جامعة الملك سعود
King Saud University

*College of Applied Computer Sciences*

(Math 151) **Discrete Mathematics**

First Semester 2021/2022

| Score |
| --- |
| |

# Final Exam

Date: 5/1/2022 Time: 01:00 P.M – 04:00 P.M Time allowed: 3 hours

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| **STUDENT NAME** | Answer Sheet (Dr Borhen) | |
| **Registration Number** | | |
| **Section** | | Number of attendance: |

- There are **10 multiple choice questions in part A** and **10 questions in part B**. The maximum score is 40 marks.
- Please do not forget to put your name and registration number on your paper.

**Put your answers in the following table.**

| QUESTION | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ANSWER | B | D | B | D | B | B | C | D | A | C |

## PART - A

**1.5 × 10 = 15**

**Q1**. The proposition $(p \wedge q) \rightarrow \neg r$ is logically equivalent to

A. $\neg r \rightarrow (p \wedge q)$  B. $\neg(p \wedge q \wedge r)$

C. $r \rightarrow (\neg p \wedge \neg r)$  D. $\neg(p \vee q \vee r)$

**Q2**. The contrapositive of the statement " If a + b=0 and a >0, then b <0 " is

A. "If $a+b \neq 0$ and $a \leq 0$, then $b \geq 0$"  B "If $b \geq 0$,then $a+b \neq 0$ and $a \leq 0$"

C. "If $b < 0$ , then $a+b \neq 0$ or $a \leq 0$"  D. "If $b \geq 0$, then $a+b \neq 0$ or $a \leq 0$"

**Q3**. The relation R defined on N= {1,2,3,4,…} by $a \; R \; b \Leftrightarrow a \leq b^2$ is

A. Reflexive and symmetric  B. Reflexive and but not symmetric

C. Symmetric but not reflexive  D. Not reflexive and not symmetric

**Q4**. The relation S={(w, x), (x, w), (x ,y), (x, z),(z, w),(z, y)} on the set E= {x, y, z, w} is

A. Antisymmetric and transitive

B. Antisymmetric but not transitive

C. Transitive but not antisymmetric

D. Not antisymmetric and not transitive

**Q5**. The relation T defined on Z = { 0, ±1, ±2, ±3, ....} by x T y ⇔ $x^2 = y^2$ is

A. An equivalence relation and a partial order
B. An equivalence relation and not a partial order
C. A partial order but not an equivalence relation
D. Not an equivalence relation not a partial order

**Q6**. If 3 is the degree of every vertex in a graph G having m vertices and 15 edges, then m=

A. 5 B. 10 C. 30 D. 45

**Q7**. Let G be a graph having 33 edges and the degree of its vertices are $x, x, x, x, 2x, 2x, 3x$ . Then the value of $x$ is

A. 2 B. 4 C. 6 D. 8

**Q8**. If G is a simple graph with 15 edges and its complement has 13 edges then the number of vertices of G is

A. 5 B. 6 C. 7 D. 8

**Q9**. The CSP form of the Boolean function $f(x,y,z) = x'z + yz'$ is

A. $x'yz + x'y'z + xyz' + x'yz'$

B. $x'yz + x'y'z' + xyz' + x'yz'$

C. $x'yz + x'y'z + xyz'$

D. $x'yz + x'y'z + x'yz' + x'y'z'$

**Q10**. The CPS form of the Boolean function $f(x,y,z) = x'z + yz'$ is

A. $(x'+y+z')(x'+y+z)(x'+y'+z')$

B. $(x'+y+z')(x+y'+z)(x'+y'+z')(x+y+z)$

C. $(x'+y+z')(x'+y+z)(x'+y'+z')(x+y+z)$

D. $(x+y'+z)(x+y'+z')(x'+y'+z')(x'+y+z')$

## PART-B

**Q1.** Using logic laws, show that $p \wedge \neg[q \rightarrow (p \vee r)]$ is a contradiction. **(2 Marks)**

$$p \wedge \neg[q \rightarrow (p \vee r)] \equiv p \wedge \neg[\neg q \vee (p \vee r)]$$
$$\equiv p \wedge [q \wedge \neg(p \vee r)]$$
$$\equiv p \wedge (q \wedge (\neg p \wedge \neg r))$$
$$\equiv (p \wedge \neg p) \wedge (q \wedge \neg r)$$
$$\equiv F \wedge (q \wedge \neg r)$$
$$\equiv F \quad \text{(contradiction)}$$

**Q2.** Let $\{a_n\}$ be a sequence defined inductively by: $\begin{cases} a_0 = 2, \quad a_1 = 4 \\ a_n = 4a_{n-1} - 3a_{n-2} \quad \text{for } n \geq 2 \end{cases}$.

Show that $a_n = 1 + 3^n$ for all integers n $\geq 0$. **(3 Marks)**

Put $P(n)$ : $a_n = 1 + 3^n$

- Basis step: $n = 0$: $2 = a_0 \stackrel{?}{=} 1 + 3^0 = 2$ | $n = 1$: $4 = a_1 \stackrel{?}{=} 1 + 3^1 = 4$

So $P(0)$ & $P(1)$ are true.

- Inductive step: let $k \geq 2$, we assume that $P(2), \ldots, P(k)$ are all true. Now we prove that $P(k+1)$ remains true.

$$a_{k+1} \stackrel{?}{=} 1 + 3^{k+1}$$

As $a_{k+1} = 4a_k - 3a_{k-1}$ and we know that $P(k)$ & $P(k-1)$ are true then $a_k = 1 + 3^k$ and $a_{k-1} = 1 + 3^{k-1}$

We get $a_{k+1} = 4(1 + 3^k) - 3(1 + 3^{k-1})$
$= 4 + 4(3^k) - 3 - (3^k)$
$= 1 + 3(3^k) = 1 + 3^{k+1}$ ✓

we deduce for $n \geq 0$; $a_n = 1 + 3^n$.

**Q3**. Let S be the relation defined on $\mathbb{N} = \{1,2,3,...\}$ as $a\,S\,b \Leftrightarrow a \mid b$. Determine whether the relation is reflexive, symmetric, antisymmetric, transitive? Justify your answer. **(4 Marks)**

(1) • S is reflexive because $a \mid a \Leftrightarrow a\,S\,a$

(1) • S is not symmetric because $2 \mid 6$ but $6 \nmid 2$

(1) • S is antisymmetric on $\mathbb{N}$ because if $a S b$ and $b S a$ then $a \mid b \Leftrightarrow b = ak$ with $k \in \mathbb{N}$ also $b \mid a \Leftrightarrow a = bk'$ with $k' \in \mathbb{N}$

So $a = (ak)k' \Leftrightarrow a(1-kk') = 0$ as $a \neq 0$ then $kk' = 1$

it follows that $k = k' = 1$ so $a = b$.

(1) • S is transitive on $\mathbb{N}$ because if $a S b$ and $b S c$

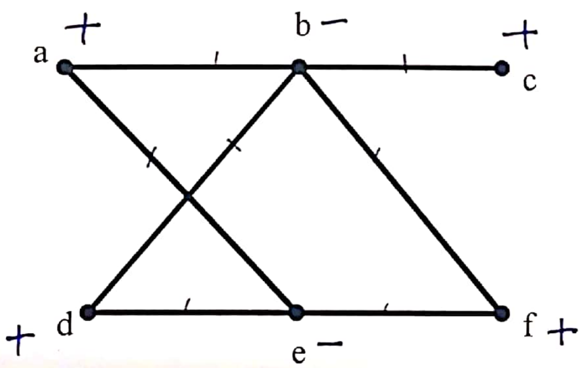
then $a \mid b \Leftrightarrow \boxed{b = ak}^{(1)}$ with $k \in \mathbb{N}$

Also $b \mid c \Leftrightarrow \boxed{c = bh}_{(2)}$ with $h \in \mathbb{N}$. By substitution (1) in (2)

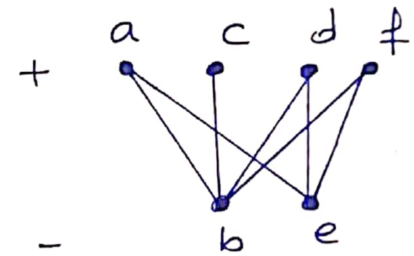
we get $c = a(kh)$ so $a \mid c \Leftrightarrow a S c$

**Q4**. Is the following graph bipartite or not? Justify your answer. **(2 Marks)**

(2)

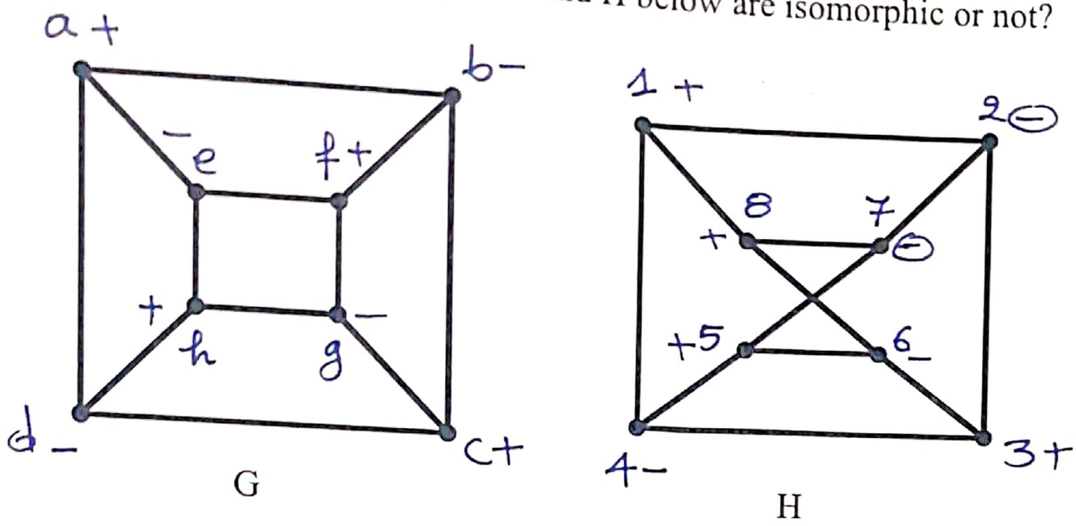



$\cong$




The following graph is bipartite because it contains only even cycles.

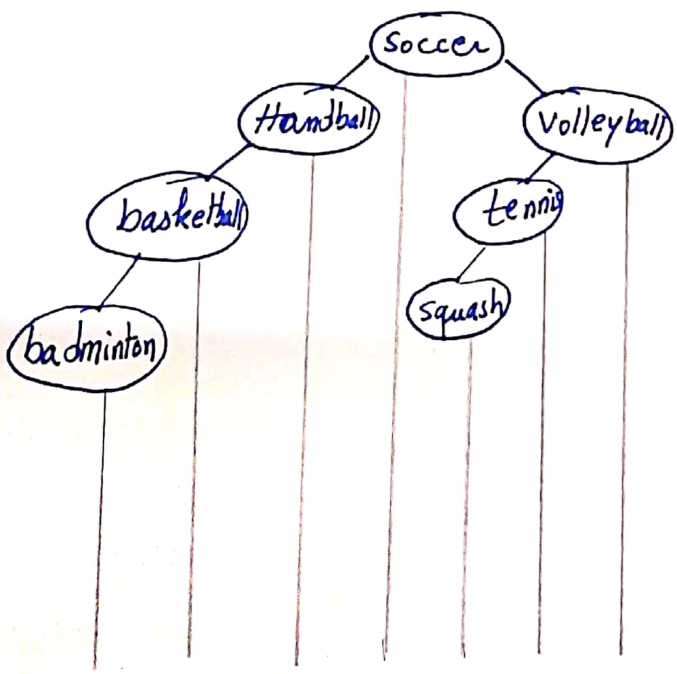
**Q5**. Determine whether the graphs G and H below are isomorphic or not? **(2 Marks)**




G is not isomorphic to H because

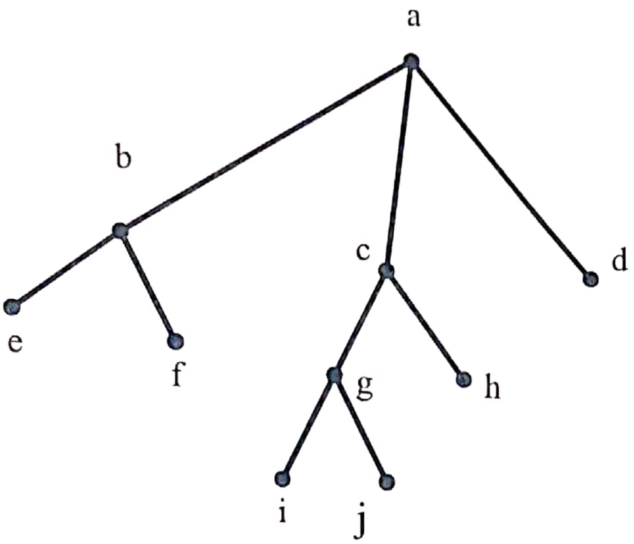
G is bipartite but H is not bipartite.

**Q6**. Form a binary search tree for the words: soccer, handball, volleyball, basketball, tennis, badminton, squash. (Using alphabetical order) **(2 Marks)**



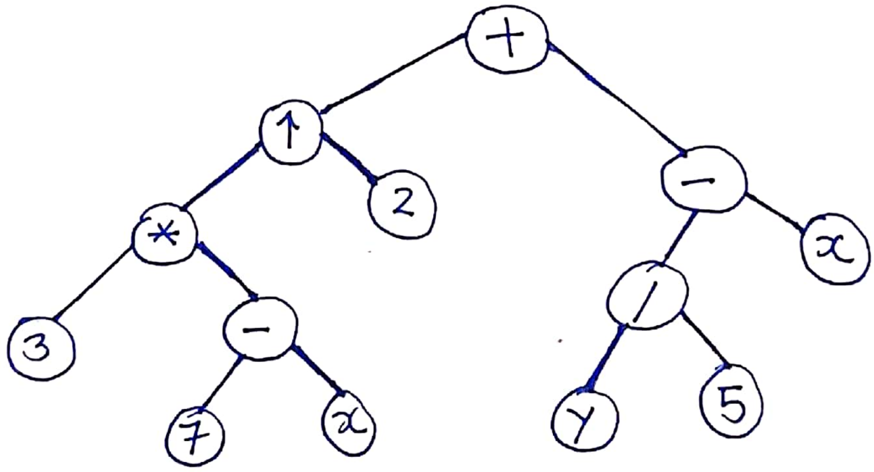

badminton / basketball / Handball / Soccer / quash / tennis / Volleyball.

**Q7**. Find the preorder, the inorder and the postorder traversals of tree as below: **(3 Marks)**




| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ·Right | PREORDER | a | b | e | f | c | g | i | j | h | d |
| eft-Root-Right | INORDER | e | b | f | a | i | g | j | c | h | d |
| eft-Right-Root | POSTORDER | e | f | b | i | j | g | h | c | d | a |

**Q8**. Represent the arithmetic expression $\left[3*\left((7-x)\uparrow 2\right)\right]+\left[(y/5)-x\right]$ by an ordered rooted tree. **(2 Marks)**




**Q9**. (a) Evaluating the prefix expression: .12.. **(1 Mark)**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| * | + | - | 5 | 3 | 2 | / | 9 | 3 |

(b) Evaluating the postfix expression: .12. **(1 Mark)**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 5 | 3 | - | 2 | + | 9 | 3 | / | | * |

**Q10.** The K-map of the Boolean function f is given by:

| | zw | zw' | z'w' | z'w |
|---|---|---|---|---|
| xy | 1 | 1 | 0 | 0 |
| xy' | 1 | 1 | 1 | 1 |
| x'y' | 0 | 1 | 0 | 0 |
| x'y | 0 | 1 | 0 | 0 |

(a) Find MSP and MPS forms.

**(2 Marks)**

① $\boxed{MSP(f) = xy' + zw' + xz}$

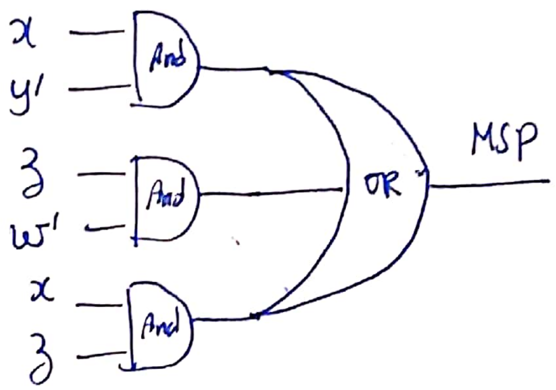
$MPS(f) = (MSP(f'))'$

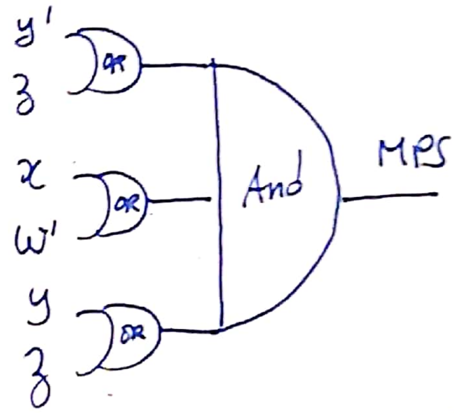
$MSP(f') = yz' + x'w + y'z'$

① So $\boxed{MPS(f) = (y'+z)\cdot(x+w')\cdot(y+z)}$

(b) Construct a least And & Or logic network for f.

**(1 Mark)**




Both are least And & OR logic network for f.