King Saud University
College of Sciences
Mathematics Department

Academic Year (G) 2018–2019
Academic Year (H) 1439–1440
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Quiz 1 ACTU 362-372, March 31, 20209 from 4:45 to 6:15 PM

Exercise 1 *For a special* 20*–year temporary life–annuity payable monthly on* (50) *we assume the following: 1.* 100 *is payable at the beginning of each month from age* 50 *for* 10 *years 2.* 400 *is payable at the beginning of each month from age* 60 *for* 10 *years Use ILT for* $i = 0.06$ *to calculate, under UDD, the APV of theses annuity payments.*

<u>**Solution**</u>:
1. Direct method Denote by a the APV of this annuity

$$\begin{aligned} a &= 100 \times 12\ddot{a}^{(12)}_{50:\overline{10}|} + 400 \times \; 12 \;{}_{10|}\ddot{a}^{(12)}_{50:\overline{10}|} \\ &= 100 \times 12 \left(\ddot{a}^{(12)}_{50:\overline{10}|} + 4 \;{}_{10}E_{50}\ddot{a}^{(12)}_{60:\overline{10}|}\right) \\ &= 100 \times 12 \left(\alpha(12)\ddot{a}_{50:\overline{10}|} - \beta(12)(1 - {}_{10}E_{50}) + 4 \;{}_{10}E_{50} \left(\alpha(12)\ddot{a}_{60:\overline{10}|} - \beta(12)(1 - {}_{10}E_{60})\right)\right). \end{aligned}$$

Moreover

$$\alpha(12) = \frac{id}{i^{(12)}d^{(12)}} = \frac{id}{12((1+i)^{1/12} - 1)(12(1 - (1-d)^{1/12}))}$$

and

$$\beta(12) = \frac{i - i^{(12)}}{i^{(12)}d^{(12)}} = \frac{i - 12((1+i)^{1/12} - 1)}{12((1+i)^{1/12} - 1)(12(1 - (1-d)^{1/12}))}.$$

Now, we pick numbers from ILT and get $a =$
2. Decomposition method: $a =$ can be written as follows $400 \times 12\ddot{a}^{(12)}_{50:\overline{20}|} - 300 \times 12\ddot{a}^{(12)}_{50:\overline{10}|}$. Now, under UDD we have

$$\ddot{a}^{(12)}_{50:\overline{20}|} = \alpha(12)\ddot{a}_{50:\overline{20}|} - \beta(12)(1 - {}_{20}E_{50}) = \alpha(12)(\ddot{a}_{50} - {}_{20}E_{50})(\ddot{a}_{70}) - \beta(12)(1 - {}_{20}E_{50})$$

and

$$\ddot{a}^{(12)}_{50:\overline{10}|} = \alpha(12)\ddot{a}_{50:\overline{10}|} - \beta(12)(1 - {}_{10}E_{50}) = \alpha(12)(\ddot{a}_{50} - {}_{10}E_{50})(\ddot{a}_{60}) - \beta(12)(1 - {}_{10}E_{50})$$

From ILT we obtain $a =$