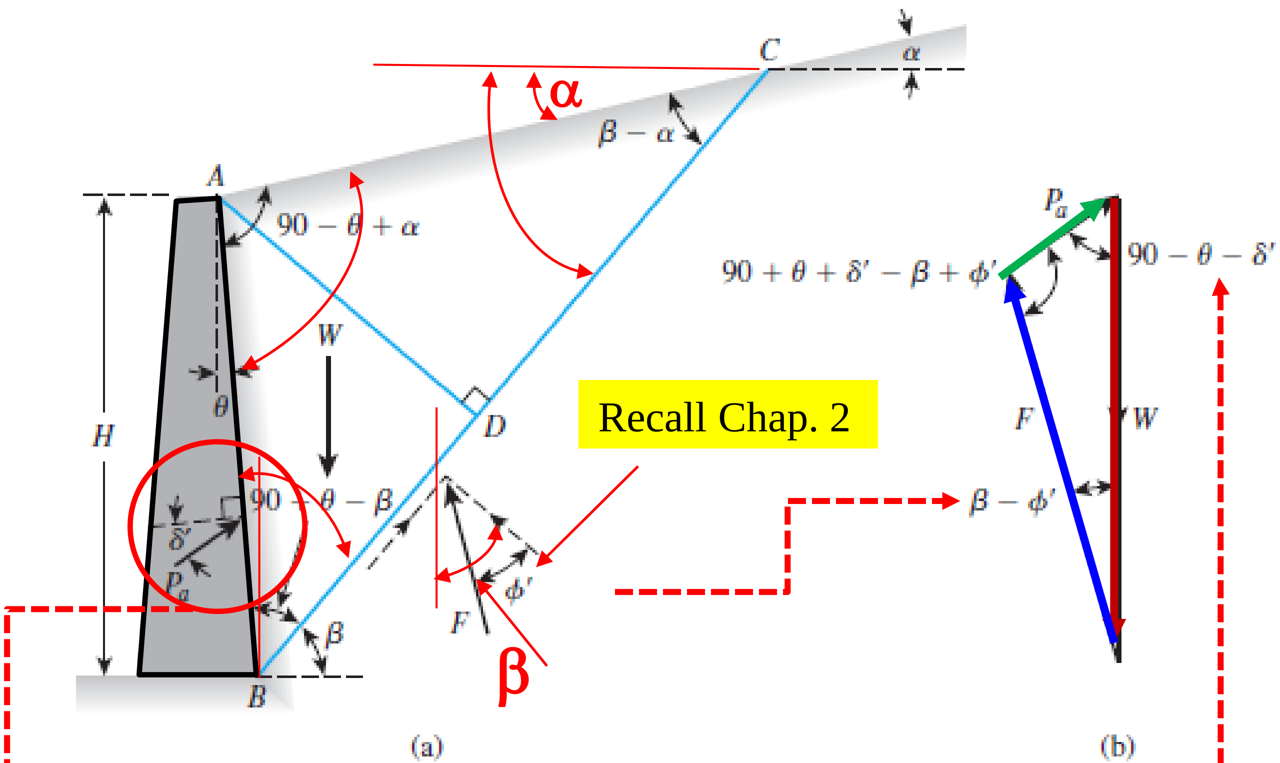
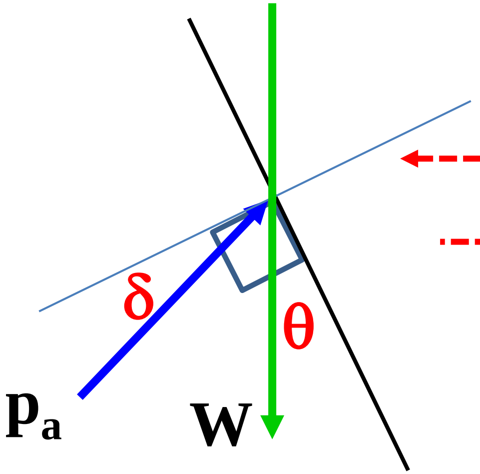


Figure 13.19 Coulomb's active pressure: (a) trial failure wedge; (b) force polygon



$$\frac{W}{\sin(90 + \theta + \delta' - \beta + \phi')} = \frac{P_a}{\sin(\beta - \phi')}$$

I. ACTIVE CASE - Granular Backfill

$$P_a = \frac{1}{2}\gamma H^2\left[\frac{\cos(\theta-\beta)\cos(\theta-\alpha)\sin(\beta-\phi')}{\cos^2\theta\sin(\beta-\alpha)\sin(90+\theta+\delta'-\beta+\phi')}\right]$$

$$\frac{dP_a}{d\beta} = 0 \quad \Rightarrow \quad P_a = \frac{1}{2}K_a\gamma H^2$$

$$K_a = \frac{\cos^2(\phi'-\theta)}{\cos^2\theta\cos(\delta'+\theta)\left[1+\sqrt{\frac{\sin(\delta'+\phi')\sin(\phi'-\alpha)}{\cos(\delta'+\theta)\cos(\theta-\alpha)}}\right]^2}$$

Note that when $\alpha = 0°$, $\theta = 0°$, and $\delta' = 0°$, Coulomb's active earth-pressure coefficient becomes equal to $(1 - \sin\phi')/(1 + \sin\phi')$, which is the same as Rankine's earth-pressure coefficient given earlier in this chapter.

I. ACTIVE CASE - Granular Backfill

$$K_a = \frac{\cos^2(\phi' - \theta)}{\cos^2\theta \cos(\delta' + \theta)\left[1 + \sqrt{\frac{\sin(\delta' + \phi')\sin(\phi' - \alpha)}{\cos(\delta' + \theta)\cos(\theta - \alpha)}}\right]^2}$$

Table 13.4 Values of K_a [Eq. (13.78)] for $\theta = 0°$, $\alpha = 0°$

↓ ϕ' (deg)	δ' (deg) → 0	5	10	15	20	25
28	0.3610	0.3448	0.3330	0.3251	0.3203	0.3186
30	0.3333	0.3189	0.3085	0.3014	0.2973	0.2956
32	0.3073	0.2945	0.2853	0.2791	0.2755	0.2745
34	0.2827	0.2714	0.2633	0.2579	0.2549	0.2542
36	0.2596	0.2497	0.2426	0.2379	0.2354	0.2350
38	0.2379	0.2292	0.2230	0.2190	0.2169	0.2167
40	0.2174	0.2089	0.2045	0.2011	0.1994	0.1995
42	0.1982	0.1916	0.1870	0.1841	0.1828	0.1831

I. ACTIVE CASE - Granular Backfill

$$K_a = \frac{\cos^2(\phi' - \theta)}{\cos^2\theta \cos(\delta' + \theta)\left[1 + \sqrt{\frac{\sin(\delta' + \phi')\sin(\phi' - \alpha)}{\cos(\delta' + \theta)\cos(\theta - \alpha)}}\right]^2}$$

Table 13.5 Values of K_a [Eq. (13.78)] (*Note:* $\delta' = \frac{2}{3}\phi'$)

		θ (deg)					
α (deg)	φ′ (deg)	0	5	10	15	20	25
0	28	0.3213	0.3588	0.4007	0.4481	0.5026	0.5662
	29	0.3091	0.3467	0.3886	0.4362	0.4908	0.5547
	30	0.2973	0.3349	0.3769	0.4245	0.4794	0.5435
	31	0.2860	0.3235	0.3655	0.4133	0.4682	0.5326
	32	0.2750	0.3125	0.3545	0.4023	0.4574	0.5220
	33	0.2645	0.3019	0.3439	0.3917	0.4469	0.5117
	34	0.2543	0.2916	0.3335	0.3813	0.4367	0.5017
	35	0.2444	0.2816	0.3235	0.3713	0.4267	0.4919
	36	0.2349	0.2719	0.3137	0.3615	0.4170	0.4824
	37	0.2257	0.2626	0.3042	0.3520	0.4075	0.4732
	38	0.2168	0.2535	0.2950	0.3427	0.3983	0.4641
	39	0.2082	0.2447	0.2861	0.3337	0.3894	0.4553
	40	0.1998	0.2361	0.2774	0.3249	0.3806	0.4468
	41	0.1918	0.2278	0.2689	0.3164	0.3721	0.4384
	42	0.1840	0.2197	0.2606	0.3080	0.3637	0.4302

I. ACTIVE CASE - Granular Backfill

$$K_a = \frac{\cos^2(\phi' - \theta)}{\cos^2\theta \cos(\delta' + \theta)\left[1 + \sqrt{\frac{\sin(\delta' + \phi')\sin(\phi' - \alpha)}{\cos(\delta' + \theta)\cos(\theta - \alpha)}}\right]^2}$$

Table 13.6 Values of K_a [Eq. (13.78)] (*Note:* $\delta' = \phi'/2$)

α (deg)	ϕ' (deg)	θ (deg)					
		0	5	10	15	20	25
0	28	0.3264	0.3629	0.4034	0.4490	0.5011	0.5616
	29	0.3137	0.3502	0.3907	0.4363	0.4886	0.5492
	30	0.3014	0.3379	0.3784	0.4241	0.4764	0.5371
	31	0.2896	0.3260	0.3665	0.4121	0.4645	0.5253
	32	0.2782	0.3145	0.3549	0.4005	0.4529	0.5137
	33	0.2671	0.3033	0.3436	0.3892	0.4415	0.5025
	34	0.2564	0.2925	0.3327	0.3782	0.4305	0.4915
	35	0.2461	0.2820	0.3221	0.3675	0.4197	0.4807
	36	0.2362	0.2718	0.3118	0.3571	0.4092	0.4702
	37	0.2265	0.2620	0.3017	0.3469	0.3990	0.4599
	38	0.2172	0.2524	0.2920	0.3370	0.3890	0.4498
	39	0.2081	0.2431	0.2825	0.3273	0.3792	0.4400
	40	0.1994	0.2341	0.2732	0.3179	0.3696	0.4304
	41	0.1909	0.2253	0.2642	0.3087	0.3602	0.4209
	42	0.1828	0.2168	0.2554	0.2997	0.3511	0.4117

EXAMPLE 13.11

Example 13.11

Refer to Figure 13.25. Given: $\alpha = 10°$; $\theta = 5°$; $H = 4$ m; unit weight of soil, $\gamma = 15$ kN/m^3; soil friction angle, $\phi' = 30°$; and $\delta' = 15°$. Estimate the active force, P_a, per unit length of the wall. Also, state the direction and location of the resultant force, P_a.

Solution

From Eq. (13.77),

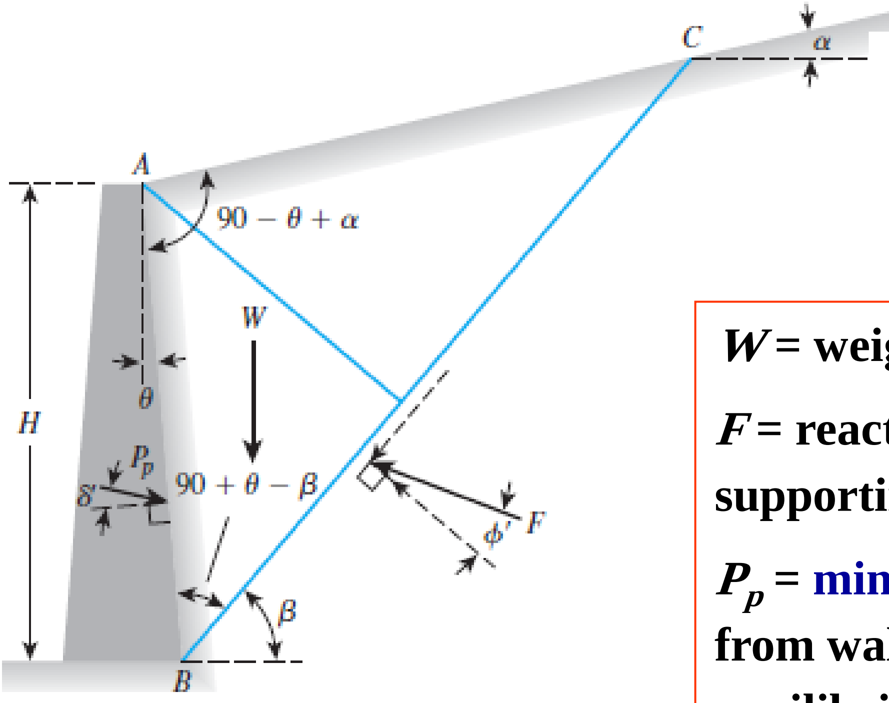
$$P_a = \frac{1}{2}\gamma H^2 K_a$$

For $\phi' = 30°$; $\delta' = 15°$—that is, $\frac{\delta'}{\phi'} = \frac{15}{30} = \frac{1}{2}$; $\alpha = 10°$; and $\theta = 5°$, the magnitude of K_a is 0.3872 (Table 13.6). So,

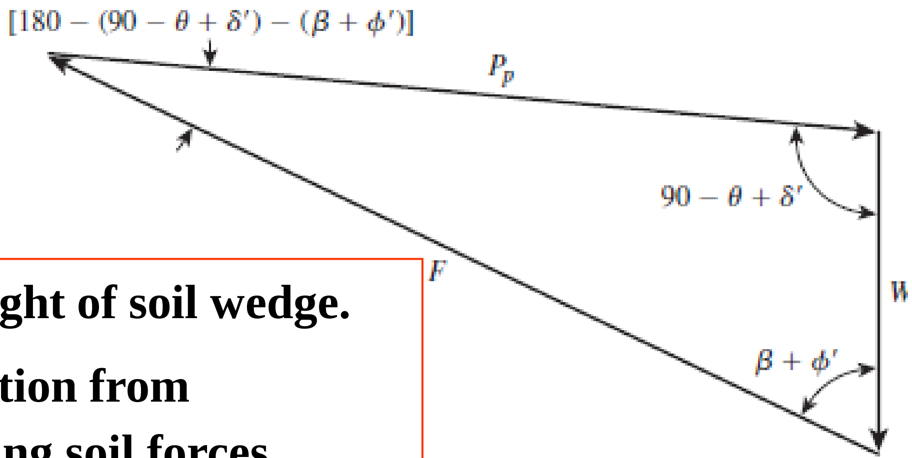
$$P_a = \frac{1}{2}(15)(4)^2(0.3872) = \mathbf{46.46\ kN/m}$$

The resultant will act at a vertical distance equal to $H/3 - 4/3 - 1.33$ m above the bottom of the wall and will be inclined at an angle of 15° ($= \delta'$) to the back face of the wall.

II. PASSIVE CASE – Granular Backfill



(a)



(b)

W **= weight of soil wedge.**

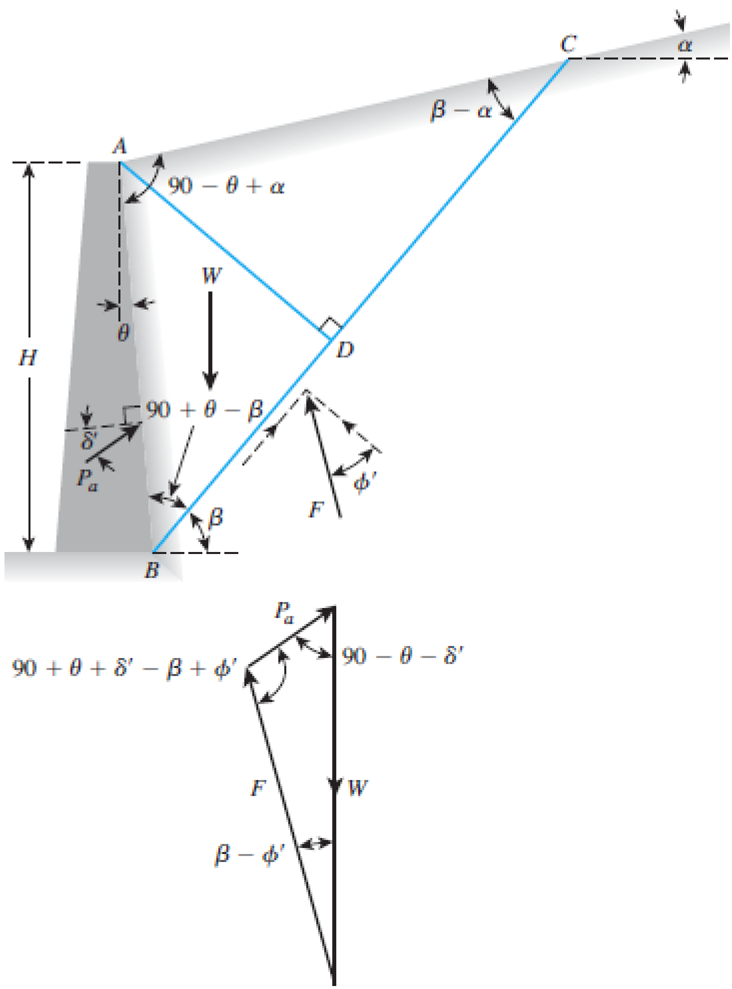
F **= reaction from supporting soil forces.**

$\boldsymbol{P_p}$ **= minimum reaction from wall required for equilibrium**

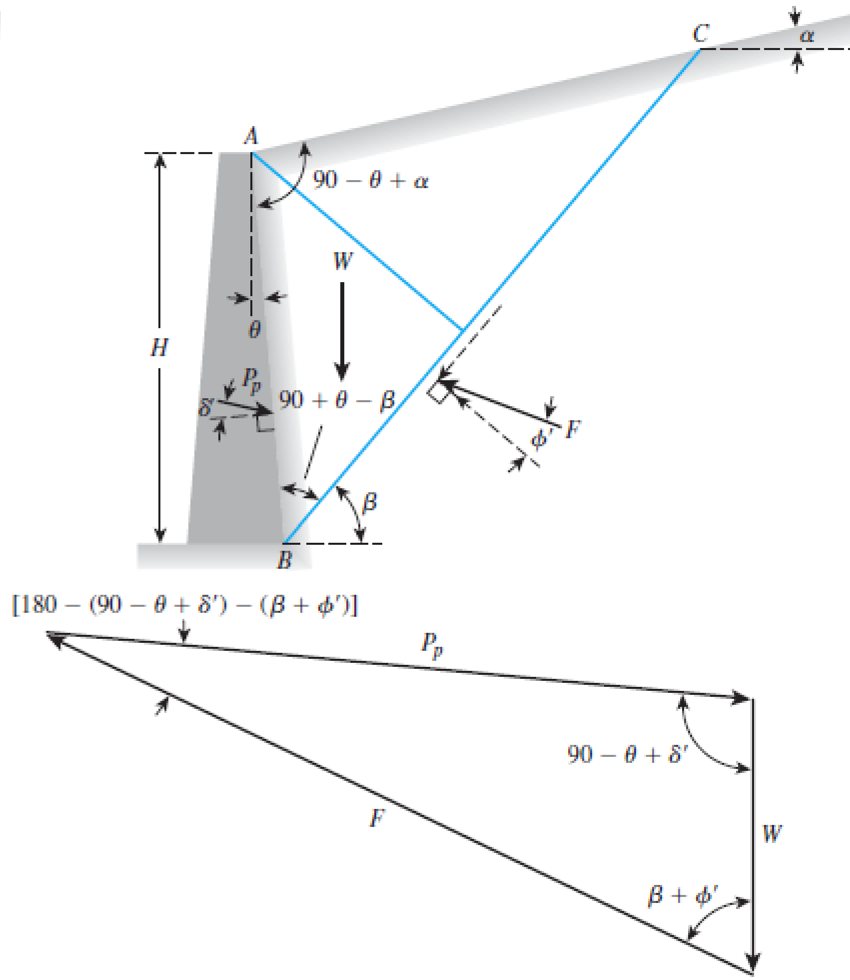
$$P_p = \tfrac{1}{2} \times K_p \times \gamma \times H^2$$

$$K_p = \frac{\cos^2(\phi' + \theta)}{\cos^2\theta \cos(\delta' - \theta)\left[1 - \sqrt{\dfrac{\sin(\phi' + \delta')\sin(\phi' + \alpha)}{\cos(\delta' - \theta)\cos(\alpha - \theta)}}\right]^2}$$

Active Vs Passive



Active Coulomb Wedge Analysis



Passive Coulomb Wedge Analysis

II. PASSIVE CASE – Granular Backfill

$$K_p = \frac{\cos^2(\phi' + \theta)}{\cos^2\theta \cos(\delta' - \theta)\left[1 - \sqrt{\frac{\sin(\phi' + \delta')\sin(\phi' + \alpha)}{\cos(\delta' - \theta)\cos(\alpha - \theta)}}\right]^2}$$

Table 13.7 Values of K_p [Eq. 13.80] for $\theta = 0°, \alpha = 0°$

	δ' (deg) →				
↓ ϕ' (deg)	0	5	10	15	20
15	1.698	1.900	2.130	2.405	2.735
20	2.040	2.313	2.636	3.030	3.525
25	2.464	2.830	3.286	3.855	4.597
30	3.000	3.506	4.143	4.977	6.105
35	3.690	4.390	5.310	6.854	8.324
40	4.600	5.590	6.946	8.870	11.772

II. PASSIVE CASE – Granular Backfill

$$K_p = \frac{\cos^2(\phi' + \theta)}{\cos^2\theta \cos(\delta' - \theta)\left[1 - \sqrt{\dfrac{\sin(\phi' + \delta')\sin(\phi' + \alpha)}{\cos(\delta' - \theta)\cos(\alpha - \theta)}}\right]^2}$$

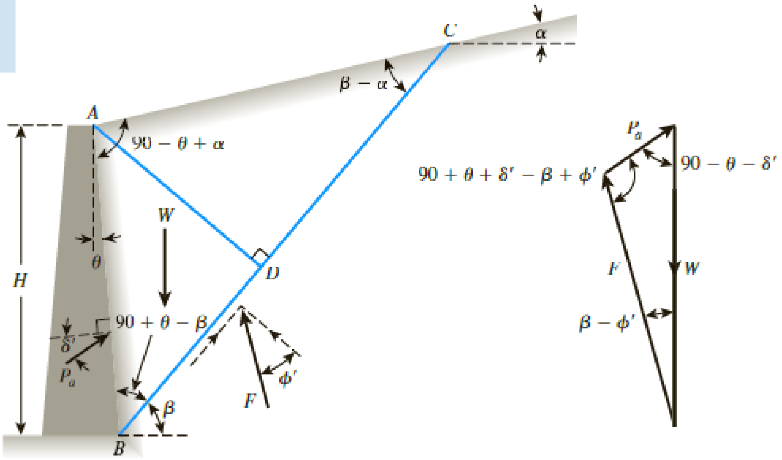
For a frictionless wall with the vertical back face supporting granular soil backfill with a horizontal surface (that is, $\theta = 0°$, $\alpha = 0°$, and $\delta' = 0°$)

$$K_p = \frac{1 + \sin\phi'}{1 - \sin\phi'} = \tan^2\left(45 + \frac{\phi'}{2}\right)$$

EXAMPLE 13.11

Example 13.11

Refer to Figure 13.25. Given: $\alpha = 10°$; $\theta = 5°$; $H = 4$ m; unit weight of soil, $\gamma = 15$ kN/m^3; soil friction angle, $\phi' = 30°$; and $\delta' = 15°$. Estimate the active force, P_a, per unit length of the wall. Also, state the direction and location of the resultant force, P_a.



Solution

From Eq. (13.77),

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

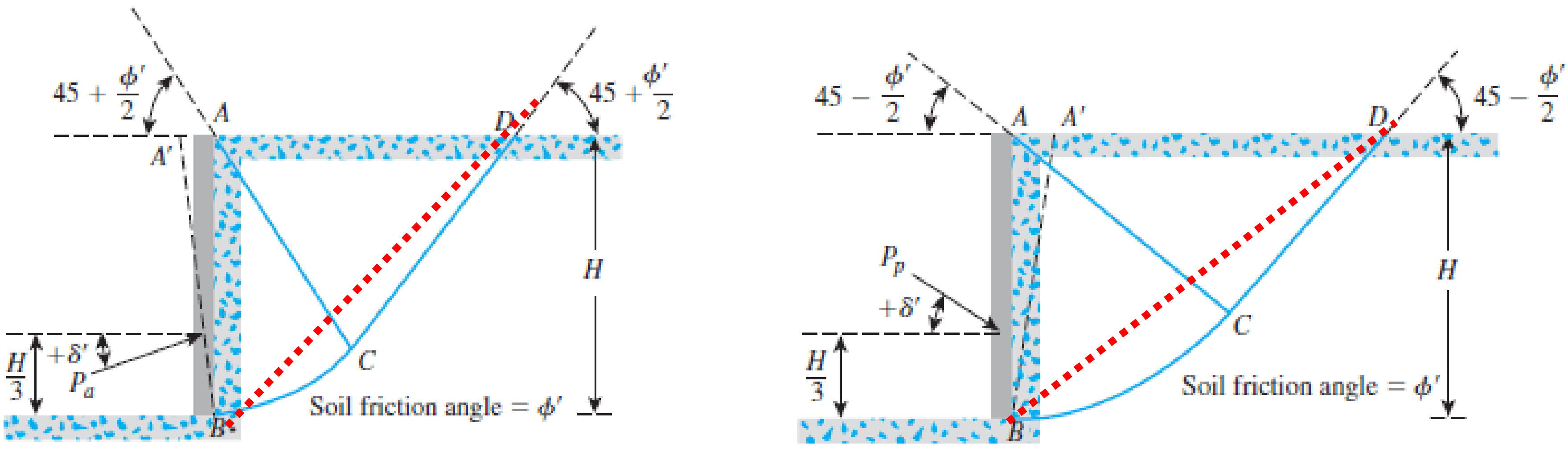
For $\phi' = 30°$; $\delta' = 15°$—that is, $\frac{\delta'}{\phi'} = \frac{15}{30} = \frac{1}{2}$; $\alpha = 10°$; and $\theta = 5°$, the magnitude of K_a is 0.3872 (Table 13.6). So,

$$P_a = \frac{1}{2}(15)(4)^2(0.3872) = \mathbf{46.46\ kN/m}$$

The resultant will act at a vertical distance equal to $H/3 = 4/3 = 1.33$ m above the bottom of the wall and will be inclined at an angle of 15° ($= \delta'$) to the back face of the wall.

REMARKS ON COULOMB's THEORY

- δ can be determined in the laboratory by means of direct shear test.
- Due to wall friction the shape of the failure surface is curved near the bottom of the wall in both the active and passive cases but Coulomb theory assumes plane surface. In the active case the curvature is light and the error involved in assuming plane surface is relatively small. This is also true in the passive case for value of $\delta < \phi/3$, but for higher value of δ the error becomes relatively large.



- The Coulomb theory is an upper bound plasticity solution. In general the theory underestimates the active pressure and overestimates the passive pressure. (Opposite of Rankine's Theory)

REMARKS ON COULOMB's THEORY

- When $\delta = 0$, $\theta = 0$, and $\alpha = 0$, Coulomb theory gives results identical to those of the Rankine theory. Thus the solution in this case is exact because the lower and upper bound results coincide.

Coulomb's Earth Pressure

$\phi = \surd$

$\theta = 0$

$\delta = 0$

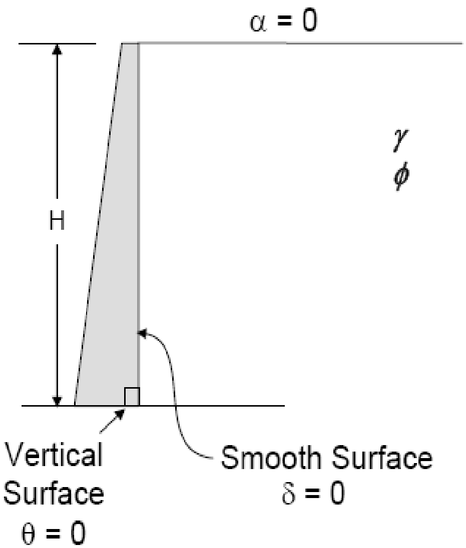
$\alpha = 0$

$$K_a = \frac{\cos^2(\phi-\theta)}{\cos^2\theta\,\cos(\delta-\theta)\left[1+\sqrt{\frac{\sin(\delta+\phi)\,\sin(\phi-\alpha)}{\cos(\delta+\theta)\,\cos(\theta-\alpha)}}\,\right]^2}$$

Under the given wall and backfill conditions, K_a of Coulomb's active earth pressure becomes equivalent to K_a of Rankine's

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$P_a = \tfrac{1}{2} K_a \gamma H^2$$

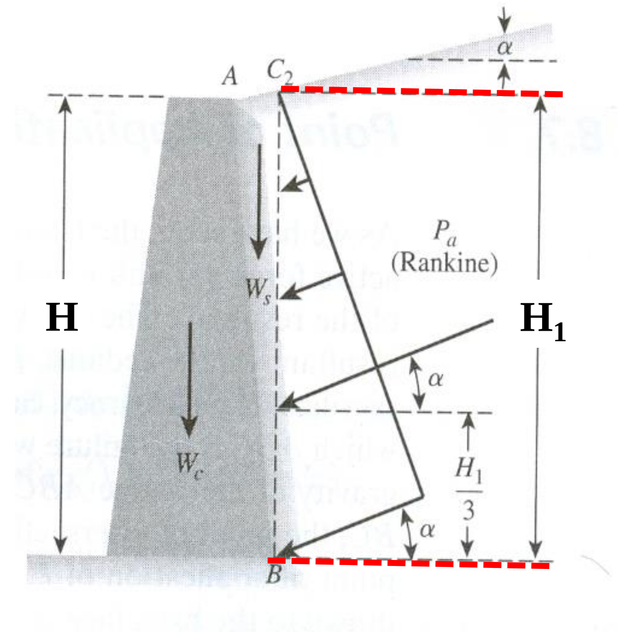


- The point of application of the total active thrust is not given by the Coulomb theory but is assumed to act at a distance of H/3 above the base of the wall.

- In Coulomb solution wall inclination (angle θ) enters in K_a and K_p. In Rankine's approximate solution θ is included into H_1 and W_s.

REMARKS ON COULOMB's THEORY

- For inclined ground surface we use H in Coulomb. However, Rankine's approximate solution uses H_1. Therefore, in Coulomb kinematic solution the effect of ground inclination enters only in K_a and K_p. In Rankine approximate solution it enters not only in K_a and K_p but also in H_1 and W_s.



- P_a Coulomb at angle δ to the normal to the wall (δ = angle of friction between the wall and the backfill). In Rankine's approximate solution P_a acts parallel to the slope of the backfill.
- In Coulomb solution wall inclination (angle θ) affects the direction of P_a and P_p. In Rankine's approximate kinematic solution wall inclination has no effect on the direction of the lateral force.

The end