TOPICS

- ❑ Introduction
- ❑ Coefficient of Lateral Earth Pressure
- ❑ Types and Conditions of Lateral Earth Pressures
- ❑ Lateral Earth pressure Theories
- ❑ **Rankine's Lateral Earth Pressure Theory**
- ❑ Lateral Earth Pressure Distribution – Cohesionless Soils
- ❑ Lateral Earth Pressure Distribution – C – f Soils
- ❑ Coulomb's Lateral Earth Pressure Theory

Lateral Earth Pressure Theories

- Since late 17th century many theories of earth of earth pressure have been proposed by various investigators. Of the theories the following two are the most popular and used for computation of active and passive earth pressures:.

  1. Rankine's Theory (No wall friction)
  2. Coulomb's Theory (With wall friction)

- Those are usually called the classical lateral earth pressure theories.

- In both theories it is required that the soil mass, or at least certain parts of the mass, is in a state of PLASTIC EQUILIBRIUM. The soil mass is on verge of failure. Failure here is defined to be the state of stress which satisfies the Mohr-Coulomb criterion.

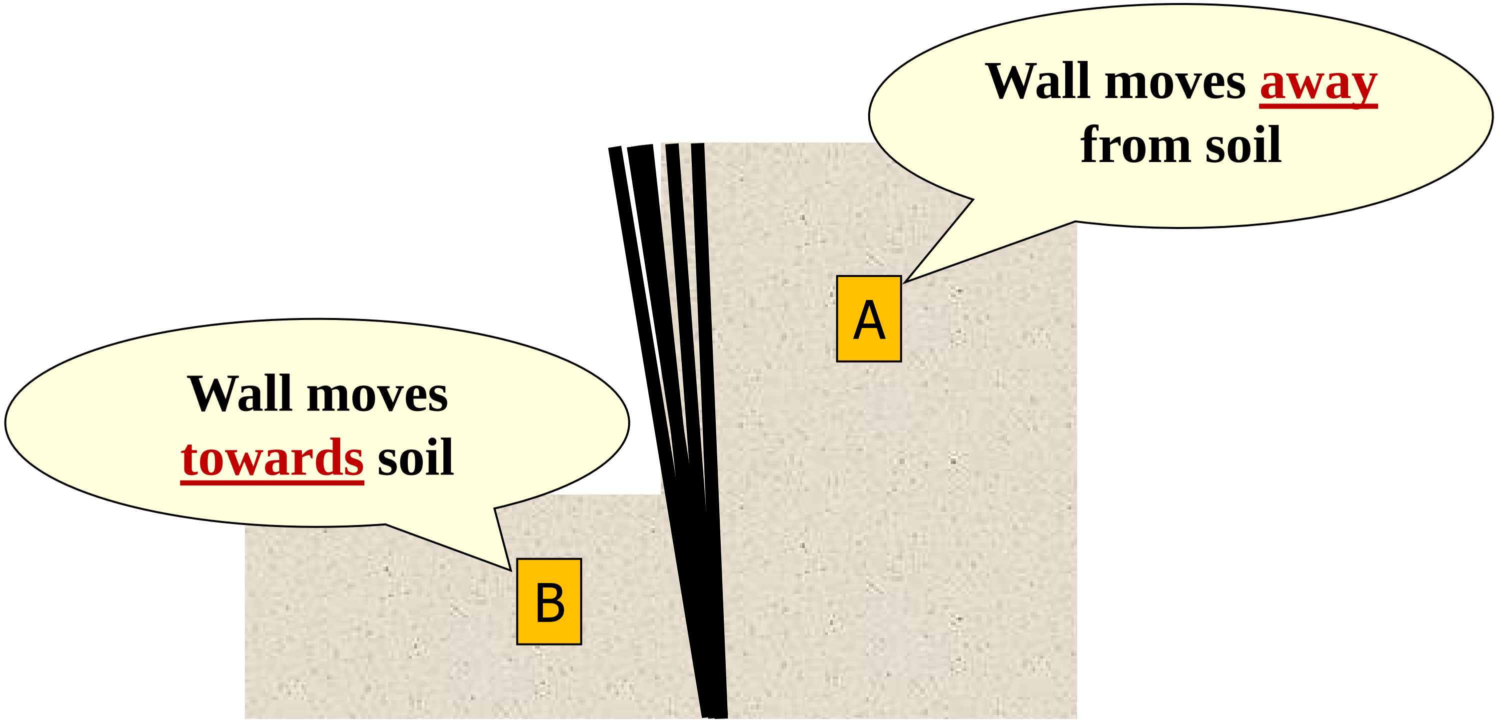
Rankine's Earth Pressure Theory

- Rankine (1857) investigated the stress condition in a soil at a state of PLASTIC EQUILIBRIUM.
- Developed based on semi infinite "loose granular" soil mass for which the soil movement is uniform.
- Used stress states of soil mass to determine lateral pressures on a frictionless wall

Assumptions:

- Vertical wall
- Smooth retaining wall
- Horizontal ground surface
- Homogeneous soil

Active vs. Passive Earth Pressures



smooth wall

Let's look at the soil elements A and B during the wall movement.

- **In most retaining walls of limited height, movement may occur by simple translation or, more frequently, by rotation about the bottom.**

Active Earth Pressure

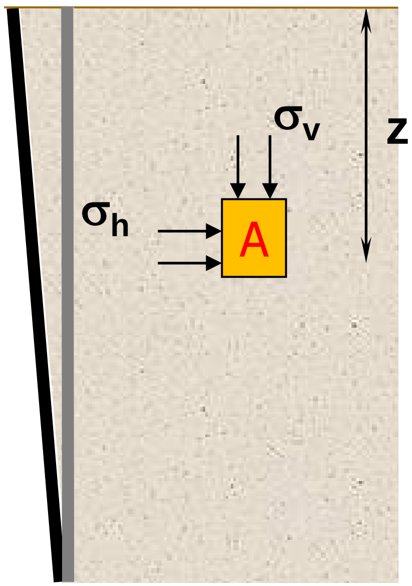
I. Active earth pressure

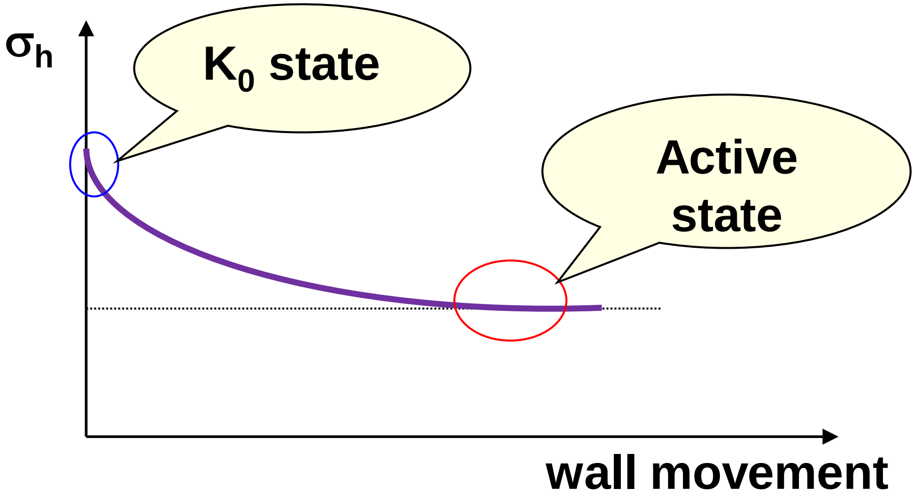
- $\sigma_v = \gamma z$
- Initially, there is no lateral movement.

  $\therefore \sigma_h = K_0 \sigma_v = K_0 \gamma z$
- As the wall moves away from the soil,
- σ_v remains the same; and
- σ_h **decreases** till failure occurs.

Active state

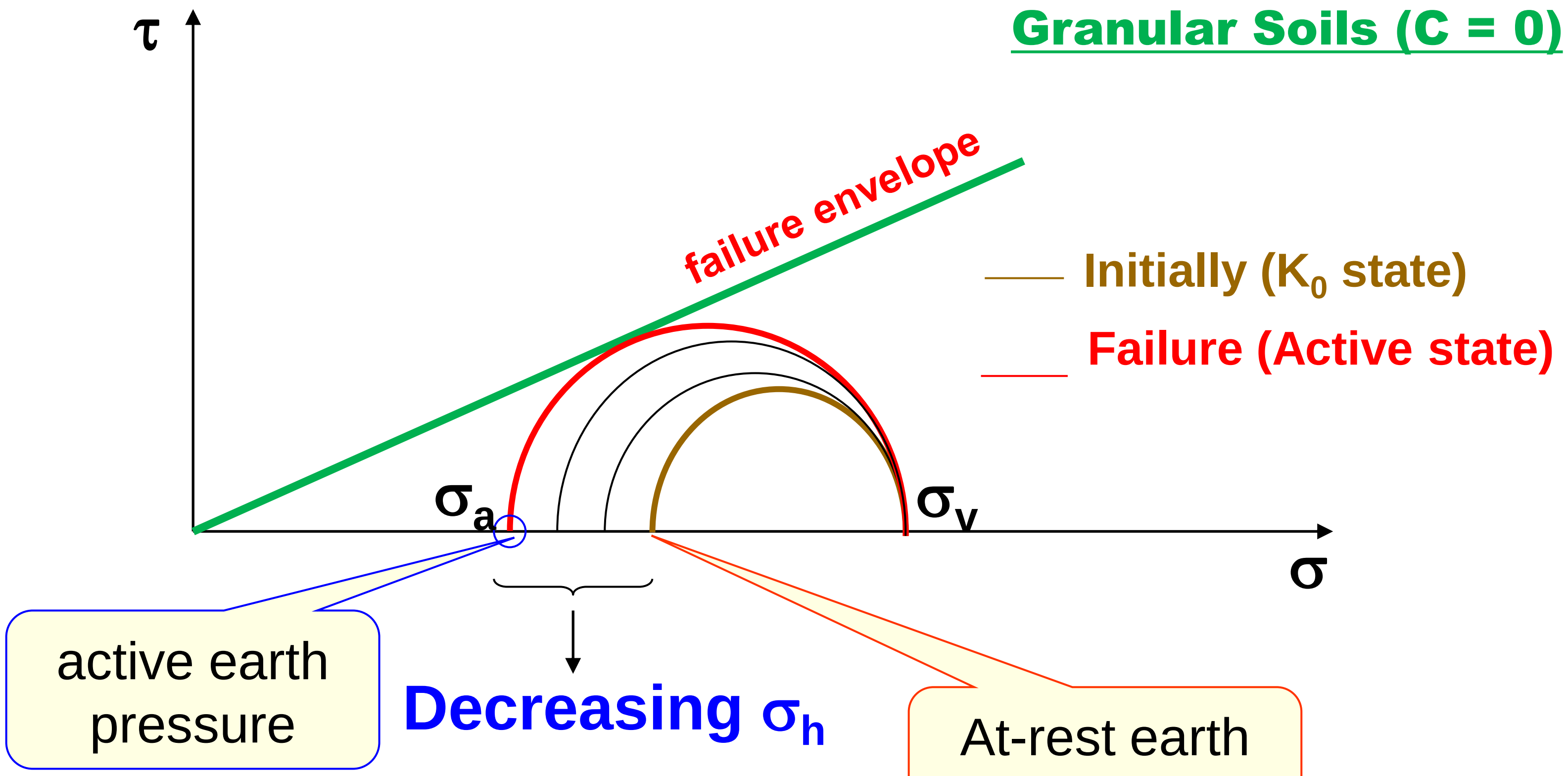
$\sigma_h \rightarrow \sigma_a$





Active Earth Pressure

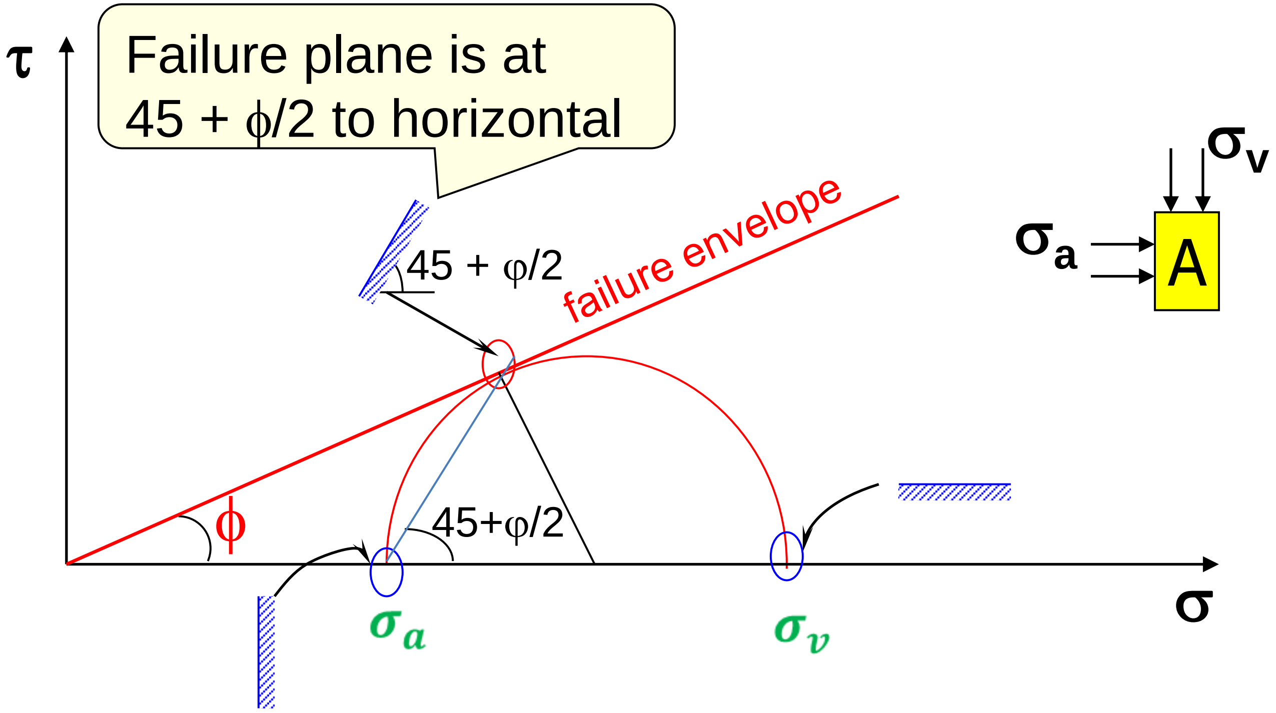
- As the wall moves away from the soil,



$$\sigma_a = K_a \sigma_v$$

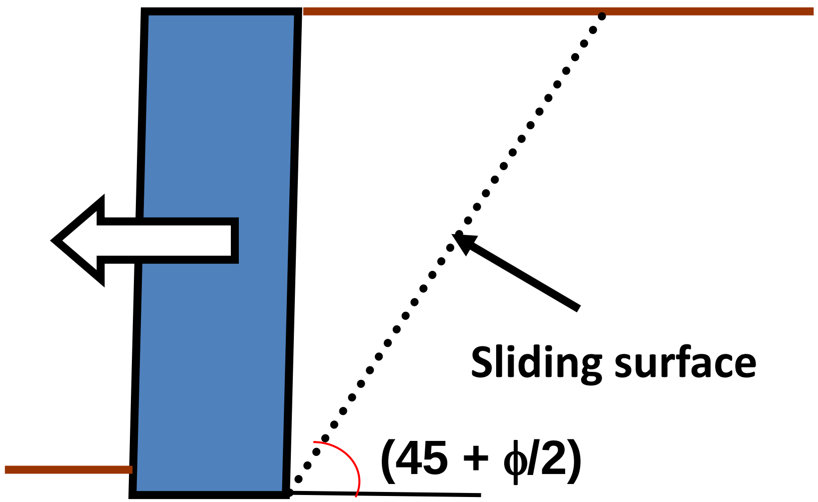
Active Earth Pressure

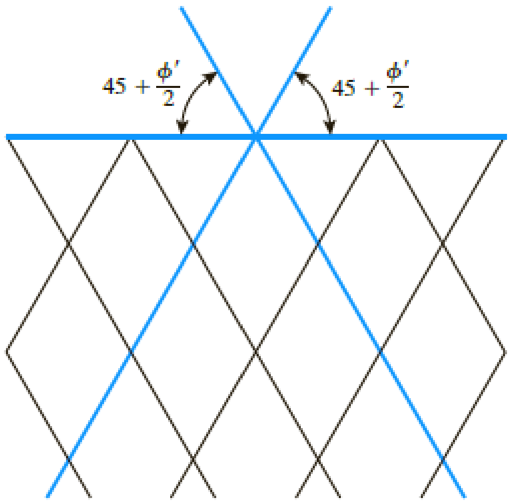
Failure plane



Orientation of Failure Planes

- From Mohr Circle the failure planes in the soil make ± (45 + ϕ/2)-degree angles with the direction of the major principal plane—that is, the horizontal.
- These are called potential ***slip planes.***

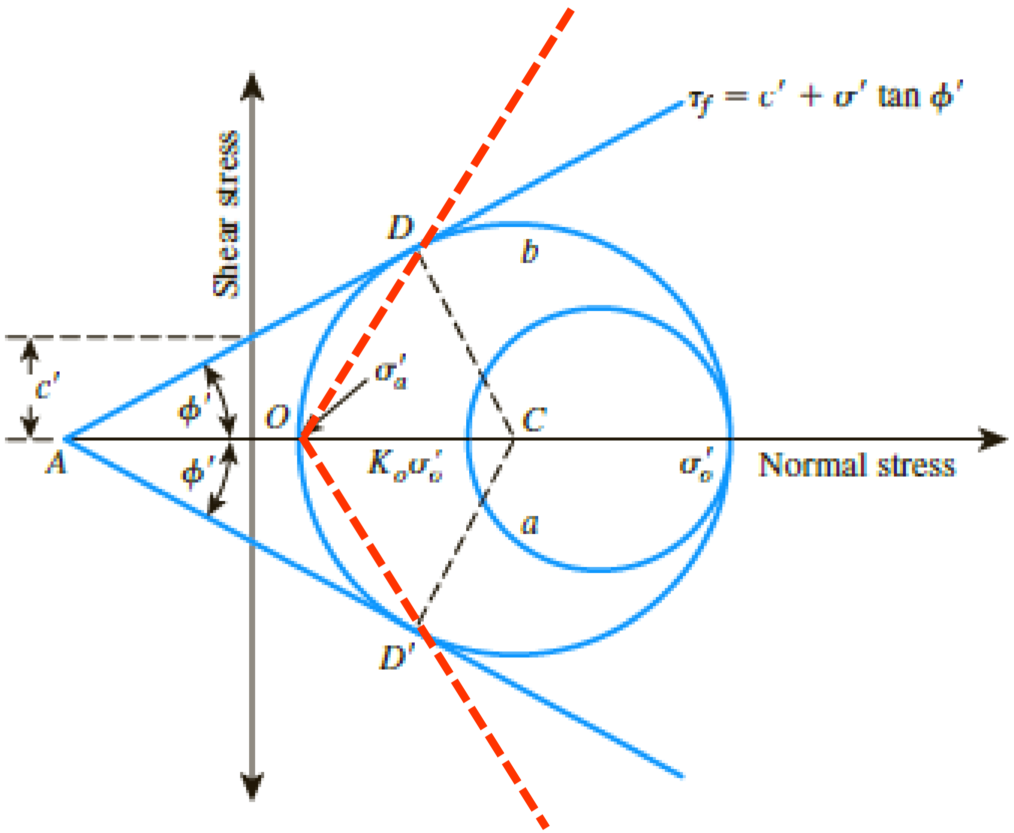




The distribution of slip planes in the soil mass.

Slip planes make angles of <u>(45 + ϕ/2)</u> degrees with the major principal plane

Orientation of Failure Planes



Active Earth Pressure

Rankine's Active Earth Pressure

$$\sigma'_a = \gamma z \tan^2\left(45 - \frac{\phi'}{2}\right) - 2c' \tan\left(45 - \frac{\phi'}{2}\right)$$

The Coefcient of Rankine's Active Earth Pressure

$$K_a = \frac{\sigma'_a}{\sigma'_o} = \tan^2\left(45 - \frac{\phi'}{2}\right)$$

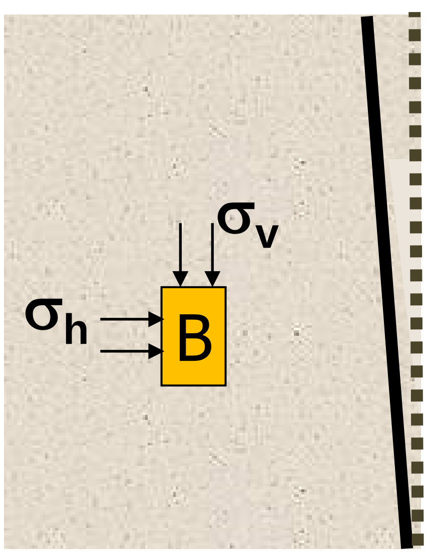
Passive Earth Pressure

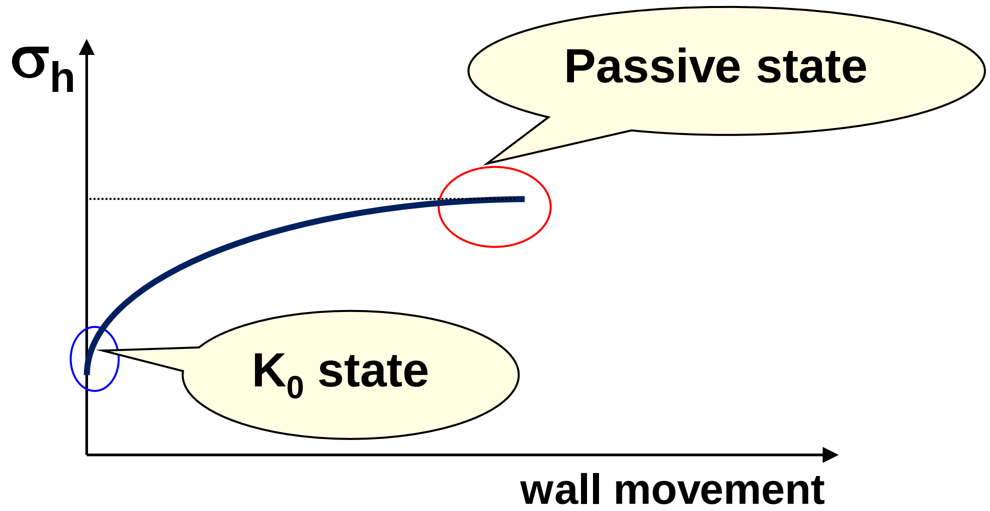
II. Passive earth pressure

- Initially, soil is in K_0 state.
- As the wall moves towards (pushed into) the soil mass,
- σ_v remains the same, and
- σ_h increases till failure occurs.

Passive state

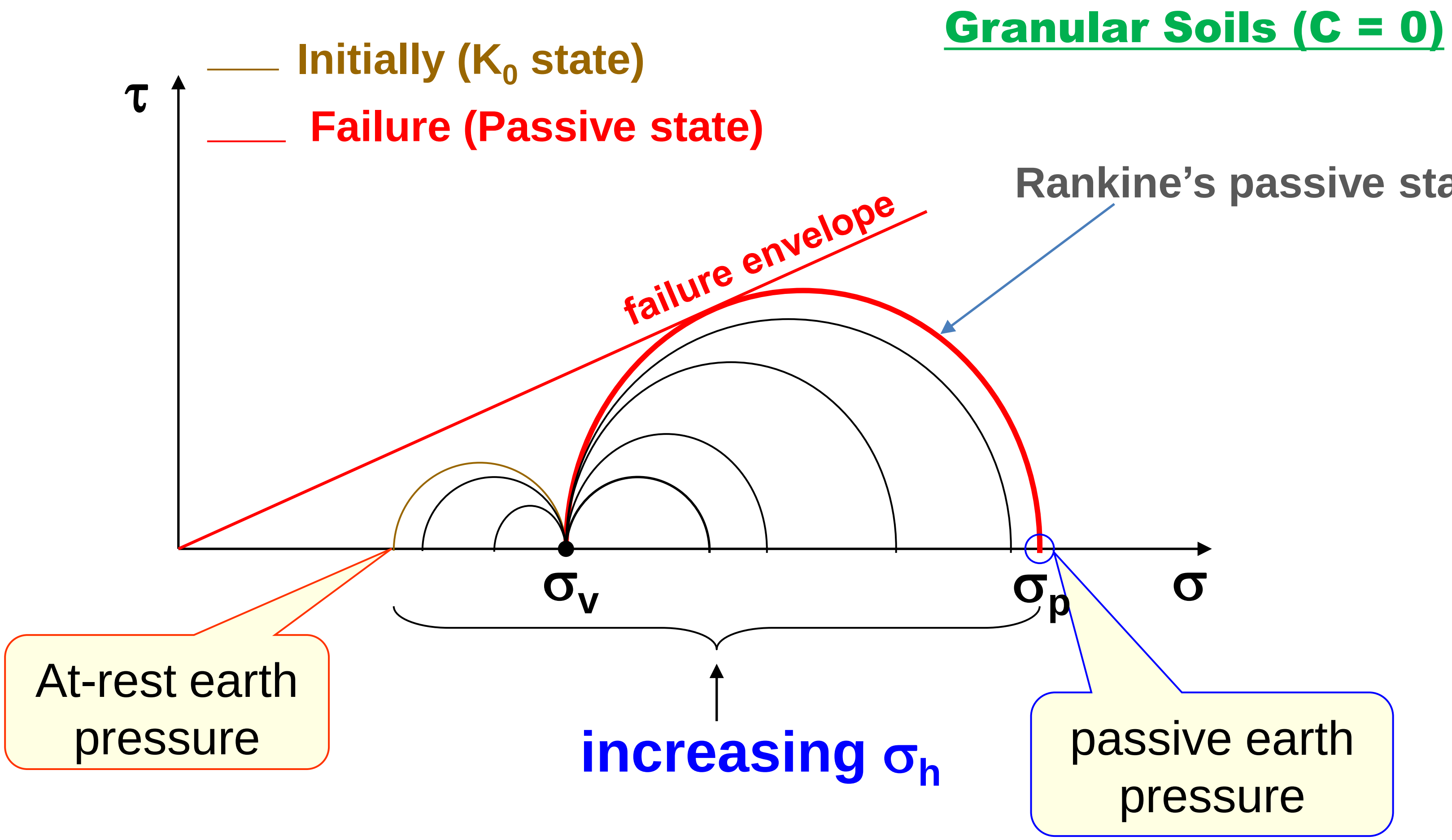
$\sigma_h \rightarrow \sigma_p$





Passive Earth Pressure

- As the wall moves towards the soil,



How do we get the expression for K_p ?

$$Sin\,\emptyset = \frac{\frac{\sigma_p - \sigma_v}{2}}{\frac{\sigma_p + \sigma_v}{2}}$$

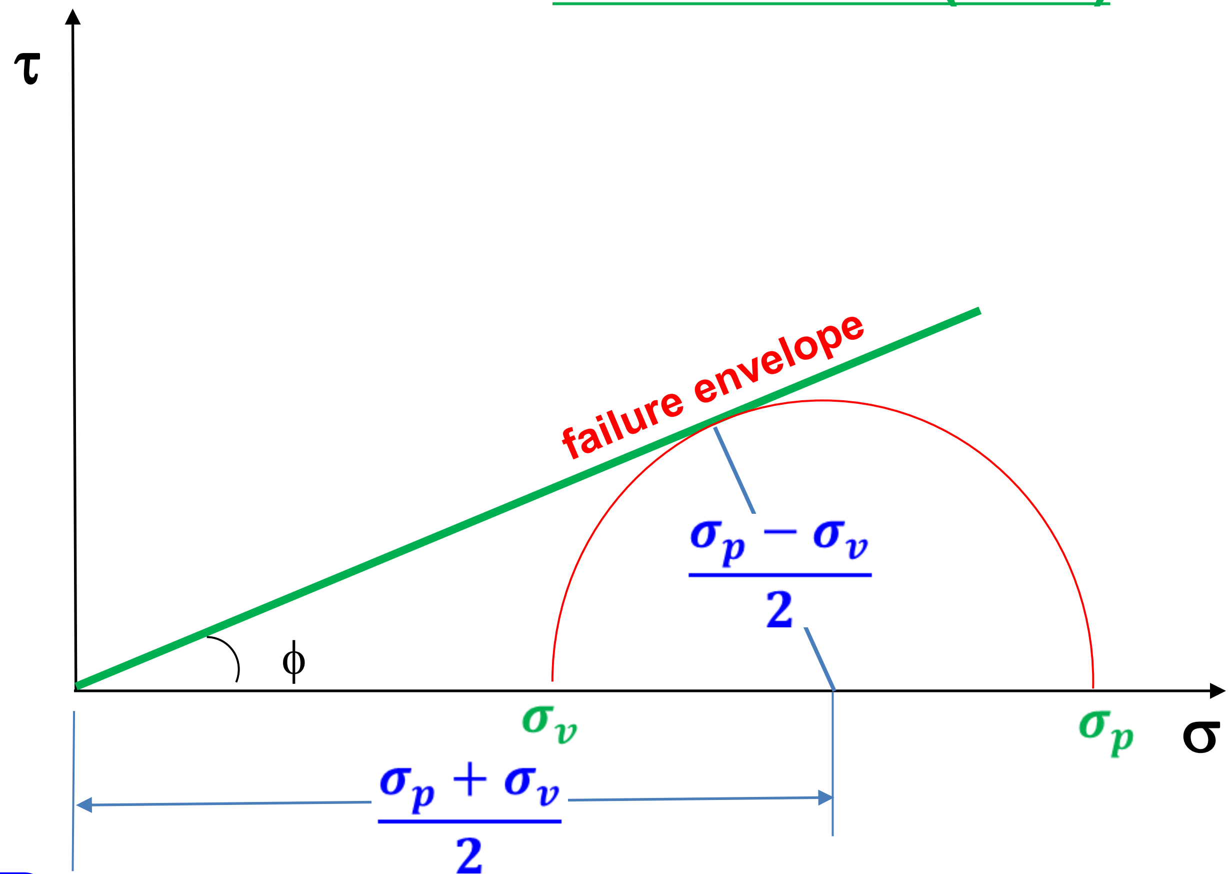
$$\sigma_p = \left(\frac{1 + sin\emptyset}{1 - sin\emptyset}\right)\sigma_v$$

$$\sigma_p = K_p \sigma_v$$

$$K_p = \left(\frac{1 + sin\emptyset}{1 - sin\emptyset}\right)$$

Rankine's coefficient of passive earth pressure

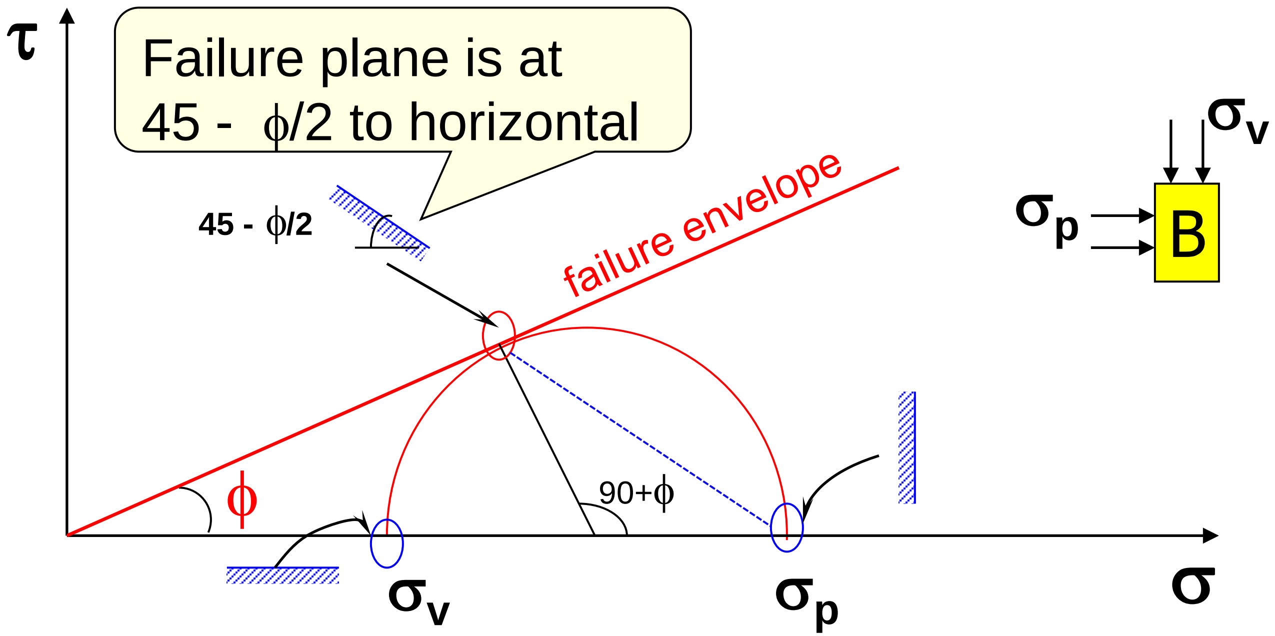
Granular Soils (C = 0)



$$K_P = \tan^2(45 + \phi/2)$$

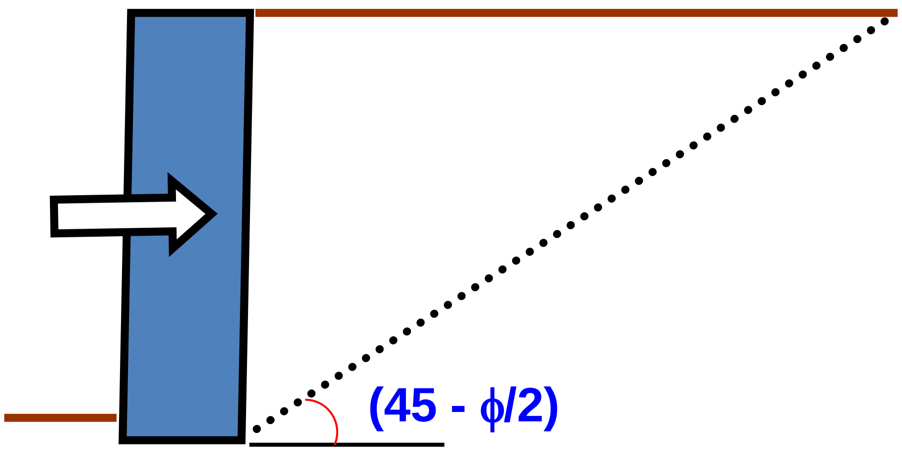
Passive Earth Pressure

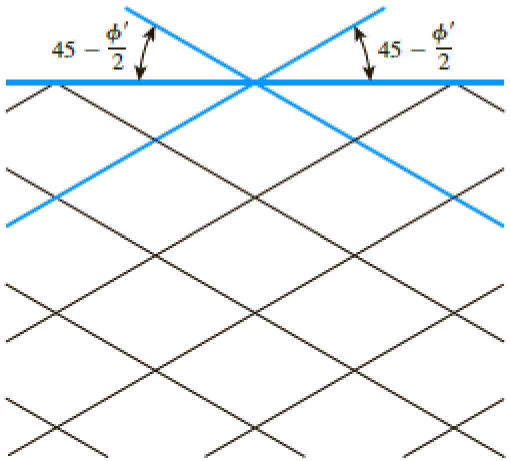
Failure plane



Orientation of Failure Planes

- **From Mohr Circle the failure planes in the soil make ± (45 - ϕ/2) degree angles with the direction of the major principal plane—that is, the horizontal.**
- **These are called potential *slip planes.***

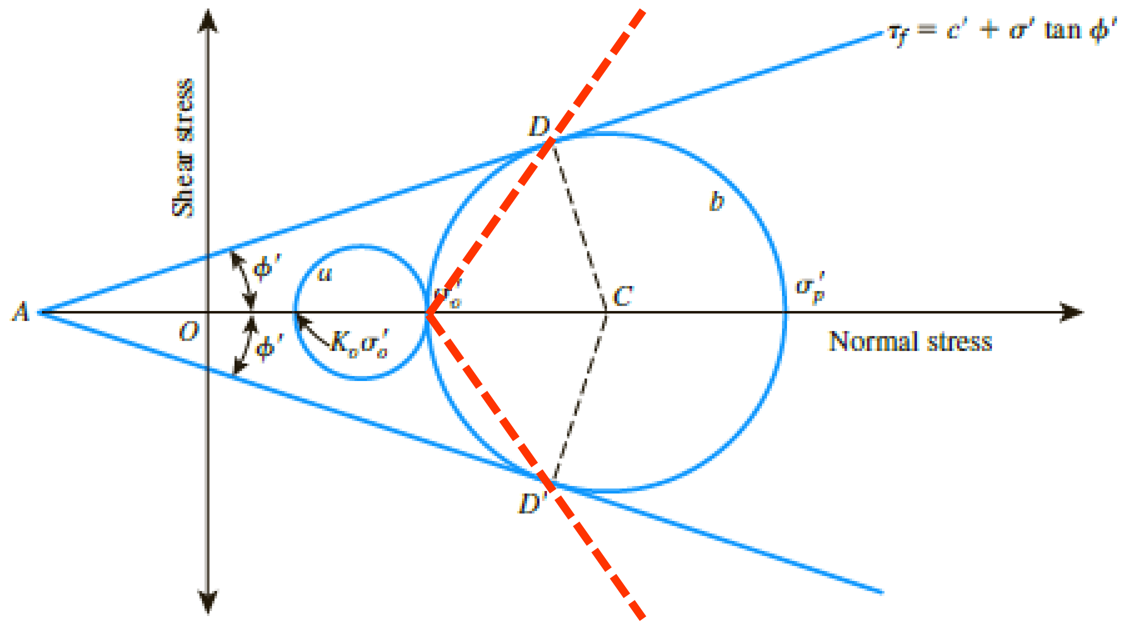




The distribution of slip planes in the soil mass.

Slip planes make angles of <u>(45 - ϕ/2)</u> degrees with the major principal plane

Orientation of Failure Planes



Passive Earth Pressure

Rankine's Passive Earth Pressure

$$\sigma'_p = \sigma'_o \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

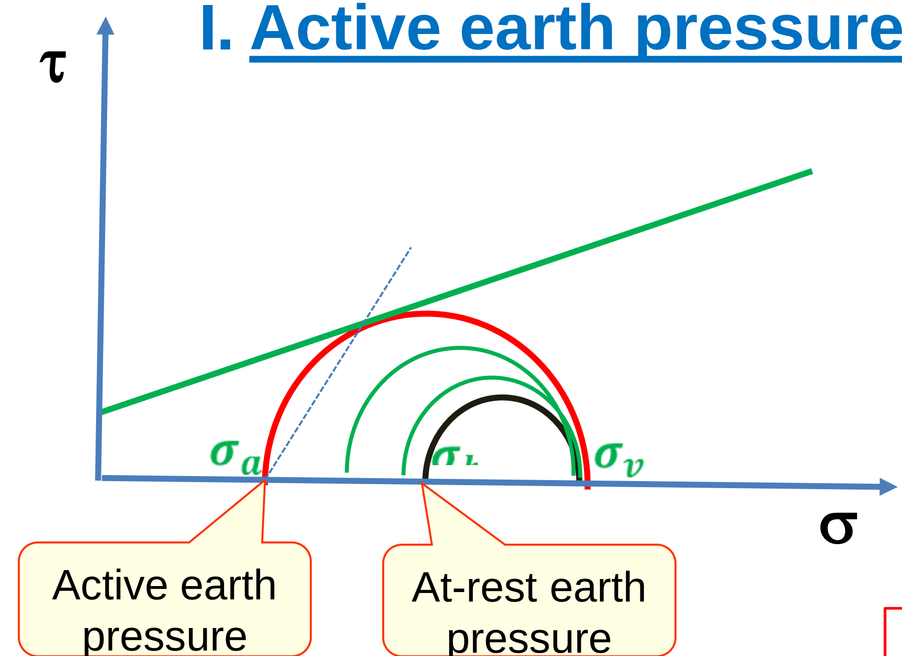
$$= \gamma z \ \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

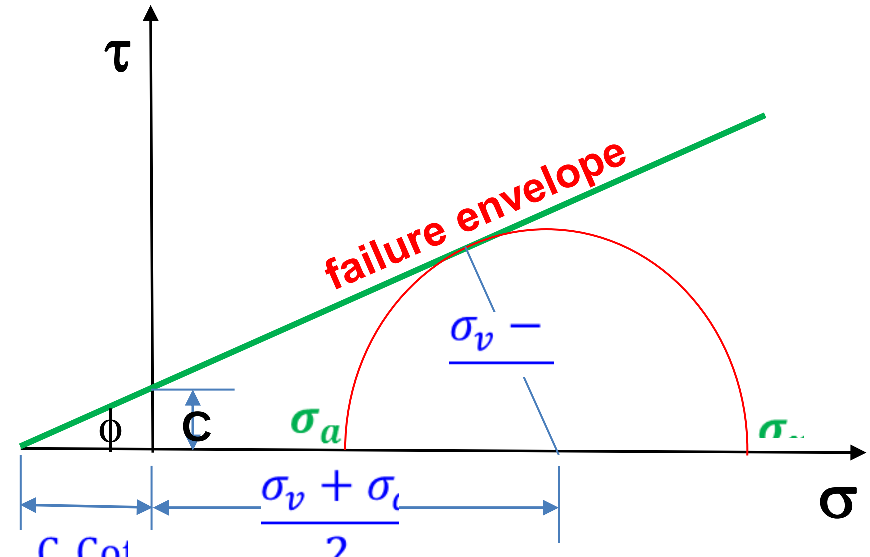
The Coefcient of Rankine's Passive Earth Pressure

$$\frac{\sigma'_p}{\sigma'_o} = K_p = \tan^2\left(45 + \frac{\phi'}{2}\right)$$

C – ϕ Soils

I. Active earth pressure





$$Sin\,\emptyset = \frac{\dfrac{\sigma_v - \sigma_a}{2}}{C.\,Cot\,\emptyset + \dfrac{\sigma_v + \sigma_a}{2}}$$

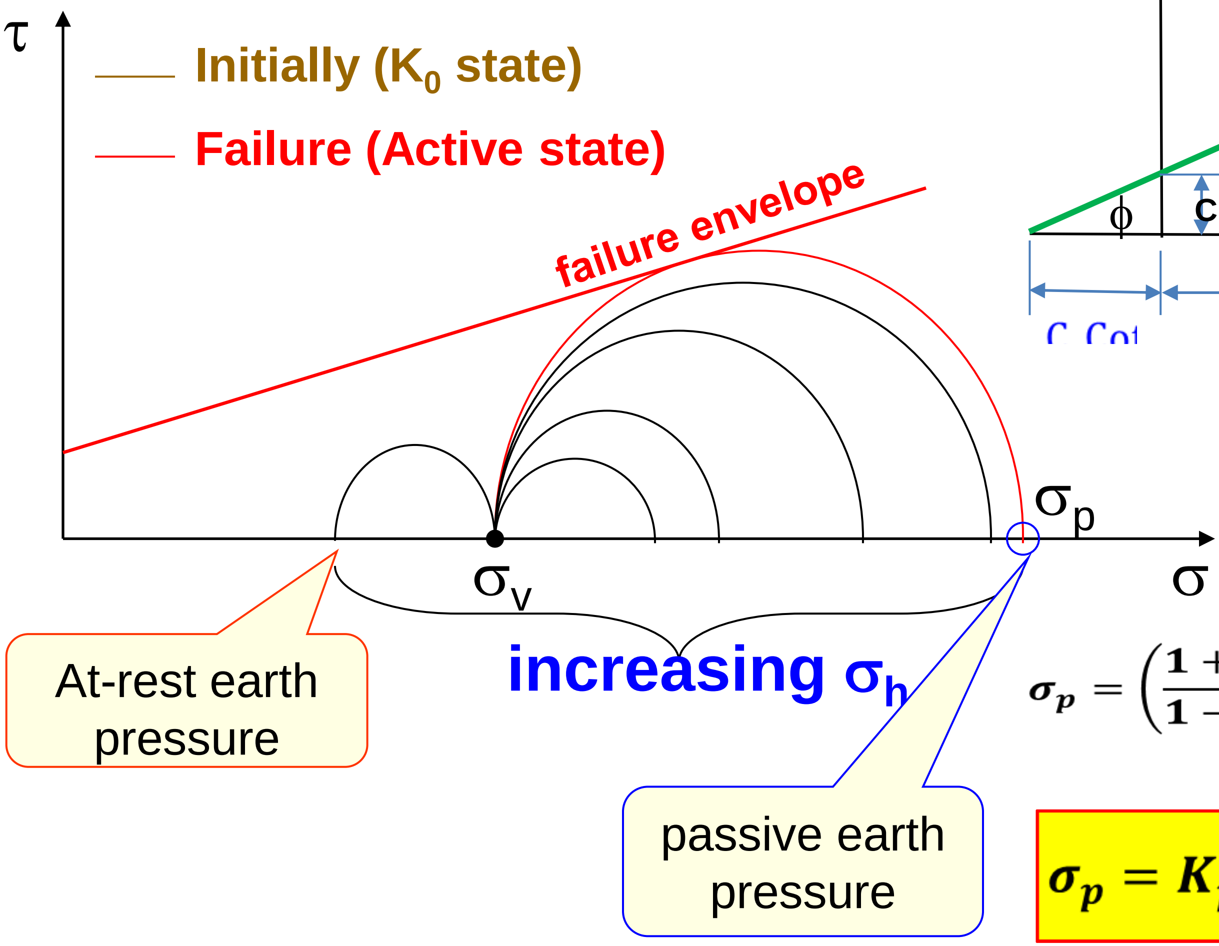
$$\sigma_a = \left(\frac{1 - sin\emptyset}{1 + sin\emptyset}\right)\sigma_v - 2\sqrt{\frac{1 - sin\emptyset}{1 + sin\emptyset}}\,C$$

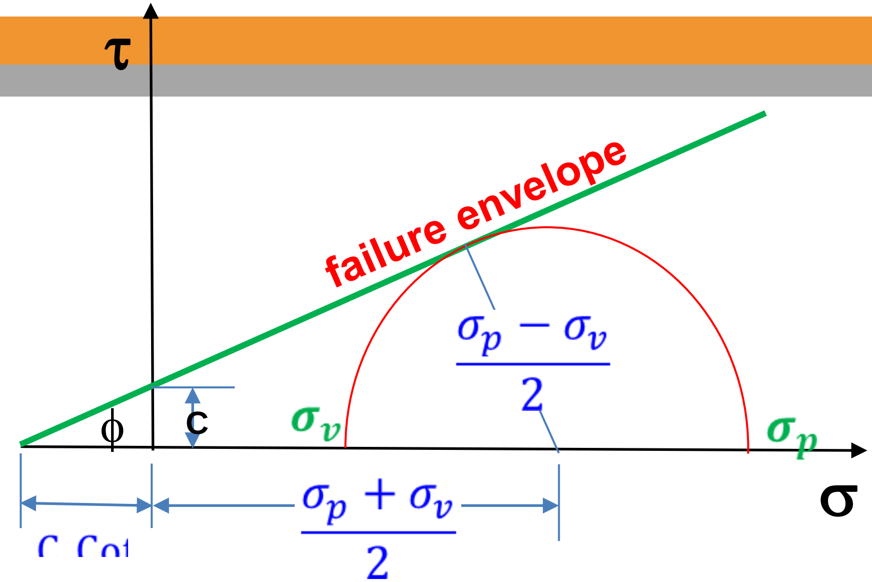
$$\sigma_a = K_a\sigma_v - 2\sqrt{K_a}C$$

$$K_a = \left(\frac{1 - sin\emptyset}{1 + sin\emptyset}\right)$$

c – ϕ Soils

II. Passive earth pressure





$$Sin\,\emptyset = \frac{\frac{\sigma_p - \sigma_v}{2}}{C.\,Cot\,\emptyset + \frac{\sigma_p + \sigma_v}{2}}$$

$$\sigma_p = \left(\frac{1 + sin\emptyset}{1 - sin\emptyset}\right)\sigma_v + 2\sqrt{\frac{1 + sin\emptyset}{1 - sin\emptyset}}\,C$$

$$\sigma_p = K_p\sigma_v + 2\sqrt{K_p}C$$

$$K_p = \left(\frac{1 + sin\emptyset}{1 - sin\emptyset}\right)$$

REMARKS

- **Expression for K_a and K_p are found theoretically using Rankine's theory or as we see later from Coulomb Theory. However, K_0 is evaluated only empirically. Therefore, the difficulty for at rest is in the evaluation of K_0.**

- **For active and passive the soil has reached limit state and we apply the failure theory. However, at-rest we could not figure out what exactly is the case of the soil.**

- **Since the at-rest condition does not involve failure of the soil (it represents a state of elastic equilibrium) the Mohr circle representing the vertical and horizontal stresses does not touch the failure envelope and the horizontal stress cannot be evaluated.**

REMARKS

- What also complicate at-rest condition is the fact that K_0 is not constant but rather change with time.

- K_0 is very sensitive to the geologic and engineering stress history. It can be as low as 0.4 or 0.5 for sedimentary deposits that have never been preloaded or up to 3.0 or greater for some very heavily preloaded deposits.

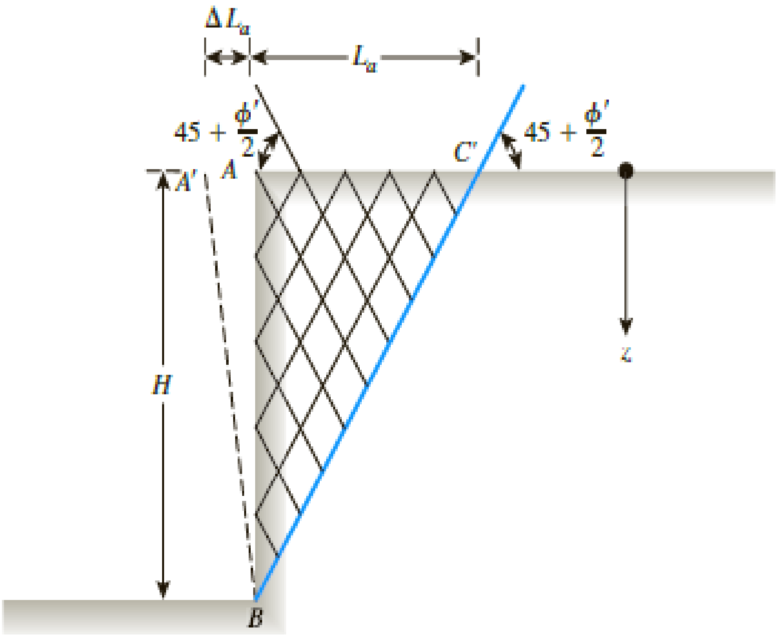
- K_a and K_p are function only of ϕ. C has no effect on them.

- $K_a < K_0 < K_p$

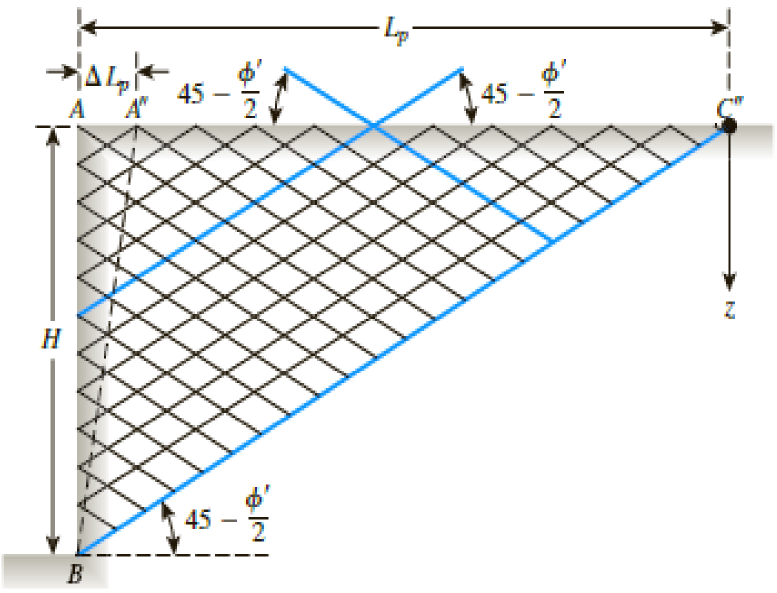
$$\frac{1-\sin\emptyset}{1+\sin\emptyset} < 1 - \sin\emptyset < \frac{1+\sin\emptyset}{1-\sin\emptyset}$$

Yielding of Wall of Limited Height

Rotation of Frictionless Wall about the Bottom



Rankine's active state



Rankine's passive state

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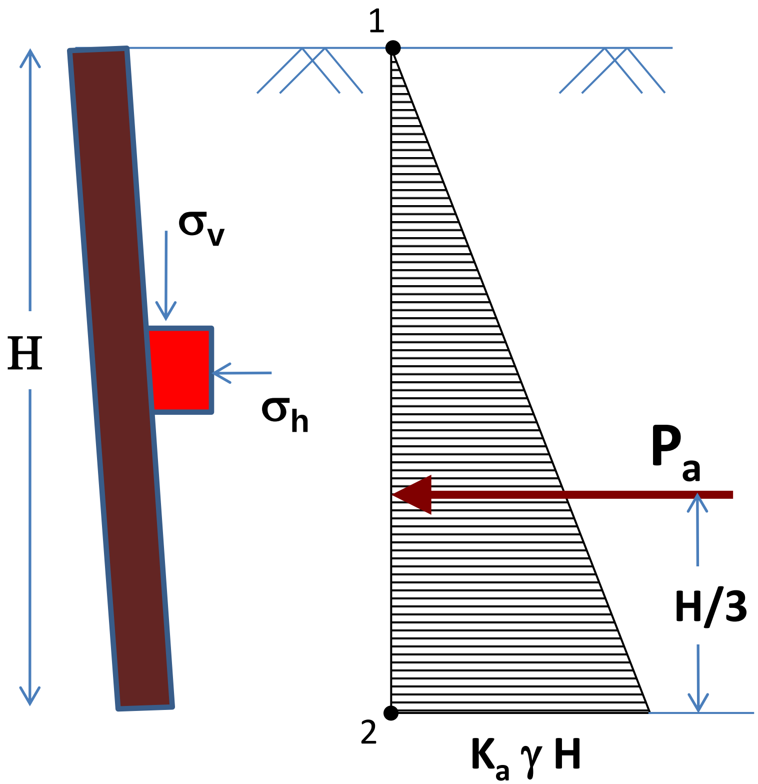
- ❑ **Lateral Earth Pressure Distribution – Cohesionless Soils**
- ❑ Lateral Earth Pressure Distribution – C – f Soils
- ❑ Coulomb's Lateral Earth Pressure Theory

Earth Pressure Distribution

I. Cohesionless soils (C=0)

1. Horizontal Ground Surface



Active Case:

The total Lateral Earth Active force per unit length of the wall (P_a)

= Area of Earth pressure diagram

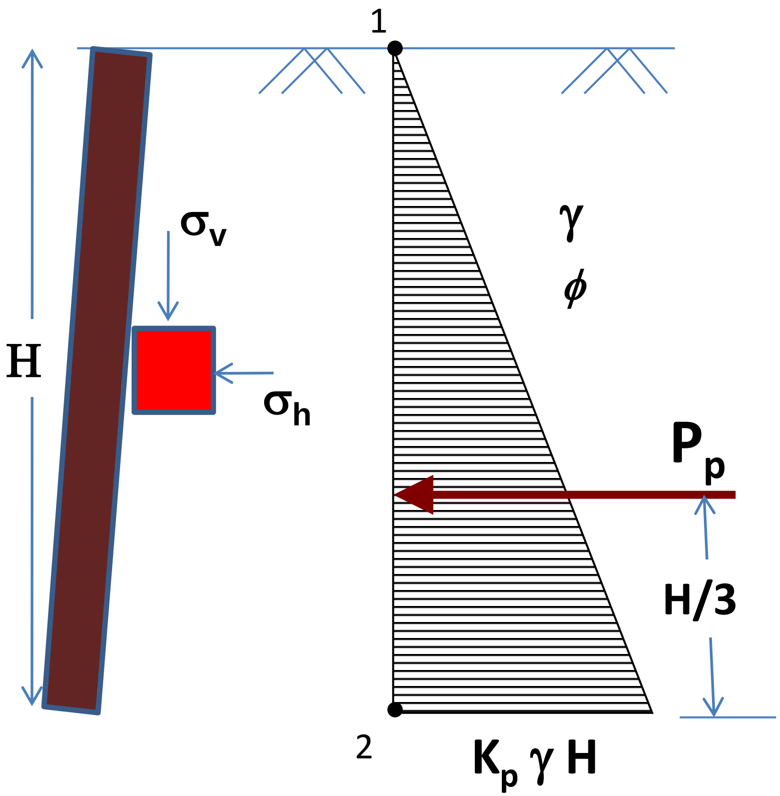
$= \frac{1}{2} \times K_a \times \gamma \times H^2$

Point of application of P_a

H/3 from the base

Earth Pressure Distribution

1. Horizontal Ground Surface



Passive Case:

The total Lateral Earth Passive force per unit length of the wall (P_p)

= Area of Earth pressure diagram

= ½ x K_p x γ x H^2

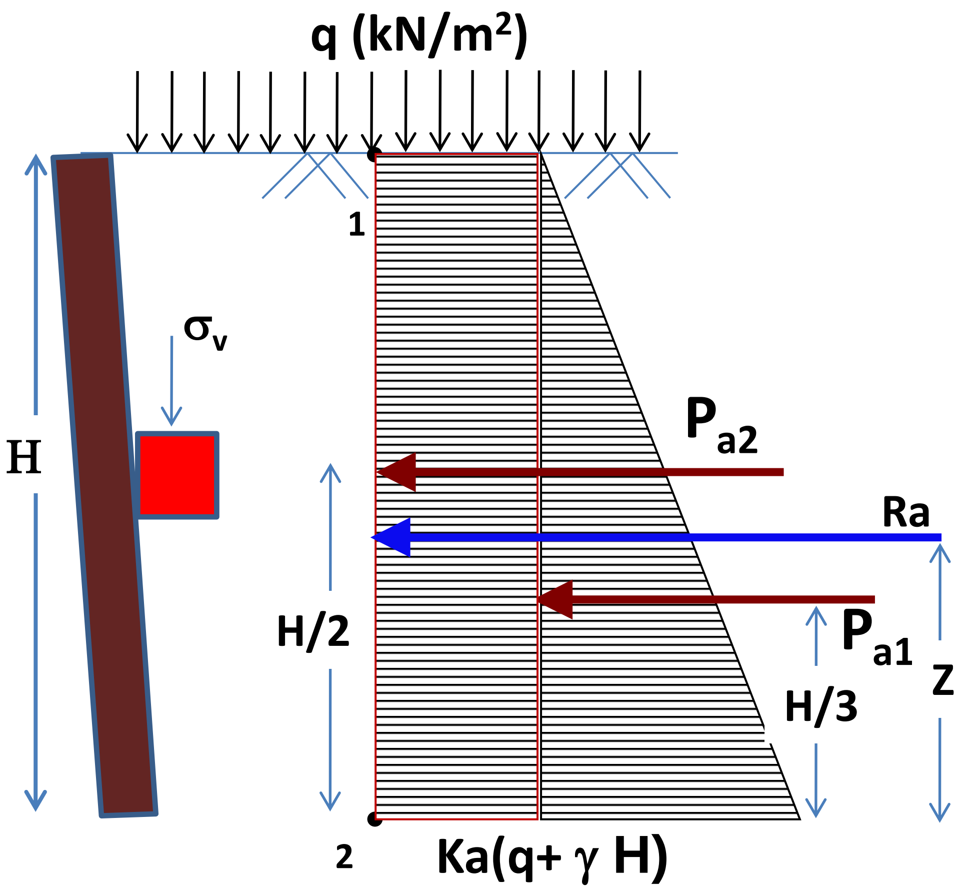
Point of application of P_p

H/3 from the base

The total lateral passive force per unit length of the wall is the area of the diagram

Earth Pressure Distribution

2. Effect of Surcharge



Active Case:

$P_{a1} = ½ \times K_a \times \gamma \times H^2$

$P_{a2} = K_a \times q \times H$

- The resultant Force acting on the wall

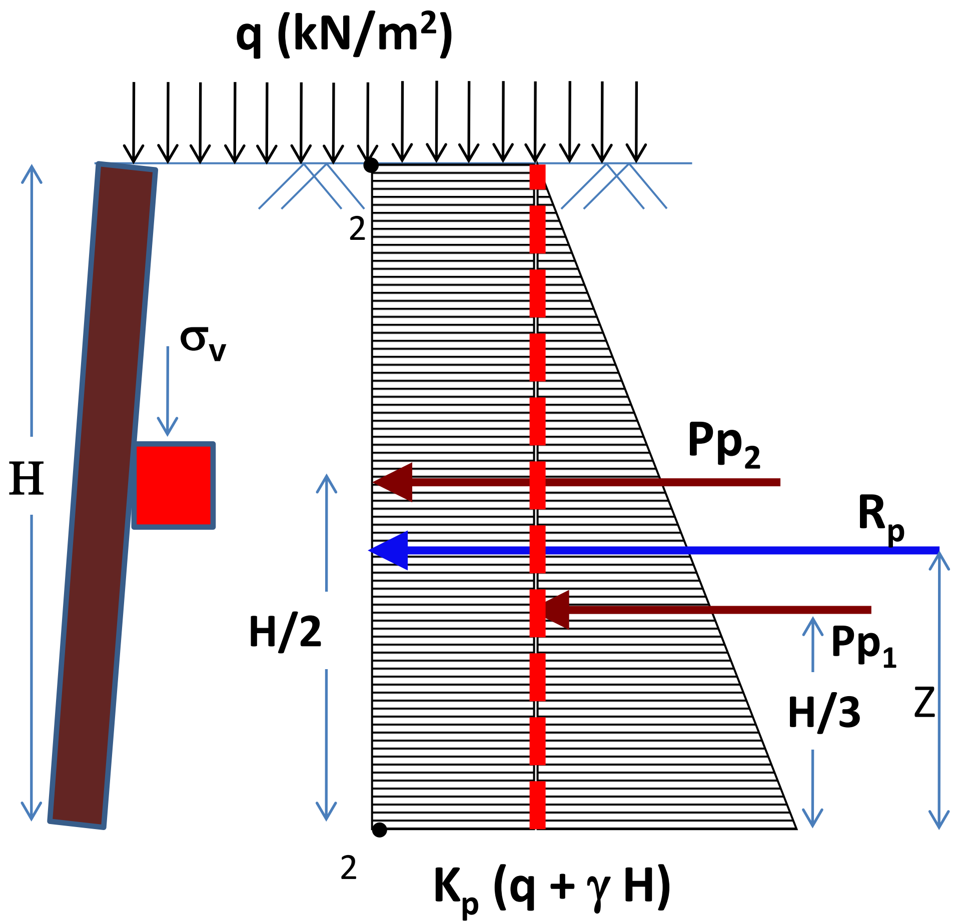
$R_a = P_{a1} + P_{a2}$

- Point of application of Resultant

$$z = \frac{Pa_1 \times \frac{H}{3} + Pa_2 \times \frac{H}{2}}{Ra}$$

Earth Pressure Distribution

2. Effect of Surcharge



Passive Case:

$Pp_1 = ½ \times K_p \times \gamma \times H^2$

$Pp_2 = K_p \times q \times H$

- The resultant Force acting on the wall

$R_p = P_{p1} + P_{p2}$

- Point of application of Resultant

$$z = \frac{Pp_1 \times \frac{H}{3} + Pp_2 \times \frac{H}{2}}{Rp}$$

Earth Pressure Distribution

3. Effect of G.W.T

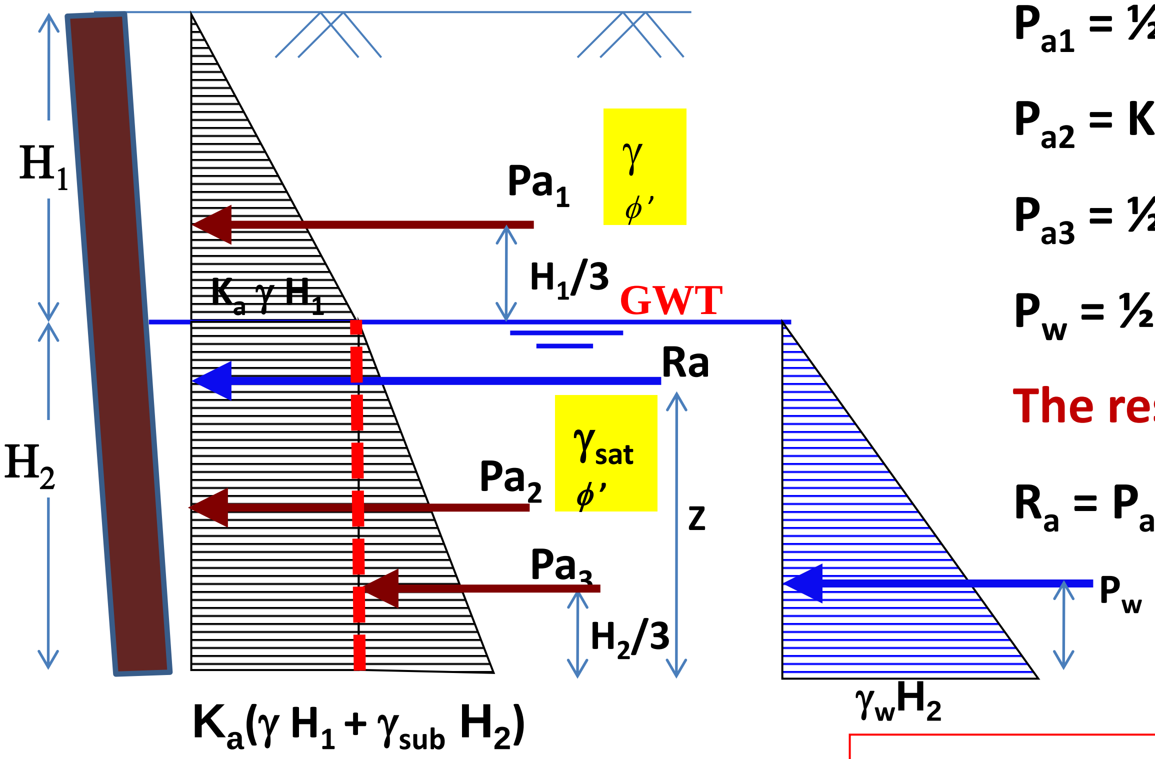
The presence of water will have two effects:

1. **The use of effective unit weight when calculating the lateral pressure for the given submerged soil.**
2. **In addition to the lateral force for the soil we add P_w.**

The effect of water is the same for at-rest, active, or passive.

Earth Pressure Distribution

3. Effect of G.W.T



Active Case:

$P_{a1} = ½ \times K_a \times \gamma \times H_1^2$

$P_{a2} = K_a \times \gamma \times H_1 \times H_2$

$P_{a3} = ½ \times K_a \times \gamma_{sub} \times H_2^2$

$P_w = ½ \times \gamma_w \times H_2^2$

The resultant Force acting on the wall

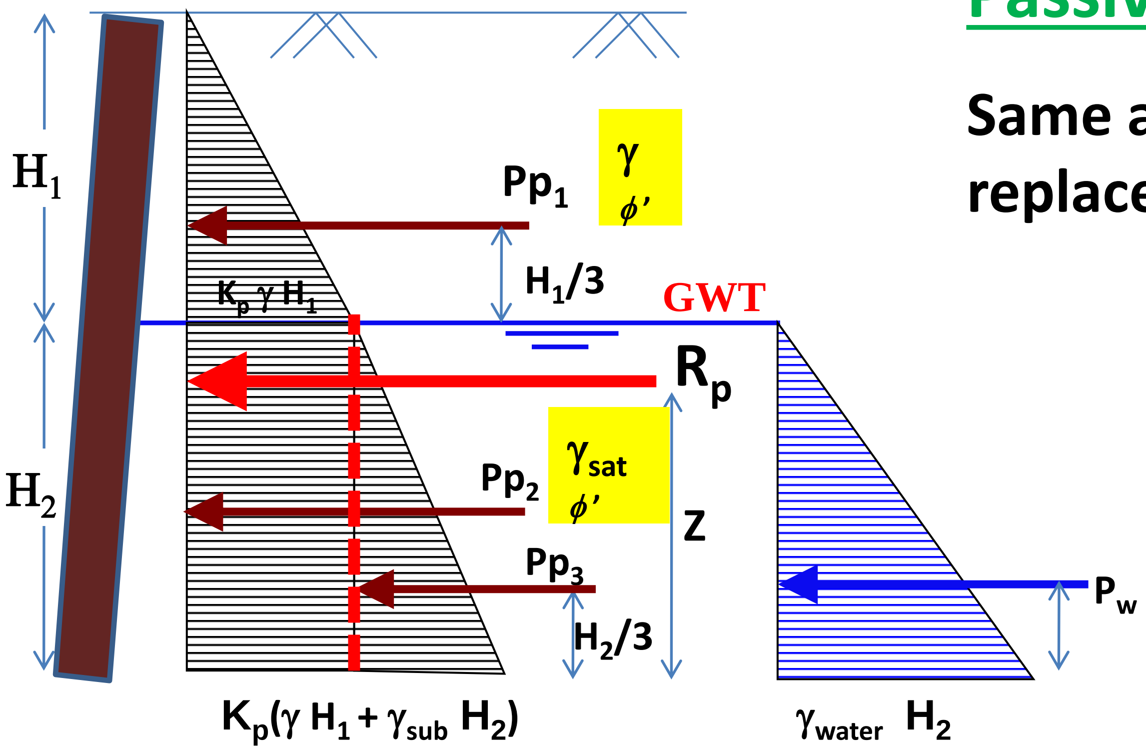
$R_a = P_{a1} + P_{a2} + P_{a3} + P_w$

Point of application of Resultant

$$z = \frac{Pa_1 \times (H_2 + \frac{H_1}{3}) + Pa_2 \times \frac{H_2}{2} + Pa_3 \times \frac{H_2}{3} + Pw \times \frac{H_2}{3}}{Ra}$$

Earth Pressure Distribution

3. Effect of G.W.T

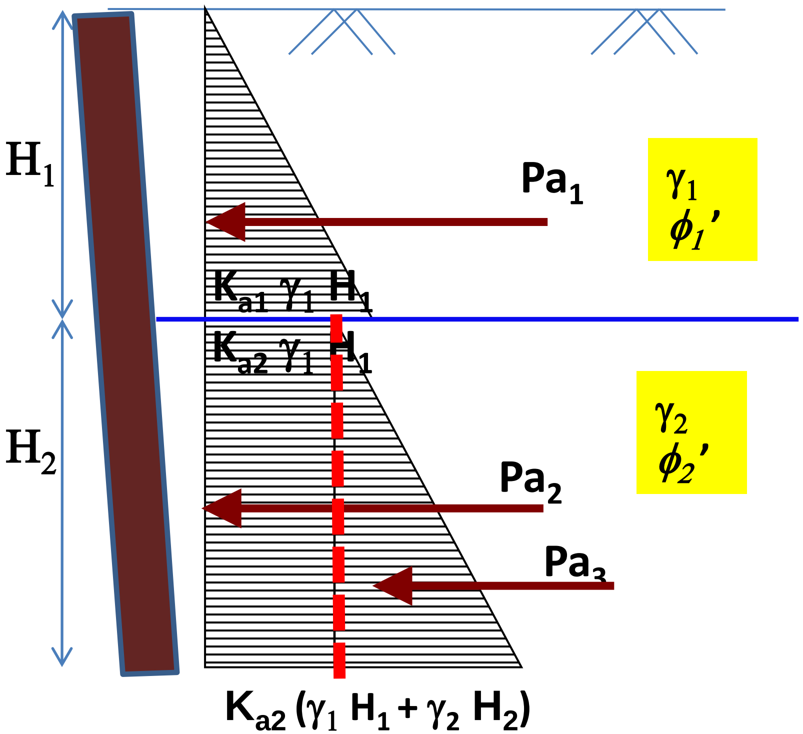


Passive Case:

Same as before except K_a is replaced with K_p

Earth Pressure Distribution

4. Layered Profile



Active Case:

Because of different ϕ, the upper and lower layer will have different lateral earth coefficients.

$$P_{a1} = ½ \times K_{a1} \times \gamma_1 \times H_1^2$$

$$P_{a2} = K_{a2} \times \gamma_1 \times H_1 \times H_2$$

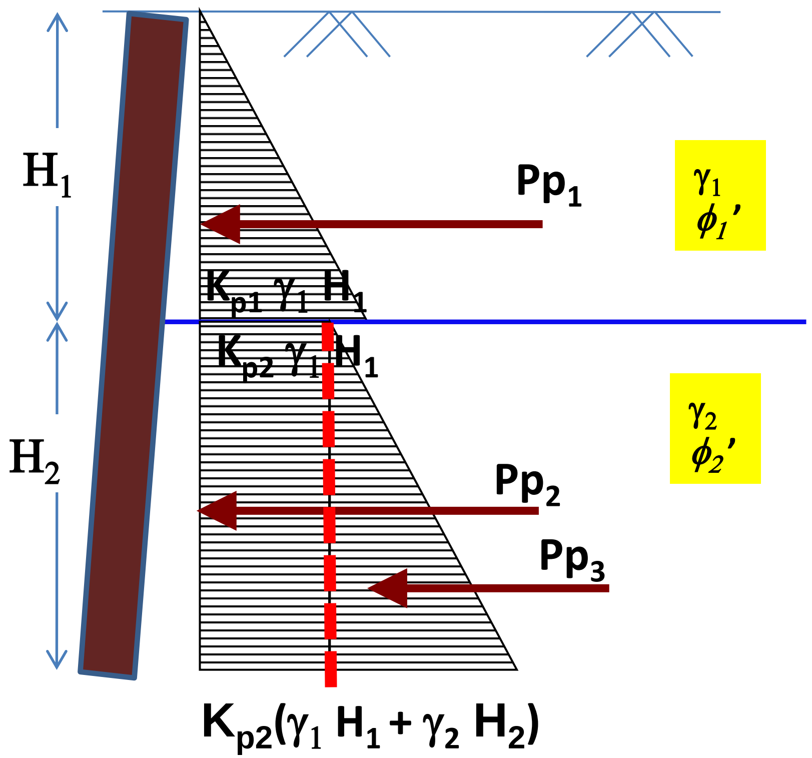
$$P_{a3} = ½ \times K_{a2} \times \gamma_2 \times H_2^2$$

The Resultant Force acting on the wall

$$R_a = P_{a1} + P_{a2} + P_{a3}$$

Earth Pressure Distribution

4. Layered Profile



Passive Case:

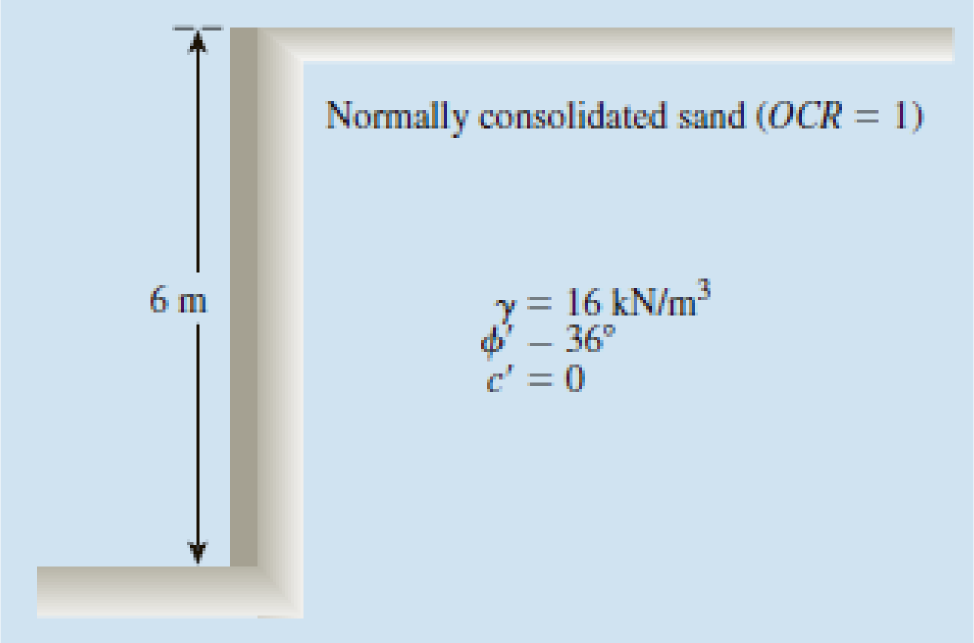
- Same as before except K_a is replaced with K_p

EXAMPLE 13.6

Example 13.6

An 6-m-high retaining wall is shown in Figure 13.21a. Determine:

a. Rankine active force per unit length of the wall and the location of the resultant
b. Rankine passive force per unit length of the wall and the location of the resultant



EXAMPLE 13.6

Part a

Because $c' = 0$, to determine the active force, we can use Eq. (13.33).

$$\sigma'_a = K_a \sigma'_o = K_a \gamma z$$

$$K_a = \frac{1 - \sin \phi'}{1 + \sin \phi'} = \frac{1 - \sin 36}{1 + \sin 36} = 0.26$$

At $z = 0$, $\sigma'_a = 0$; at $z = 6$ m,

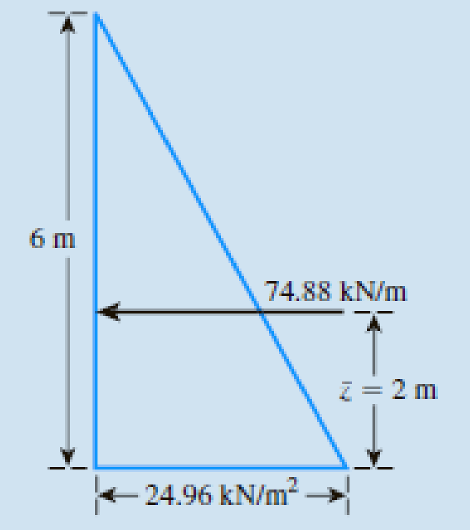
$$\sigma'_a = (0.26)(16)(6) = 24.96 \text{ kN/m}^2$$

The pressure-distribution diagram is shown in Figure 13.21b. The active force per unit length of the wall is

$$P_a = \frac{1}{2}(6)(24.96) = \mathbf{74.88\ kN/m}$$

Also,

$$\bar{z} = \mathbf{2\ m}$$



EXAMPLE 13.6

Part b

To determine the passive force, we are given that $c' = 0$. So, from Eq. (13.36),

$$\sigma'_p = K_p\sigma'_o = K_p\gamma z$$

$$K_p = \frac{1 + \sin\phi'}{1 - \sin\phi'} = \frac{1 + \sin 36}{1 - \sin 36} = 3.85$$

At $z = 0$, $\sigma'_p = 0$; at $z = 6$ m,

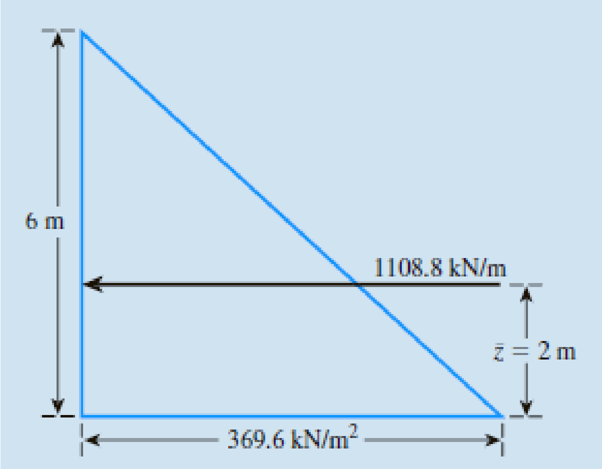
$$\sigma'_p = (3.85)(16)(6) = 369.6 \text{ kN/m}^2$$

The pressure-distribution diagram is shown in Figure 13.21c. The passive force per unit length of the wall is

$$P_p = \frac{1}{2}(6)(369.6) = \mathbf{1108.8\ kN/m}$$

Also,

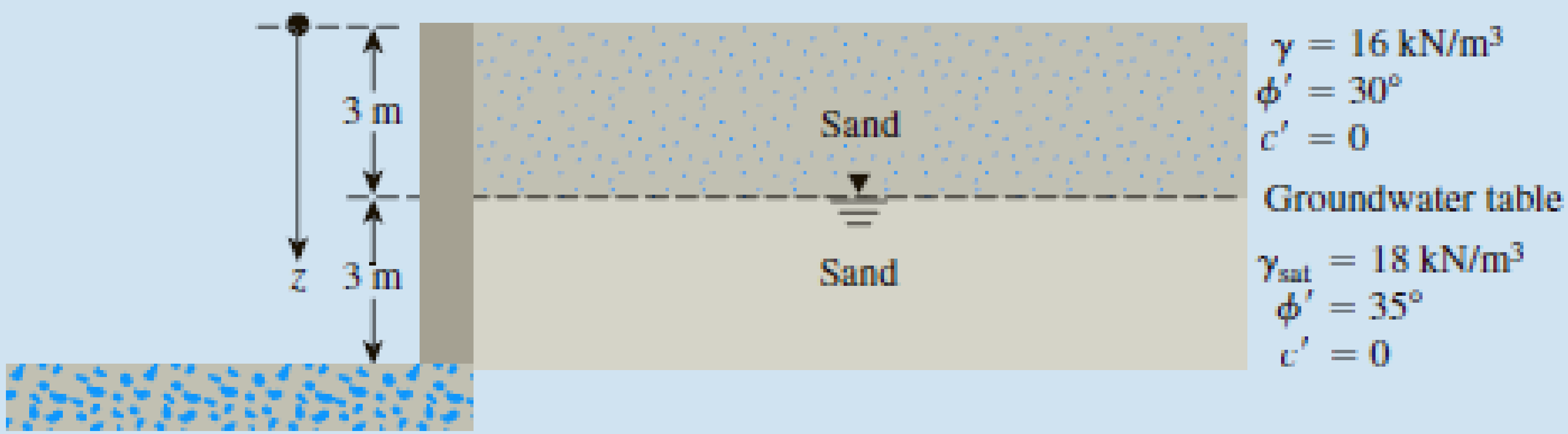
$$\bar{z} = \frac{6}{3} = \mathbf{2\ m}$$



EXAMPLE 13.7

Example 13.7

For the retaining wall shown in Figure 13.22a, determine the force per unit length of the wall for Rankine's active state. Also find the location of the resultant.



EXAMPLE 13.7

Solution

Given that $c' = 0$, we know that $\sigma'_a = K_a \sigma'_o$. For the upper layer of the soil, Rankine's active earth-pressure coefficient is

$$K_a = K_{a(1)} = \frac{1 - \sin 30°}{1 + \sin 30°} = \frac{1}{3}$$

For the lower layer,

$$K_a = K_{a(2)} = \frac{1 - \sin 35°}{1 + \sin 35°} = 0.271$$

At $z = 0$, $\sigma'_o = 0$. At $z = 3$ m (just inside the bottom of the upper layer), $\sigma'_o = 3 \times 16 = 48$ kN/m². So

$$\sigma'_a = K_{a(1)}\sigma'_o = \frac{1}{3} \times 48 = 16 \text{ kN/m}^2$$

Again, at $z = 3$ m (in the lower layer), $\sigma'_o = 3 \times 16 = 48$ kN/m², and

$$\sigma'_a = K_{a(2)}\sigma'_o = (0.271)(48) = 13.0 \text{ kN/m}^2$$

At $z = 6$ m,

$$\sigma'_o = 3 \times 16 + 3(18 - \underset{\substack{\uparrow \\ \gamma_w}}{9.81}) = 72.57 \text{ kN/m}^2$$

and

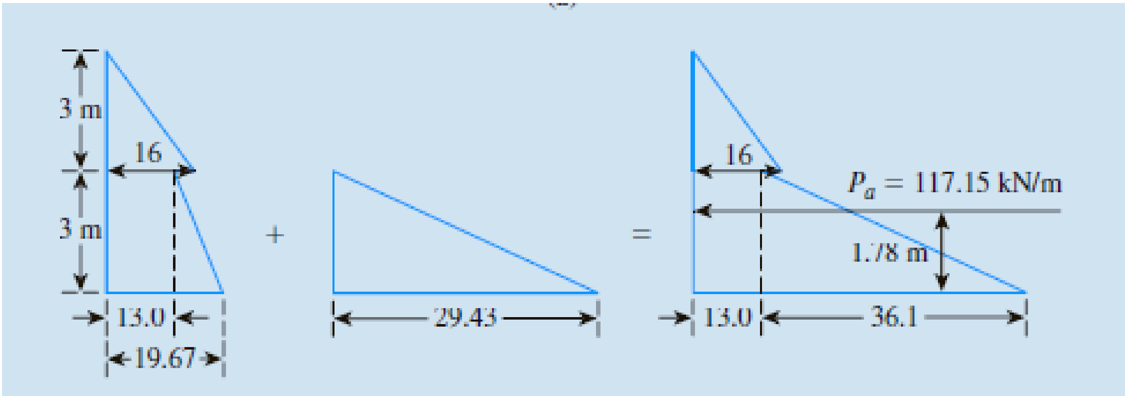
$$\sigma'_a = K_{a(2)}\sigma'_o = (0.271)(72.57) = 19.67 \text{ kN/m}^2$$

The variation of σ'_a with depth is shown in Figure 13.22b.

The lateral pressures due to the pore water are as follows.

At $z = 0$: $u = 0$
At $z = 3$ m: $u = 0$
At $z = 6$ m: $u = 3 \times \gamma_w = 3 \times 9.81 = 29.43$ kN/m²

EXAMPLE 13.7



The variation of u with depth is shown in Figure 13.22c, and that for σ_a (total active pressure) is shown in Figure 13.22d. Thus,

$$P_a = \left(\tfrac{1}{2}\right)(3)(16) + 3(13.0) + \left(\tfrac{1}{2}\right)(3)(36.1) = 24 + 39.0 + 54.15 = \mathbf{117.15\ kN/m}$$

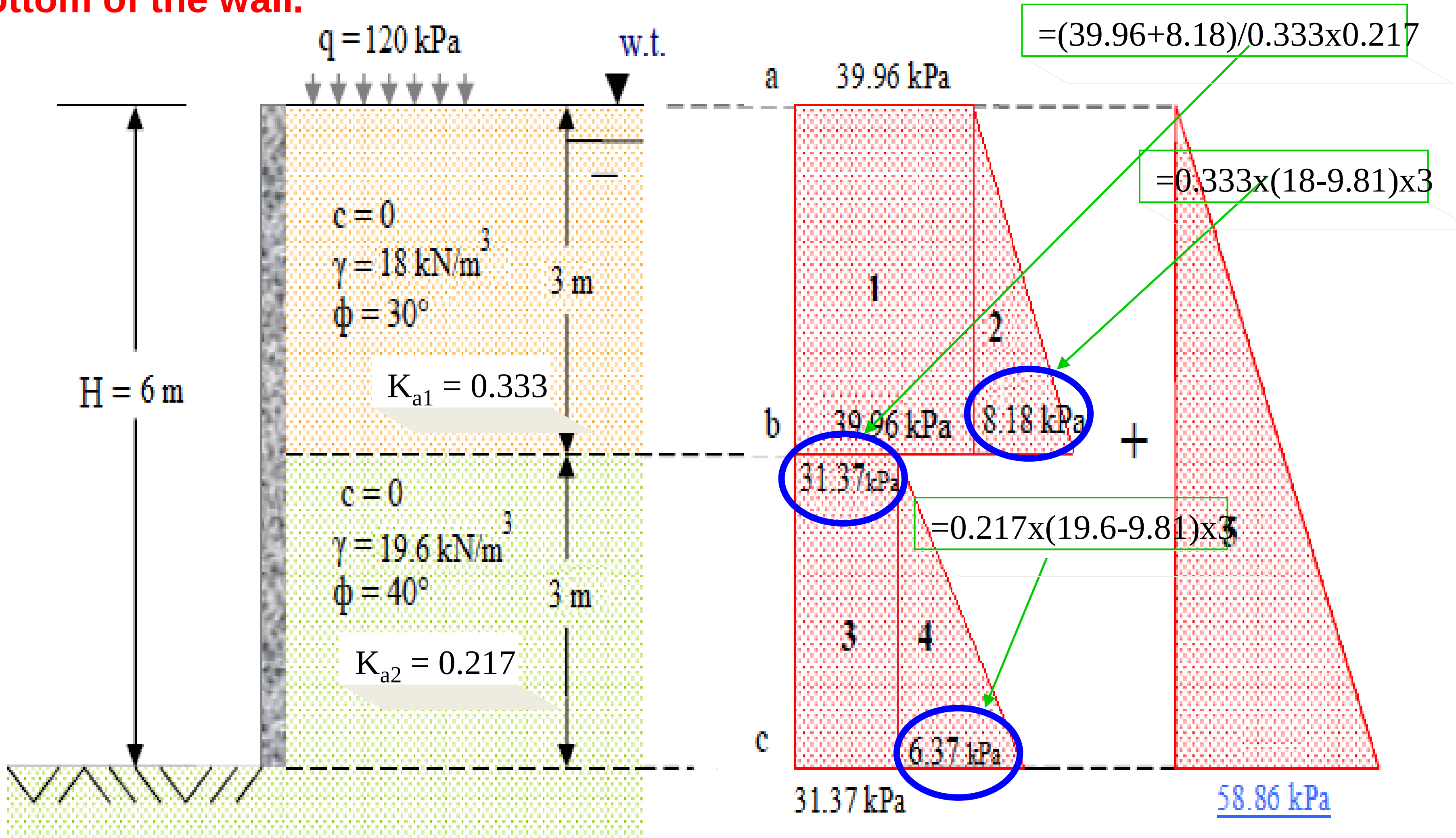
The location of the resultant can be found by taking the moment about the bottom of the wall:

$$\bar{z} = \frac{24\left(3 + \frac{3}{3}\right) + 39.0\left(\frac{3}{2}\right) + 54.15\left(\frac{3}{3}\right)}{117.15}$$

$$= \mathbf{1.78\ m}$$

EXAMPLE

Draw the pressure diagram on the wall in an active pressure condition, and find the total resultant F on the wall and its location with respect to the bottom of the wall.



SOLUTION

Step 1

$K_{a1} = \tan^2(45° - 30°/2) = 0.333$

$K_{a2} = \tan^2(45° - 40°/2) = 0.217$

Step 2

The stress on the wall at point *a* is:

$p_a = q\,K_{a1} = (120)\,(0.333) = \underline{39.96\ kPa}$

The stress at *b* (within the top stratum) is:

$p_{b-} = (q + \gamma' h)\,K_{a1}$

$= [120 + (18 - 9.81)\,(3)]\,[0.333] = \underline{48.14\ kPa}$

The stress at *b* (within bottom stratum) is:

$p_{b+} = (q + \gamma' h)\,K_{a2}$

$= [120 + (18 - 9.81)\,(3)]\ [0.217] = \underline{31.37\ kPa}$

The stress at point *c* is:

$p_{c'} = [q + (\gamma' h)_1 + (\gamma' h)_2]\,K_{a2}$

$= [120 + (18 - 9.81)\,(3) + (19.6 - 9.81)\,(3)]\,[0.217] = \underline{37.75\ kPa}$

The pressure of the water upon the wall is:

$p_w = \gamma_w h = (9.81)\,(6) = \underline{58.86\ kPa}$

SOLUTION

Step 3

The forces from each area:

$F_1 = (3)(39.96) = 119.88$ kN/m

$F_2 = ½ (3)(8.18) = 12.27$ kN/m

$F_3 = (3)(31.37) = 94.11$ kN/m

$F_4 = ½ (3)(6.37) = 9.555$ kN/m

$F_5 = ½ (58.86)(6) = 176.58$ kN/m

$\mathbf{F_{total}} = \mathbf{412.395}$ kN/m

Step 4

The location of forces ŷ is at:

ŷ **412.395** =119.88 (4.5) + 12.27 (4) + 94.11 (1.5) + 9.555 (1) + 176.58 (2)

=539.46+ 49.08 + 141.165 + 9.555 + 353.16 = 1092.42 kN

ŷ = 2.65 m from bottom of wall

NOTES

- The distribution of lateral pressure will be one triangle (no bends) if:
  - K is the same for all layers behind the wall.
  - No water or water is at the surface.
  - The same g

- If we have K_0 (or even K_a and K_p) different, then we calculate the lateral pressure at the interface of the two layers twice. First we use K of the upper layer then K of the lower layer. In both we have the same σ_v, but since K is different we will have two different lateral pressures.

- Since there can be no lateral transfer of weight if the surface is horizontal no shear stresses exist on horizontal and vertical planes. The vertical and horizontal stresses, therefore are principal stresses.

- Rankine's theory overestimates active pressure and underestimates passive pressure because it assumes frictionless wall.

The end