King Saud University
College of Sciences
Mathematics Department

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Quiz 1 ACTU 362, October 13, 2019 from 4:45 to 6:15 PM

Problem

1. If $\ell_x = 100\left(k - \frac{1}{2}x\right)^{\frac{2}{3}}$ and $\mu_{50} = \frac{1}{48}$ what is the value of k?

2. You are given $\mu_x = \dfrac{2x}{(10,000 - x^2)}$ for $0 \leq x < 100$. Determine q_x.

3. Calculate $\mathring{e}_{20}$ given that
$$S_0(t) = \left(\frac{50}{50+t}\right)^3$$

4. Express the probabilities associated with the following events in actuarial notation.
   (a) A person age 10 now survives to age 35.
   (b) A new born infant dies no later than age 40.
   (c) A person age 40 now survives to age 50 but dies before attaining age 65.

Solution:

1. We have
$$S_0(t) = {}_tp_0 = \frac{\ell_x}{\ell_0} = \frac{100\left(k - \frac{1}{2}t\right)^{\frac{2}{3}}}{100k^{\frac{2}{3}}} = \frac{\left(k - \frac{1}{2}t\right)^{\frac{2}{3}}}{k^{\frac{2}{3}}}$$
and
$$-S_0'(t) = \frac{1}{2}\frac{2}{3}\frac{\left(k - \frac{1}{2}t\right)^{\frac{2}{3}-1}}{k^{\frac{2}{3}}} = \frac{1}{3}\frac{\left(k - \frac{1}{2}t\right)^{\frac{-1}{3}}}{k^{\frac{2}{3}}}.$$
Therefore
$$\mu_t = \frac{-S_0'(t)}{S_0(t)} = \frac{\frac{1}{3}\frac{\left(k-\frac{1}{2}t\right)^{\frac{-1}{3}}}{k^{\frac{2}{3}}}}{\frac{\left(k-\frac{1}{2}t\right)^{\frac{2}{3}}}{k^{\frac{2}{3}}}} = \frac{1}{3\left(k - \frac{1}{2}t\right)} = \frac{2}{6k - 3t}.$$
Hence
$$\mu_{50} = \frac{1}{48} = \frac{2}{6k - 3 \times 50} \iff 6k - 150 = 96 \iff k = 123.$$

2. We know
$$q_x = 1 - p_x \quad \text{and} \quad p_x = e^{\int_0^1 \frac{-2(x+t)}{(10000-(x+t)^2)}dt} = e^{\left[\ln(10000-(x+t)^2)\right]_0^1} = e^{\ln(10000-(x+1)^2) - \ln(10000-x^2)}$$
Thus
$$p_x = \frac{10000 - (x+1)^2}{10000 - x^2}$$
hence
$$q_x = 1 - \frac{10000 - (x+1)^2}{10000 - x^2} = \frac{2x+1}{10000 - x^2}.$$

3. By definiton we have

$$\overset{\circ}{e}_{20} = \int_0^\infty {}_tp_{20}dt = \int_0^\infty S_{20}(t)dt = \int_0^\infty \frac{S_0(20+t)}{S_0(20)}dt = \int_0^\infty \frac{S_0(20+t)}{S_0(20)}dt$$

and

$$\frac{S_0(20+t)}{S_0(20)} = \frac{\left(\frac{50}{70+t}\right)^3}{\left(\frac{50}{70}\right)^3} = \frac{70^3}{(t+70)^3}$$

therefore

$$\begin{aligned}\overset{\circ}{e}_{20} &= \int_0^\infty S_0(t) = \int_0^\infty \frac{70^3}{(t+70)^3}dt = 70^3 \int_0^\infty \frac{dt}{(t+70)^3} \\ &= 70^3 \left[\frac{1}{2}\frac{-1}{(t+70)^2}\right]_0^\infty = \frac{1}{2}\frac{70^3}{70^2} = 35.\end{aligned}$$

4. We have

(a) The probability that a person age $x = 10$ now survives to age $35 = 10+25$ can be expressed as

$$S_{10}(25) = \mathrm{P}(T_{10} \geq 25) = {}_{25}p_{10}.$$

(b) The probability that a new born infant ($x = 0$) dies no later than age $t = 40$ can be expressed as

$$\mathrm{P}(T_0 \leq 40) = {}_{40}q_0.$$

(c) The probability that a person age 40 now survives to age 50 but dies before attaining age 65 can be expressed as ($x = 40$, $t = 50 - 40 = 10$, and $u = 65 - 50 = 15$)

$$\mathrm{P}(10 \leq T_{40} \leq 10+15) = {}_{10|15}q_{40}.$$