Brouwer's conjecture

From Wikipedia, the free encyclopedia

[Jump to navigation](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#mw-head)[Jump to search](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#searchInput)

In the mathematical field of [spectral graph theory](https://en.wikipedia.org/wiki/Spectral_graph_theory), **Brouwer's conjecture** is a conjecture by [Andries Brouwer](https://en.wikipedia.org/wiki/Andries_Brouwer" \o "Andries Brouwer) on upper bounds for the intermediate sums of the [eigenvalues](https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors) of the [Laplacian](https://en.wikipedia.org/wiki/Laplacian_matrix) of a [graph](https://en.wikipedia.org/wiki/Graph_(discrete_mathematics)) in term of its number of edges.[[1]](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_note-:0-1)

The conjecture states that if *G* is a simple undirected graph and *L*(*G*) its Laplacian matrix, then its eigenvalues *λn*(*L*(*G*)) ≤ *λn*−1(*L*(*G*)) ≤ ... ≤ *λ*1(*L*(*G*)) satisfy

{\displaystyle \sum \_{i=1}^{t}\lambda \_{i}(L(G))\leq m(G)+\left({\begin{array}{c}t+1\\2\end{array}}\right),\quad t=1,\ldots ,n}

Graphical user interface, application, Word

Description automatically generated

 where *m*(*G*) is the number of edges of *G*.

State of the art[[edit](https://en.wikipedia.org/w/index.php?title=Brouwer%27s_conjecture&action=edit&section=1)]

Brouwer has confirmed by computation that the conjecture is valid for all graphs with at most 10 vertices.[[1]](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_note-:0-1) It is also known that the conjecture is valid for any number of vertices if *t* = 1, 2, *n* − 1, and *n*.

For certain types of graphs, Brouwer's conjecture is known to be valid for all *t* and for any number of vertices. In particular, it is known that is valid for trees,[[2]](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_note-2) and for unicyclic and bicyclic graphs.[[3]](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_note-3) It was also proved that Brouwer’s conjecture holds for two large families of graphs; the first family of graphs is obtained from a clique by identifying each of its vertices to a vertex of an arbitrary c-cyclic graph, and the second family is composed of the graphs in which the removal of the edges of the maximal complete bipartite subgraph gives a graph each of whose non-trivial components is a c-cyclic graph.[[4]](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_note-4) For certain sequences of random graphs, Brouwer's conjecture holds true with probability tending to one as the number of vertices tends to infinity.[[5]](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_note-5)

References[[edit](https://en.wikipedia.org/w/index.php?title=Brouwer%27s_conjecture&action=edit&section=2)]

* 1. ^ [Jump up to:***a***](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_ref-:0_1-0) [***b***](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_ref-:0_1-1) *Brouwer, Andries E.; Haemers, Willem H. (2012). Spectra of Graphs. Universitext. New York, NY: Springer New York.*[*doi*](https://en.wikipedia.org/wiki/Doi_(identifier))*:*[*10.1007/978-1-4614-1939-6*](https://doi.org/10.1007%2F978-1-4614-1939-6)*.*[*ISBN*](https://en.wikipedia.org/wiki/ISBN_(identifier))[*978-1-4614-1938-9*](https://en.wikipedia.org/wiki/Special:BookSources/978-1-4614-1938-9)*.*
  2. [**^**](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_ref-2) *Haemers, W.H.; Mohammadian, A.; Tayfeh-Rezaie, B. (2010). "On the sum of Laplacian eigenvalues of graphs". Linear Algebra and Its Applications.****432****(9): 2214–2221.*[*doi*](https://en.wikipedia.org/wiki/Doi_(identifier))*:*[*10.1016/j.laa.2009.03.038*](https://doi.org/10.1016%2Fj.laa.2009.03.038)*.*
  3. [**^**](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_ref-3) *Du, Zhibin; Zhou, Bo (2012).*[*"Upper bounds for the sum of Laplacian eigenvalues of graphs"*](https://doi.org/10.1016%2Fj.laa.2012.01.007)*. Linear Algebra and Its Applications.****436****(9): 3672–3683.*[*doi*](https://en.wikipedia.org/wiki/Doi_(identifier))*:*[*10.1016/j.laa.2012.01.007*](https://doi.org/10.1016%2Fj.laa.2012.01.007)*.*
  4. [**^**](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_ref-4) *Ganie, Hilal A.; Pirzada, S.; Rather, Bilal A.; Trevisian, V (2020).*[*"Further developments on Brouwer's conjecture for the sum of Laplacian eigenvalues of graphs"*](https://www.sciencedirect.com/science/article/abs/pii/S0024379519304963)*. Linear Algebra and Its Applications.****588****(1): 1–18.*[*doi*](https://en.wikipedia.org/wiki/Doi_(identifier))*:*[*10.1016/j.laa.2019.11.020*](https://doi.org/10.1016%2Fj.laa.2019.11.020)*.*
  5. [**^**](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_ref-5) *Rocha, Israel (2020). "Brouwer's conjecture holds asymptotically almost surely". Linear Algebra and Its Applications.****597****: 198–205.*[*arXiv*](https://en.wikipedia.org/wiki/ArXiv_(identifier))*:*[*1906.05368*](https://arxiv.org/abs/1906.05368)*.*[*doi*](https://en.wikipedia.org/wiki/Doi_(identifier))*:*[*10.1016/j.laa.2020.03.019*](https://doi.org/10.1016%2Fj.laa.2020.03.019)*.*