كلية العلوم
قسم الإحصاء وبحوث العمليات

College of Science.
Department of Statistics & Operations Research

Final Exam
Academic Year 1442-1443 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	توزيعات الخسارة		اسم المقرر
Course Code	**ACTU 466**		رمز المقرر
Exam Date	2021-05-05	1442-09-23	تاريخ الامتحان
Exam Time	**09: 00 AM**		وقت الامتحان
Exam Duration	3 hours	ثلاث ساعات	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 1 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 1 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف و الساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

Exercise 1 *You may use calculator (no need to calculate primitives). Assume the following data set:*

$$8 \quad 12 \quad 15 \quad 25 \quad 40$$

If the data is smoothed using a triangular kernel with a bandwidth of 6*, calculate the variance of the smoothed distribution.*

A) 37.6 *B)* 237.6 *C)* 137.6 *D)* 337.6 *E)* 437.6

Exercise 2 *Losses are distributed as a Pareto distribution with parameters* $\alpha = 2$ *and* θ*. If losses are subject to an ordinary deductible of* 20,000*, the expected cost per payment is* 30,000*. If losses are subject to a franchise deductible of* 20,000*, calculate the the expected cost per-loss.*

Exercise 3 *You are given the following ages at time of death of* 10 *individuals*

$$10 \quad 20 \quad 30 \quad 40 \quad 50 \quad 60 \quad 70 \quad 80 \quad 90 \quad 100$$

Using a triangular kernel with bandwidth 20*, find the kernel estimate of* $F(51)$*.*

A) 0.40 *B)* 0.42 *C)* 0.44 *D)* 0.46 *E)* 0.48

Exercise 4 *An insurance Company is completing a mortality study on a 3 year term insurance policy.*

Life	*Date of Entry*	*Date of Exit*	*Reason for Exit*
1	0	0.2	*Lapse (zawal)*
2	0	0.3	*Death*
3	0	0.4	*Lapse*
4	0	0.5	*Death*
5	0	0.5	*Death*
6	0	0.5	*Lapse*
7	0	1.0	*Death*
8	0	3.0	*Expiry of Policy*
9	0	3.0	*Expiry of Policy*
10	0	3.0	*Expiry of Policy*
11	0	3.0	*Expiry of Policy*
12	0	3.0	*Expiry of Policy*
13	0	3.0	*Expiry of Policy*
14	0	3.0	*Expiry of Policy*
15	0	3.0	*Expiry of Policy*
16	0.5	2.0	*Lapse*
17	0.5	1.0	*Death*
18	1.0	3.0	*Expiry of Policy*
19	1.0	3.0	*Expiry of Policy*
20	2.0	2.5	*Death*

Calculate $S_{20}(1.0)$ *using the Nelson-Aalen estimate where death is the decrement of interest.*

Loss - Final
Solution

(1

Ex 1 (5 marks)

$y-b$	y	$y+b$
2	8	14
6	12	18
9	15	21
19	25	31
34	40	46

$$f(x) = \frac{1}{5} \sum_{i=1}^{5} k_{y_i}(x)$$

where

$$k_{y_i}(x) = \begin{cases} \frac{1}{36}\left(x-(y_i-b)\right) & \text{for } x \text{ in } (y_i-b, y) \\ \frac{1}{36}\left(y_i+b-x\right) & \text{for } x \text{ in } (y, y_i+b) \end{cases}$$

$$E(X) = \frac{1}{5\times 36}\Big[\int_2^8 x(x-2)\,dx + \int_8^{14} x(14-x)\,dx$$

$$+ \int_6^{12} x(x-6)\,dx + \int_{12}^{18} x(18-x)\,dx$$

$$+ \int_9^{15} x(x-9)\,dx + \int_{15}^{21} x(21-x)\,dx$$

$$+ \int_{19}^{25} x(x-19)\,dx + \int_{25}^{31} x(31-x)\,dx$$

$$+ \int_{34}^{40} x(x-34)\,dx + \int_{40}^{46} x(46-x)\,dx$$

$$= \frac{3600}{5\times 36} = 20.$$

$$E(X^2) = \frac{1}{5\times 36}\Big[\int_2^8 x^2(x-2)\,dx + \int_8^{14} x^2(14-x)\,dx + \cdots\Big]$$

$$= \frac{96768}{5\times 36} = 537.6$$

$$V(X) = 537.6 - 400 = 137.6.$$

Or

$\therefore$ X is a 5-point mixture distribution

let Y the discrete uniform with support $\{8, 12, 15, 25, 40\}$

$X|_{Y=y_i}$ is the r.v with density $f_{y_i}(x)$.

let $Z_i = X|_{Y=y_i}$

We have $X = \sum_{i=1}^{5} \mathbb{1}_{Y=y_i} \times Z_i$ (1)

We show that

$$E(Z_i) = \frac{1}{b^2}\left[\int_{y_i-b}^{y_i} x(x-y_i+b)\,dx + \int_{y_i}^{y_i+b} x(y_i+b-x)\,dx\right.$$

$$= y_i$$

$$E(Z_i^2) = \frac{1}{b^2}\left[\int_{y_i-b}^{y_i} x^2(x-y_i+b)\,dx + \int_{y_i}^{y_i+b} x^2(y_i+b-x)\,dx\right]$$

$$= \frac{b^2}{6} + y_i^2.$$

$$(1) \Rightarrow E(X) = \sum_{i=1}^{5} \underbrace{P(Y=y_i)}_{1/5} \cdot E(Z_i) = \frac{\sum y_i}{5} = 20$$

$$E(X^2) = \sum_{i=1}^{5} P(Y=y_i)\, E(Z_i^2) = \frac{b^2}{6} + \frac{\sum y_i^2}{5}$$

$$= \frac{36}{6} + 531.6 = 537.6$$

$$\Rightarrow VX = \underbrace{\left[\frac{\sum y_i^2}{5} - \bar{y}^2\right]}_{V_e(X)} + \frac{b^2}{6} = 137.6$$

(3

Ex 2 (4 marks)

$X \sim \text{Pareto}(\alpha = 2, \theta)$; $d = 20{,}000$; $E(Y_1^P) = 30{,}000$

franchise $d = 20{,}000$ $E(Y_2^L)$?

$$E(Y_1^L) = E(X) - E(X \wedge d)$$
$$= \frac{\theta}{\alpha-1} - \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{d+\theta}\right)^{\alpha-1}\right]$$
$$= \frac{\theta}{\alpha-1}\left(\frac{\theta}{d+\theta}\right)^{\alpha-1}$$

$$E(Y_1^P) = \frac{E(Y_1^L)}{P(X>d)} = \frac{\frac{\theta}{\alpha-1}\left(\frac{\theta}{d+\theta}\right)^{\alpha-1}}{\left(\frac{\theta}{d+\theta}\right)^{\alpha}} = \frac{d+\theta}{\alpha-1}$$

$$= \frac{20{,}000+\theta}{2-1} = 30{,}000 \Rightarrow \theta = 10{,}000$$

franchise $E(Y_2^L) = \underbrace{E(X) - E(X \wedge d)}_{E(Y_1^L)} + d\,P(X>d)$

$$= 30{,}000 \times P(X>d) + d\,P(X>d)$$
$$= (30{,}000 + 20{,}000)\left(\frac{\theta}{d+\theta}\right)^{\alpha}$$
$$= 50{,}000\left(\frac{10{,}000}{30{,}000}\right)^{2} = 5{,}555.556$$

Ex 3 (5 marks)

$y_i - b$	y_i	$y_i + b$	$K_{y_i}(51)$
-10	10	30	1
0	20	40	1
10	30	50	1
20	40	60	
30	50	70	
40	60	80	
50	70	90	
60	80	100	0
70	90	110	0
80	100	120	0

$$F_{y_i}(51) = \begin{cases} \frac{1}{2b^2}\left(51-(y_i-b)\right)^2 & \text{for } 51 \text{ in } (y_i-b, y_i) \\ 1 - \frac{(y_i+b-51)^2}{2b^2} & \text{for } 51 \text{ in } (y_i, y_i+b) \end{cases}$$

$$\hat{F}(51) = \frac{1}{10}\Bigg[1+1+1+\left(1-\frac{(60-51)^2}{2b^2}\right) + \left(1-\frac{(70-51)^2}{2b^2}\right) + \frac{(51-40)^2}{2b^2} + \frac{(51-50)^2}{2b^2}\Bigg] = \frac{4.6}{10}$$

$$= 0.46.$$

Ex 4 (5 marks)

$u = 15$

$$E(X \wedge u) = \int_0^u S(x)\,dx = \int_0^u \frac{\theta - x}{\theta}\,dx$$

$$= \frac{1}{\theta}\left[\theta x - \frac{x^2}{2}\right]_0^u = \frac{1}{\theta}\left[\theta u - u^2/2\right]$$

A method of moment estimate of θ is $\hat{\theta}$ solution of

$$\frac{1}{\theta}\left(\theta u - u^2/2\right) = \bar{x} \quad (= 9.33)$$

$$(\Leftrightarrow) \quad \theta u - u^2/2 = \theta \bar{x}$$

$$\Rightarrow \quad \theta(u - \bar{x}) = u^2/2$$

$$\Rightarrow \quad \hat{\theta} = \frac{u^2}{2(u - \bar{x})} = \frac{15^2}{2(15 - 9.33)} = 1985$$

Ex 5 (1 + 2 marks)

$N \sim Bin(2, 0.1)$

$Y \sim$ Zero-truncated $Bin(5, 0.4)$

$$\begin{cases} S = \sum_{1}^{N} Y_i \\ S = 0 \text{ if } N = 0 \end{cases}$$

(a) $P(S_1 + S_2 = 0) = P(S_1 = 0) \times P(S_2 = 0)$

$$= P(S_1 = 0)^2$$

$P(S_1 = 0) = P(N_1 = 0) = 0.9^2 = 0.81$

$\Rightarrow P(S_1 + S_2 \neq 0) = 0.81^2 = 0.6561$

(b) $$P(S_1+S_2=1) = P(S_1=1, S_2=0) + P(S_1=0, S_2=1)$$

$$= 2 \times P(S_1=1)\, \underbrace{P(S_2=0)}_{0.81}$$

$$P(S_1=1) = \underbrace{P(S_1=1 \mid N=0)}_{=0} P(N=0)$$

$$+ P(S_1=1 \mid N=1)\, \underbrace{P(N=1)}_{=2\times 0.1\times 0.9} = 0.18$$

$$+ \underbrace{P(S_1=1 \mid N=2)}_{=0} P(N=2)$$

$$= P(Y=1)\, P(N=1) = P_1^T\,(0.18)$$

with $$P_1^T = \frac{m(1-q)^{m-1}q}{1-(1-q)^m} = 0.281 \quad (m=5, q=0.4)$$

$$\Rightarrow P(S_1+S_2=1) = 2 \times 0.281 \times 0.18 \times 0.81$$

$$= 0.0819.$$