College of Science.
Department of Statistics & Operations Research

كلية العلوم
قسم الإحصاء وبحوث العمليات

First Midterm Exam
Academic Year 1442-1443 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	توزيعات الخسارة		اسم المقرر
Course Code	466 ريك		رمز المقرر
Exam Date	2020-10-19	1442-03-03	تاريخ الامتحان
Exam Time	10: 00 AM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 1 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 1 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف و الساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

Exercise 1 *The random variable X is distributed as a Pareto distribution given a parameter θ and a parameter $\alpha = 3$. θ follows a gamma distribution with parameters $\alpha = 2$ and $\theta = 3$. Calculate $Var(X)$.*

$a)$ 67 $b)$ 45 $c)$ 12 $d)$ 70

Exercise 2 *Losses are distributed as a two point mixture distribution of Gamma distributions.*
The first Gamma distribution with parameters of $\alpha = 4$ and $\theta = 2$ has a weight of 0.75.
The second Gamma distribution with parameters of $\alpha = 4$ and $\theta = 10$ has a weight of 0.25.
Use the normal approximation to determine the probability that the sum of 100 *independent claims will exceed* 1750.

$a)$ $1 - \Phi(0.15)$ $b)$ $1 - \Phi(0.64)$ $c)$ $1 - \Phi(0.45)$ $d)$ $1 - \Phi(0.86)$

Exercise 3 *The number of claims for dental insurance is distributed as a Poisson distribution. The amount of each individual claim is follows a gamma distribution with $\alpha = 2$ and $\theta = 100$. The variance of the aggregate claims is $Var[S] = 138,000$. Calculate the expected value of the aggregate claims.*

$a)$ 460 $b)$ 220 $c)$ 165 $d)$ 13

Exercise 4 *For an insurance:*
(i) Losses have a density function $f_X(x) = \begin{cases} 0.02x, & 0 < x < 10 \\ 0, & elsewhere \end{cases}$
(ii) The insurance has an ordinary deductible of 4 *per loss.*
(iii) Y^P is the claim payment per payment random variable.
Claculate $E[Y^P]$.

$a)$ 3.4 $b)$ 3.6 $c)$ 3.8 $d)$ 4.0 $e)$ 4.2

Exercise 5 *The number of claims in a period has a geometric distribution with mean* 4. *The amount of each claim X follows $Pr(X = x) = 0.25$, $x = 1, 2, 3, 4$. The number of claims and the claim amounts are independent. S is the aggregate claim amount in the period. Claculate $F_S(3) = \Pr(S \leq 3)$.*

$a)$ 0.27 $b)$ 0.29 $c)$ 0.31 $d)$ 0.33 $e)$ 0.35

Exercise 6 *The variable S has a compound Poisson claims distribution with the following:*
(i) Individual claim amounts equal to 2 *or* 3.
(ii) $var(S) = 126$.
(iii) $\lambda = 29$.
Determine the expected number of claims of size 2.

$a)$ 25 $b)$ 27 $c)$ 29 $d)$ 31 $e)$ 33

Mid 1.
loss 2020/S2

Ex1 4 marks $X|\theta \sim \text{Pareto}(\theta, \alpha=3)$

(1) $\theta \sim \text{Gamma}(\alpha=2, \theta=3)$

$$\text{Var}\, X = \text{Var}(E(X|\theta)) + E(\text{Var}(X|\theta))$$

(1)
$$E(X|\theta) = \theta/(\alpha-1) = \theta/2.$$
$$E(X^2|\theta) = \frac{2\theta^2}{(\alpha-1)(\alpha-2)} = \theta^2 \Rightarrow \text{Var}(X|\theta) = \theta^2 - \theta^2/4 = 3/4\,\theta^2.$$

$$\text{Var}\, X = \text{Var}(\theta/2) + E(3/4\,\theta^2)$$
$$= 1/4\, V(\theta) + 3/4\, E(\theta^2)$$

(1)
$$E(\theta) = \alpha\theta = 6$$
$$E(\theta^2) = \alpha(\alpha+1)\theta^2 = 54 \Rightarrow \text{Var}\,\theta = 18$$

(2) $$\text{Var}\, X = \frac{18}{4} + \frac{3}{4}(54) = \boxed{45}$$

Ex2 (4) $$f(x) = a_1 f_1(x) + a_2 f_2(x).$$

$$S = \sum_1^{100} X_i \Rightarrow \begin{cases} \mu_S = E(S) = 100\,\mu_X \\ \sigma_S^2 = \text{Var}(S) = 100\,\sigma_X^2. \end{cases}$$

$$E(X) = a_1 \overbrace{\mu_{X_1}}^{8} + a_2 \overbrace{\mu_{X_2}}^{40} = 3/4(8) + 1/4(40) = 16 \quad (1)$$

$$E(X^2) = a_1 E(X_1^2) + a_2 E(X_2^2) = 3/4(80) + 1/4(2000) = 560 \quad (1)$$

$$\Rightarrow \text{Var}\, X = 560 - 16^2 = 304. \quad (1)$$

$$P(S > 1750) = P\left(\frac{S-\mu_S}{\sigma_S} > \frac{1750 - (16\times 100)}{\sqrt{304 \times 100}}\right)$$

$$\simeq P(Z > 0.86) = \boxed{1 - \Phi(0.86)}$$

(1)

Ex 4 4 marks

$$E(Y^P) = \frac{E(Y^L)}{S(d)}.$$

$$S(x) = P(X > x) = \int_x^{\infty} f(t)dt = \int_x^{10} 0.02t\,dt$$

$$= 0.02\left[t^2/2\right]_x^{10} = 0.01(100 - x^2)$$

$$E(Y^L) = \int_d^{10} S(x)dx = \int_d^{10} 0.01(100 - x^2)dx$$

$$= 0.01\left[100 - x^3/3\right]_4^{10} = \dots = 2.88$$

$$S(4) = 0.01(100 - 16) = 0.84$$

$$\Rightarrow \quad E(Y^P) = \frac{2.88}{0.84} = \boxed{3.428571}$$

Ex 5 5 marks

$N \sim$ Geom (mean $= \beta = 4$)

X: 1, 2, 3, 4 — $1/4$.

$$P(S \leq 3) = \sum_0^3 P(s)$$

$$\left\{ P_0 = \frac{1}{1+\beta} = 1/5; \quad P_1 = \frac{\beta}{(1+\beta)^2} = 0.16; \quad P_2 = \frac{\beta^2}{(1+\beta)^3} = 0.128, \quad P_3 = \frac{\beta^3}{(1+\beta)^4} = 0.1024 \right.$$

$$P(S=0) = P(N=0) = P_0 = 0.2$$

$$P(S=1) = P(N=1)P(X=1) = P_1/4 = 0.04$$

$$P(S=2) = P(N=1)P(X=2) + P(N=2)P(X=1)^2$$

$$= P_1/4 + P_2/16 = 0.048$$

$$P(S=3) = P(N=1)P(X=3) + P(N=2)\,2P(X=1)P(X=2)$$

$$+ P(N=3)P(X=1)^3$$

$$= P_1/4 + 2P_2/16 + P_3/64$$

$$P(S \leq 3) = P_0 + P_1/4 + P_2/16 + P_1/4 + P_1/4 + P_2/8 + P_3/64$$

$$= \boxed{0.3456}$$

Ex 6

$$S = \sum_{i=1}^{N} Y_i$$

$$Y = \begin{cases} 2 & \text{Prob } p \\ 3 & \text{Prob } q = 1-p \end{cases}$$

$\text{Var } S = 126 \; ; \; \lambda = 29.$

Number of claims equal 2 $\sim$ Bin(N, p)

Extected number of claims equal 2 is λp.

p?

$\mu_N = \sigma_N^2 = \lambda = 29$

$E(Y) = 2p + 3(1-p) = 3-p$

$E(Y^2) = 4p + 9(1-p) = 9-5p$

$\sigma_Y^2 = p - p^2.$

$V(S) = \lambda\left[\mu_Y^2 + \sigma_Y^2\right] = 29(9-5p) = 126$

$\Rightarrow p = \dfrac{9 - 126/29}{5} = 0.931$

$\Rightarrow \lambda p = 29 \dfrac{9 - 129/29}{5} = \boxed{27}$

Ex 3

$N \sim P(\lambda)$

$X \sim \text{Gamma}(\alpha = 2, \theta = 100)$

$S = \sum_{i=1}^{N} X_i$ (1) $\mu_X = 200$ $E(X^2) = 60{,}000$

$\Rightarrow V(X) = 20{,}000$ (1)

$\text{Var } S = \sigma_N^2 \mu_X^2 + \mu_N \sigma_X^2$

$= \lambda(\mu_X^2 + \sigma_X^2) = \lambda(\cancel{E(X^2}\, 60{,}000) = 138{,}000$

$\Rightarrow \lambda = \dfrac{138{,}000}{60{,}000} = 2.3$ (1)

$E(S) = \lambda \mu_X = 2.3(200) = 460$ (1)