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Model Answer of the Quiz 1 ACTU–462 (40%)

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For a fully discrete whole life insurance of 50000 on (30), you are given

$$\mu_{30+t} = \begin{cases} 0.01 & \text{for } t < 10 \\ 0.02 & \text{for } t \geq 10. \end{cases} \quad \text{and } i = 4\%.$$

Calculate ${}_{2.5}V$.
<u>**Solution**</u>:

By prospective approach we have ${}_{2}V = \text{APV(F.B.)}_2 - \text{APV(F.P.)}_2 = A_{32} - P_{30}\ddot{a}_{32}$ where

$$P_{30} = \frac{A_{30}}{\ddot{a}_{30}} = \frac{dA_{30}}{1-A_{30}} = \frac{1-d\ddot{a}_{30}}{\ddot{a}_{30}} = \frac{1}{\ddot{a}_{30}} - d.$$

Therefore

$${}_{2}V_{30} = A_{32} - \left(\frac{1}{\ddot{a}_{30}} - d\right)\ddot{a}_{32} = 1 - d\ddot{a}_{32} - \left(\frac{1}{\ddot{a}_{30}} - d\right)\ddot{a}_{32} = 1 - \frac{a_{*} * 32}{\ddot{a}_{30}}.$$

Now,

$$\begin{aligned} \ddot{a}_{30} &= \ddot{a}_{55:\overline{10}|} + {}_{10|}\ddot{a}_{30} = \ddot{a}_{30:\overline{10}|} + {}_{10}E_{55}\ddot{a}_{10} \\ &= \sum_{k=0}^{9} v^k \; {}_{k}p_{30} + v^{10} \; {}_{10}p_{30} \sum_{k=0}^{\infty} v^k \; {}_{k}p_{30} = \sum_{k=0}^{9} \frac{e^{-0.01k}}{(1.04)^k} + \frac{e^{-0.01\times 10}}{(1.04)^{10}} \sum_{k=0}^{\infty} \frac{e^{-0.02k}}{(1.04)^k} \\ &= \sum_{k=0}^{9} \left(\frac{e^{-0.01}}{1.04}\right)^k + \frac{e^{-0.01\times 10}}{(1.04)^{10}} \sum_{k=0}^{\infty} \left(\frac{e^{-0.02}}{1.04}\right)^k = 8.0935 + 10.631 = 18.725 \end{aligned}$$

and

$$\begin{aligned} \ddot{a}_{32} &= \ddot{a}_{32:\overline{8}|} + {}_{8|}\ddot{a}_{32} = \ddot{a}_{57:\overline{8}|} + {}_{8}E_{57}\ddot{a}_{40} \\ &= \sum_{k=0}^{7} v^k \; {}_{k}p_{32} + v^{8} \; {}_{8}p_{32} \sum_{k=0}^{\infty} v^k \; {}_{k}p_{40} = \sum_{k=0}^{7} \frac{e^{-0.01k}}{(1.04)^k} + \frac{e^{-0.01\times 8}}{(1.04)^{8}} \sum_{k=0}^{\infty} \frac{e^{-0.02k}}{(1.04)^k} \\ &= \sum_{k=0}^{7} \left(\frac{e^{-0.01}}{1.04}\right)^k + \frac{e^{-0.01\times 8}}{(1.04)^{8}} \sum_{k=0}^{\infty} \left(\frac{e^{-0.02}}{1.04}\right)^k = 6.776\,9 + 11.73 = 18.507 \end{aligned}$$

Finally

$${}_{2}V = 50000 \; {}_{2}V_{30} = 50000\left(1 - \frac{18.507}{18.725}\right) = \mathbf{582.11}.$$

One can also remark that $P = 50000\left(\frac{1}{18.725} - \frac{0.04}{1.04}\right) = \mathbf{747.15}$.
One can also calculate

$$\begin{aligned} A_{30} &= \frac{q}{q+i}\left(1 - v^{10} \; {}_{10}p_{30}\right) + v^{10} \; {}_{10}p_{30}A_{40} \\ &= \frac{1-e^{-0.01}}{1.04 - e^{-0.01}}\left(1 - \left(\frac{1}{1.04}\right)^{10} e^{-0.1}\right) + \left(\frac{1}{1.04}\right)^{10} e^{-0.1}\frac{1-e^{-0.02}}{1.04-e^{-0.02}} = 0.27984 \end{aligned}$$

so $P = 50000\frac{\frac{0.04}{1.04}0.27984}{1-0.27984} = \mathbf{747.27}$

By recursion formula we have

$$({}_2V + P_2)(1+i)^{0.5} = 50000v^{0.5}\ {}_{0.5}q_{32} + \ {}_{2.5}V\ {}_{0.5}p_{32},$$

then

$$\begin{aligned}
{}_{2.5}V &= \frac{({}_2V + P_2)(1+i)^{0.5} - v^{0.5}\ {}_{0.5}q_{32}}{{}_{0.5}p_{32}} = 50000\frac{({}_2V_{30} + \frac{1}{\ddot{a}_{30}} - d)(1+i)^{0.5} - v^{0.5}\left(1 - \ {}_{0.5}p_{32}\right)}{{}_{0.5}p_{32}} \\
&= \frac{50000({}_2V_{30} + \frac{1}{\ddot{a}_{30}} - d)(1+i)^{0.5} - v^{0.5}\left(1 - p_{32}^{0.5}\right)}{p_{32}^{0.5}} \\
&= 50000\frac{(0.011642 + \frac{1}{18.725} - \frac{0.04}{1.04})(1.04)^{0.5} - (1.04)^{-0.5}\left(1 - e^{-0.01\times 0.5}\right)}{e^{-0.01\times 0.5}} \\
&= 50000 \times 0.022332 = \mathbf{1116.6}.
\end{aligned}$$