College of Science.
Department of Statistics & Operations Research

كلية العلوم
قسم الإحصاء وبحوث العمليات

Second Midterm Exam
Academic Year 1443-1444 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	**Loss**		**اسم المقرر**
Course Code	**Actu 466**		**رمز المقرر**
Exam Date	**2021-11-17**	**1443-04-12**	**تاريخ الامتحان**
Exam Time	**10: 00 AM**		**وقت الامتحان**
Exam Duration	**2 hours**	**ساعتان**	**مدة الامتحان**
Classroom No.			**رقم قاعة الاختبار**
Instructor Name			**اسم استاذ المقرر**

Student Information معلومات الطالب		
Student's Name		**اسم الطالب**
ID number		**الرقم الجامعي**
Section No.		**رقم الشعبة**
Serial Number		**الرقم التسلسلي**

General Instructions:

- Your Exam consists of 1 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 1 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف و الساعات الذكية خارج قاعة الامتحان.
-

هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

Exercise 1 *Auto liability losses for a group of insureds (Group R) follow a Pareto distribution with* $\alpha = 2$ *and* $\theta = 2,000$*. Losses from a second group (Group S) follow a Pareto distribution with* $\alpha = 2$ *and* $\theta = 3,000$*. Group R has an ordinary deductible of* 500*, while Group S has a franchise deductible of* 200*. Calculate the amount that the expected cost* ***per payment*** *for Group S exceeds that for Group R.*

700   800   900   1000

Exercise 2 *Determine the loss elimination ratio for the Pareto distribution with* $\alpha = 3$ *and* $\theta = 2,000$ *with an ordinary deductible of* 500*, and interpret this number.*

0.26   0.36   0.46   0.56

Exercise 3 *a) Consider a Poisson distribution with mean* $\lambda = 1.2$*. Evaluate the probabilities* p_k *where* $k = 0, 1, 2, 3, 4, 5$*.*
b) Consider the corresponding zero-truncated Poisson distribution. Evaluate the probabilities p_k^T *where* $k = 1, 2, 3, 4, 5$*.*
c) Consider the corresponding zero-modified Poisson distribution with $p_0^M = 0.4$*. Evaluate the probabilities* p_k^M *where* $k = 1, 2, 3, 4, 5$*.*

Exercise 4 *Losses follow an exponential distribution with mean* $5,000$*. An insurance policy covers losses subject to a franchise deductible of* $2,000$*. Determine the expected insurance payment per loss.*

$2,692$ $3,692$ $4,692$ $5,692$

A.3.3 One-parameter distributions

A.3.3.1 Exponential—θ

$$
\begin{aligned}
f(x) &= \frac{e^{-x/\theta}}{\theta} \qquad F(x) = 1 - e^{-x/\theta} \\
M(t) &= (1-\theta t)^{-1} \qquad \mathrm{E}[X^k] = \theta^k \Gamma(k+1), \quad k > -1 \\
\mathrm{E}[X^k] &= \theta^k k!, \quad \text{if } k \text{ is an integer} \\
\mathrm{VaR}_p(X) &= -\theta \ln(1-p) \\
\mathrm{TVaR}_p(X) &= -\theta \ln(1-p) + \theta \\
\mathrm{E}[X \wedge x] &= \theta(1 - e^{-x/\theta}) \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k \Gamma(k+1)\Gamma(k+1; x/\theta) + x^k e^{-x/\theta}, \quad k > -1 \\
&= \theta^k k! \Gamma(k+1; x/\theta) + x^k e^{-x/\theta}, \quad k \text{ an integer} \\
\text{mode} &= 0
\end{aligned}
$$

A.3.3.2 Inverse exponential—θ

$$
\begin{aligned}
f(x) &= \frac{\theta e^{-\theta/x}}{x^2} \qquad F(x) = e^{-\theta/x} \\
\mathrm{E}[X^k] &= \theta^k \Gamma(1-k), \quad k < 1 \\
\mathrm{VaR}_p(X) &= \theta(-\ln p)^{-1} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k G(1-k; \theta/x) + x^k(1 - e^{-\theta/x}), \quad \text{all } k \\
\text{mode} &= \theta/2
\end{aligned}
$$

A.5 Other distributions

A.5.1.1 Lognormal—μ,σ (μ can be negative)

$$
\begin{aligned}
f(x) &= \frac{1}{x\sigma\sqrt{2\pi}} \exp(-z^2/2) = \phi(z)/(\sigma x), \quad z = \frac{\ln x - \mu}{\sigma} \qquad F(x) = \Phi(z) \\
\mathrm{E}[X^k] &= \exp(k\mu + k^2\sigma^2/2) \\
\mathrm{E}[(X \wedge x)^k] &= \exp(k\mu + k^2\sigma^2/2)\Phi\left(\frac{\ln x - \mu - k\sigma^2}{\sigma}\right) + x^k[1 - F(x)] \\
\text{mode} &= \exp(\mu - \sigma^2)
\end{aligned}
$$

A.2.3 Two-parameter distributions

A.2.3.1 Pareto (Pareto Type II, Lomax)—α, θ

$$
\begin{aligned}
f(x) &= \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}} \qquad F(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}, \quad -1 < k < \alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k k!}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer} \\
\mathrm{VaR}_p(X) &= \theta[(1-p)^{-1/\alpha} - 1] \\
\mathrm{TVaR}_p(X) &= \mathrm{VaR}_p(X) + \frac{\theta(1-p)^{-1/\alpha}}{\alpha-1}, \quad \alpha > 1 \\
\mathrm{E}[X \wedge x] &= \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right], \quad \alpha \neq 1 \\
\mathrm{E}[X \wedge x] &= -\theta \ln\left(\frac{\theta}{x+\theta}\right), \quad \alpha = 1 \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)} \beta[k+1, \alpha-k; x/(x+\theta)] + x^k \left(\frac{\theta}{x+\theta}\right)^\alpha, \quad \text{all } k \\
\text{mode} &= 0
\end{aligned}
$$

A.2.3.2 Inverse Pareto—τ, θ

$$
\begin{aligned}
f(x) &= \frac{\tau\theta x^{\tau-1}}{(x+\theta)^{\tau+1}} \qquad F(x) = \left(\frac{x}{x+\theta}\right)^\tau \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(\tau+k)\Gamma(1-k)}{\Gamma(\tau)}, \quad -\tau < k < 1 \\
\mathrm{E}[X^k] &= \frac{\theta^k (-k)!}{(\tau-1)\cdots(\tau+k)}, \quad \text{if } k \text{ is a negative integer} \\
\mathrm{VaR}_p(X) &= \theta[p^{-1/\tau} - 1]^{-1} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k \tau \int_0^{x/(x+\theta)} y^{\tau+k-1}(1-y)^{-k} dy + x^k \left[1 - \left(\frac{x}{x+\theta}\right)^\tau\right], \quad k > -\tau \\
\text{mode} &= \theta \frac{\tau-1}{2}, \quad \tau > 1, \text{ else } 0
\end{aligned}
$$

A.2.3.3 Loglogistic (Fisk)—γ, θ

$$
\begin{aligned}
f(x) &= \frac{\gamma(x/\theta)^\gamma}{x[1+(x/\theta)^\gamma]^2} \qquad F(x) = u, \quad u = \frac{(x/\theta)^\gamma}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \theta^k \Gamma(1+k/\gamma)\Gamma(1-k/\gamma), \quad -\gamma < k < \gamma \\
\mathrm{VaR}_p(X) &= \theta(p^{-1} - 1)^{-1/\gamma} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k \Gamma(1+k/\gamma)\Gamma(1-k/\gamma)\beta(1+k/\gamma, 1-k/\gamma; u) + x^k(1-u), \quad k > -\gamma \\
\text{mode} &= \theta\left(\frac{\gamma-1}{\gamma+1}\right)^{1/\gamma}, \quad \gamma > 1, \text{ else } 0
\end{aligned}
$$

Appendix B

An Inventory of Discrete Distributions

B.1 Introduction

The 16 models fall into three classes. The divisions are based on the algorithm by which the probabilities are computed. For some of the more familiar distributions these formulas will look different from the ones you may have learned, but they produce the same probabilities. After each name, the parameters are given. All parameters are positive unless otherwise indicated. In all cases, p_k is the probability of observing k losses.

For finding moments, the most convenient form is to give the factorial moments. The jth factorial moment is $\mu_{(j)} = \mathrm{E}[N(N-1)\cdots(N-j+1)]$. We have $\mathrm{E}[N] = \mu_{(1)}$ and $\mathrm{Var}(N) = \mu_{(2)} + \mu_{(1)} - \mu_{(1)}^2$.

The estimators which are presented are not intended to be useful estimators but rather for providing starting values for maximizing the likelihood (or other) function. For determining starting values, the following quantities are used [where n_k is the observed frequency at k (if, for the last entry, n_k represents the number of observations at k or more, assume it was at exactly k) and n is the sample size]:

$$\hat{\mu} = \frac{1}{n}\sum_{k=1}^{\infty} k n_k, \quad \hat{\sigma}^2 = \frac{1}{n}\sum_{k=1}^{\infty} k^2 n_k - \hat{\mu}^2.$$

When the method of moments is used to determine the starting value, a circumflex (e.g., $\hat{\lambda}$) is used. For any other method, a tilde (e.g., $\tilde{\lambda}$) is used. When the starting value formulas do not provide admissible parameter values, a truly crude guess is to set the product of all λ and β parameters equal to the sample mean and set all other parameters equal to 1. If there are two λ and/or β parameters, an easy choice is to set each to the square root of the sample mean.

The last item presented is the probability generating function,

$$P(z) = \mathrm{E}[z^N].$$

B.2 The $(a, b, 0)$ class

B.2.1.1 Poisson—λ

$$\begin{aligned} p_0 &= e^{-\lambda}, \quad a = 0, \quad b = \lambda \qquad p_k = \frac{e^{-\lambda}\lambda^k}{k!} \\ \mathrm{E}[N] &= \lambda, \quad \mathrm{Var}[N] = \lambda \qquad P(z) = e^{\lambda(z-1)} \end{aligned}$$

466 — Pd 2

Ex 2

$X_R \sim Pareto(\alpha = 2, \theta = 2000)$

$X_S \sim Pareto(\alpha = 2, \theta = 3000)$

$d_R = 500 \qquad d_S = 200$

$$E(Y_P^R) = \frac{E(Y_L^R)}{S(d_R)} = \frac{E(X) - E(X \wedge d)}{S(d)}$$

$$= \frac{\frac{\theta}{\alpha-1}\left[\frac{\theta}{d_R+\theta}\right]^{\alpha-1}}{\left(\frac{\theta}{d_R+\theta}\right)^{\alpha}} = \frac{d_R+\theta}{\alpha-1} = 2500$$

$$E(Y_P^S) = \frac{E(X) - E(X \wedge d) + d\,S(d)}{S(d)}$$

$$= \frac{d_S+\theta}{\alpha-1} + d_S = 3200 + 200 = 3400$$

$$E(Y_P^S) - E(Y_P^R) = 3400 - 2500 = 900$$

Ex 2 $X \sim Pareto(\alpha = 3, \theta = 2000)$, $d = 500$

$$CER = \frac{E(X) - E(Y^L)}{E(X)} = \frac{E(X \wedge d)}{E(X)}$$

$$= \frac{\frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{d+\theta}\right)^{\alpha-1}\right]}{\theta/(\alpha-1)} = 1 - \left(\frac{2000}{2500}\right)^2$$

$$= 0.36$$

Ex 3 $X \sim P(\lambda = 1.2)$

(3)(a) $p_k = e^{-\lambda}\frac{\lambda^k}{k!}$: $p_0 = 0.301$, $p_1 = 0.361$, $p_2 = 0.2168$, $p_3 = 0.0867$, $p_4 = 0.026$, $p_5 = 0.0062$

(3)(b) $p_k^T = \frac{1-0}{1-p_0}p_k$ = $p_1^T = 0.517$, $p_2^T = 0.310$, $p_3^T = 0.124$, $p_4^T = 0.037$, $p_5^T = 0.0089$

(3)(c) $p_0^M = 0.4$

$p_k^M = \frac{1-p_0^M}{1-p_0}p_k$ = $p_1^M = 0.31032$, $p_2^M = 0.1861$, $p_3^M = 0.0744$, $p_4^M = 0.0223$, $p_5^M = 0.00536$

Ex 4 $X \sim Exp(\text{mean} = 5{,}000)$ $f(x) = \frac{1}{\theta} e^{-x/\theta}$

$d = 2{,}000$ $\theta = 5{,}000$

$E(Y^L) = E(X) - E(X \wedge d) + d\, S(d)$

$= \theta - \theta(1 - e^{-d/\theta}) + d\, e^{-d/\theta}$

$= (\theta + d)\, e^{-d/\theta} = 7000\, e^{-2/5} = 4{,}692$