College of Science.
Department of Statistics & Operations Research

كلية العلوم
قسم الإحصاء وبحوث العمليات

First Midterm Exam
Academic Year 1442-1443 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	Loss		اسم المقرر
Course Code	Actu 466		رمز المقرر
Exam Date	2021-11-01	1442-03-15	تاريخ الامتحان
Exam Time	10: 00 AM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 1 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 1 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف و الساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

Exercise 1 *For a house insurance policy, the loss amount in the event of a fire, is being modeled by a Pareto distribution with* $\theta = 10,000$ *and* $\alpha = 7$*. For a policy with a deductible amount of $5,000, calculate the mean excess loss* $E(Y^P)$*.*

Exercise 2 *The random variable* X *is the loss under a medical insurance policy and is distributed as a 2 point-mixture distribution. The 2 point-mixture distribution is a combination of a gamma distribution with a weight of* 0.4 *and a Pareto distribution with a weight of* 0.6*. The parameters for the gamma distribution are* $\alpha = 2$ *and* $\theta = 100$ *. The parameters for the Pareto distribution are* $\alpha = 5$ *and* $\theta = 5,000$*.*
Calculate the standard deviation of X*.*

$A)$ 1054.66 $B)$ 1354.66 $C)$ 1654.66 $D)$ 19545.66

Exercise 3 *The distribution of a loss* X *is a two-point mixture: i) With probability* 0.8*,* X *has a Pareto distribution with* $\alpha = 2$ *and* $\theta = 100$*. ii) With probability* 0.2*,* X *has a Pareto distribution with* $\alpha = 4$ *and* $\theta = 3,000$*. Calculate* $\Pr(X \leq 200)$*.*

$A)$ 0.88 $B)$ 0.85 $C)$ 0.82 $D)$ 0.76

Exercise 4 *A spliced distribution is defined to have the following function*

$$f(x) = \begin{cases} a.f_1(x) & 0 < x < 100 \\ b.f_2(x) & 100 \leq x < 200 \end{cases}$$

$f_1(x)$ *is the density function of a uniform random variable on the interval* $(0, 100)$*,* $a = 0.4$*,* $b = 0.6$*, and* $f_2(x)$ *is the density function of a uniform variable on the interval* $[100, 200)$*.*
Find the variance of the spliced distribution.

$A)$ 3033 $B)$ 3233 $C)$ 3433 $D)$ 3633

A.2.3.4 Paralogistic—α, θ

This is a Burr distribution with $\gamma = \alpha$.

$$
\begin{aligned}
f(x) &= \frac{\alpha^2 (x/\theta)^\alpha}{x[1+(x/\theta)^\alpha]^{\alpha+1}} \qquad F(x) = 1 - u^\alpha, \quad u = \frac{1}{1+(x/\theta)^\alpha} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(1+k/\alpha)\Gamma(\alpha - k/\alpha)}{\Gamma(\alpha)}, \quad -\alpha < k < \alpha^2 \\
\mathrm{VaR}_p(X) &= \theta[(1-p)^{-1/\alpha} - 1]^{1/\alpha} \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(1+k/\alpha)\Gamma(\alpha - k/\alpha)}{\Gamma(\alpha)} \beta(1+k/\alpha, \alpha - k/\alpha; 1-u) + x^k u^\alpha, \quad k > -\alpha \\
\text{mode} &= \theta \left(\frac{\alpha - 1}{\alpha^2 + 1} \right)^{1/\alpha}, \quad \alpha > 1, \text{ else } 0
\end{aligned}
$$

A.2.3.5 Inverse paralogistic—τ, θ

This is an inverse Burr distribution with $\gamma = \tau$.

$$
\begin{aligned}
f(x) &= \frac{\tau^2 (x/\theta)^{\tau^2}}{x[1+(x/\theta)^\tau]^{\tau+1}} \qquad F(x) = u^\tau, \quad u = \frac{(x/\theta)^\tau}{1+(x/\theta)^\tau} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(\tau + k/\tau)\Gamma(1 - k/\tau)}{\Gamma(\tau)}, \quad -\tau^2 < k < \tau \\
\mathrm{VaR}_p(X) &= \theta(p^{-1/\tau} - 1)^{-1/\tau} \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\tau + k/\tau)\Gamma(1 - k/\tau)}{\Gamma(\tau)} \beta(\tau + k/\tau, 1 - k/\tau; u) + x^k[1 - u^\tau], \quad k > -\tau^2 \\
\text{mode} &= \theta\,(\tau - 1)^{1/\tau}, \quad \tau > 1, \text{ else } 0
\end{aligned}
$$

A.3 Transformed gamma family

A.3.2 Two-parameter distributions

A.3.2.1 Gamma—α, θ

$$
\begin{aligned}
f(x) &= \frac{(x/\theta)^\alpha e^{-x/\theta}}{x\Gamma(\alpha)} \qquad F(x) = \Gamma(\alpha; x/\theta) \\
M(t) &= (1-\theta t)^{-\alpha}, \quad t < 1/\theta \qquad \mathrm{E}[X^k] = \frac{\theta^k \Gamma(\alpha+k)}{\Gamma(\alpha)}, \quad k > -\alpha \\
\mathrm{E}[X^k] &= \theta^k (\alpha + k - 1) \cdots \alpha, \quad \text{if } k \text{ is an integer}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\alpha+k)}{\Gamma(\alpha)} \Gamma(\alpha+k; x/\theta) + x^k[1 - \Gamma(\alpha; x/\theta)], \quad k > -\alpha \\
&= \alpha(\alpha+1)\cdots(\alpha+k-1)\theta^k \Gamma(\alpha+k; x/\theta) + x^k[1 - \Gamma(\alpha; x/\theta)], \quad k \text{ an integer} \\
\text{mode} &= \theta(\alpha - 1), \quad \alpha > 1, \text{ else } 0
\end{aligned}
$$

A.2.3 Two-parameter distributions

A.2.3.1 Pareto (Pareto Type II, Lomax)—α, θ

$$
\begin{aligned}
f(x) &= \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}} \qquad F(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}, \quad -1 < k < \alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k k!}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer} \\
\mathrm{VaR}_p(X) &= \theta[(1-p)^{-1/\alpha} - 1] \\
\mathrm{TVaR}_p(X) &= \mathrm{VaR}_p(X) + \frac{\theta(1-p)^{-1/\alpha}}{\alpha-1}, \quad \alpha > 1 \\
\mathrm{E}[X \wedge x] &= \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right], \quad \alpha \neq 1 \\
\mathrm{E}[X \wedge x] &= -\theta \ln\left(\frac{\theta}{x+\theta}\right), \quad \alpha = 1 \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k\Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}\beta[k+1, \alpha-k; x/(x+\theta)] + x^k\left(\frac{\theta}{x+\theta}\right)^\alpha, \quad \text{all } k \\
\text{mode} &= 0
\end{aligned}
$$

A.2.3.2 Inverse Pareto—τ, θ

$$
\begin{aligned}
f(x) &= \frac{\tau\theta x^{\tau-1}}{(x+\theta)^{\tau+1}} \qquad F(x) = \left(\frac{x}{x+\theta}\right)^\tau \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(\tau+k)\Gamma(1-k)}{\Gamma(\tau)}, \quad -\tau < k < 1 \\
\mathrm{E}[X^k] &= \frac{\theta^k(-k)!}{(\tau-1)\cdots(\tau+k)}, \quad \text{if } k \text{ is a negative integer} \\
\mathrm{VaR}_p(X) &= \theta[p^{-1/\tau} - 1]^{-1} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k\tau\int_0^{x/(x+\theta)} y^{\tau+k-1}(1-y)^{-k}dy + x^k\left[1 - \left(\frac{x}{x+\theta}\right)^\tau\right], \quad k > -\tau \\
\text{mode} &= \theta\frac{\tau-1}{2}, \quad \tau > 1, \text{ else } 0
\end{aligned}
$$

A.2.3.3 Loglogistic (Fisk)—γ, θ

$$
\begin{aligned}
f(x) &= \frac{\gamma(x/\theta)^\gamma}{x[1+(x/\theta)^\gamma]^2} \qquad F(x) = u, \quad u = \frac{(x/\theta)^\gamma}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \theta^k\Gamma(1+k/\gamma)\Gamma(1-k/\gamma), \quad -\gamma < k < \gamma \\
\mathrm{VaR}_p(X) &= \theta(p^{-1}-1)^{-1/\gamma} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k\Gamma(1+k/\gamma)\Gamma(1-k/\gamma)\beta(1+k/\gamma, 1-k/\gamma; u) + x^k(1-u), \quad k > -\gamma \\
\text{mode} &= \theta\left(\frac{\gamma-1}{\gamma+1}\right)^{1/\gamma}, \quad \gamma > 1, \text{ else } 0
\end{aligned}
$$

Mid 1

loss

Solution

Ex1 $X \sim \text{Pareto}(\alpha = 7, \theta = 10{,}000)$

$d = 5{,}000$.

$$E(Y^P) = \frac{E(Y^L)}{S(d)} = \frac{E(X) - E(X \wedge d)}{S(d)}$$

$$S(d) = \left(\frac{\theta}{d+\theta}\right)^{\alpha}$$

$$E(X) - E(X \wedge d) = \frac{\theta}{\alpha-1} - \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{d+\theta}\right)^{\alpha-1}\right]$$

$$= \frac{\theta}{\alpha-1}\left(\frac{\theta}{d+\theta}\right)^{\alpha-1}$$

$$\Rightarrow E(Y^P) = \frac{\frac{\theta}{\alpha-1}}{\left(\frac{\theta}{d+\theta}\right)^{-1}} = \frac{\theta}{\alpha-1} \; \frac{d+\theta}{\theta} = \frac{d+\theta}{\alpha-1}$$

$$= 2{,}500$$

Ex2 $X_1 \sim \text{Gamma}$ $E(X_1) = \alpha\theta$ $E(X_1^2) = \alpha(\alpha+1)\theta^2$

(7) $X_2 \sim \text{Pareto}$ $E(X_2) = \frac{\theta}{\alpha-1}$ $E(X_2^2) = \frac{2\theta^2}{(\alpha-1)(\alpha-2)}$

$$E(X) = a_1 E(X_1) + a_2 E(X_2) = 0.4(200) + 0.6\left(\frac{5000}{4}\right) = 830$$

$$E(X^2) = a_1 E(X_1^2) + a_2 E(X_2^2) = 0.4 \times 6 \times 100^2 + 0.6 \times 2 \times \frac{5000^2}{4 \times 3}$$

$$= 2{,}524{,}000$$

$$\sigma = \sqrt{E(X^2) - (E(X))^2} = 1{,}354.66$$

Ex3 $P(X \leq 200) = F_X(200) = a_1 F_{X_1}(200) + a_2 F_{X_2}(200)$

$$= 0.8\left(1 - \left(\frac{100}{300}\right)^2\right) + 0.2\left(1 - \left(\frac{3000}{3200}\right)^4\right)$$

$$= 0.7566$$

Ex 4

$$f(x) = \begin{cases} \frac{0.4}{100} & 0 < x < 100 \\ \frac{0.6}{100} & 100 < x < 200 \end{cases}$$

$$E(X) = \int_0^{200} x f(x)\,dx = \int_0^{100} x\left(\frac{0.4}{100}\right)dx + \int_{100}^{200} x\left(\frac{0.6}{100}\right)dx$$

$$= 110 \qquad (= 0.4 \times 50 + 0.6 \times 150)$$

$$E(X^2) = \int_0^{200} x^2 f(x)\,dx = \int_0^{100} x^2\left(\frac{0.4}{100}\right)dx + \int_{100}^{200} x^2\left(\frac{0.6}{100}\right)dx$$

$$= 15,333.33$$

$$\sigma^2 = E(X^2) - (E(X))^2 = 3,233.33$$