

King Saud University  
College of Sciences  
Mathematics Department

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Quiz 1 ACTU 362-372, February 20, 2020 from 4:45 to 6:15 PM

**Exercise 1** You are given the following select-and-ultimate life table with a **two** year select period.

$x$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{x+2}$
90			
91	1250		920
92	1000	900	

$$\text{and } q_{[x]+t} = \frac{t+1}{3} q_{x+t}$$

Calculate  $\ell_{[90]+1}$

**Solution:** By definition

$$\ell_{[90]+1} = \frac{\ell_{[90]+2}}{1 - q_{[90]+1}} = \frac{\ell_{92}}{1 - q_{[90]+1}} = \frac{\ell_{92}}{1 - \frac{2}{3}q_{91}}.$$

So we need to find  $q_{91}$ . For  $t = 0$ ,  $q_{[91]} = \frac{1}{3}q_{91}$  and

$$\begin{aligned} q_{[91]} &= 1 - \frac{\ell_{[91]+1}}{\ell_{[91]}} = 1 - \frac{1}{\ell_{[91]}} \frac{\ell_{[91]+2}}{1 - q_{[91]+1}} \\ &= 1 - \frac{1}{1250} \frac{920}{1 - \frac{2}{3}q_{92}}. \end{aligned}$$

Moreover  $q_{92} = 3q_{[92]} = 3 \left(1 - \frac{\ell_{[92]+1}}{\ell_{[92]}}\right) = 3 \left(1 - \frac{900}{1000}\right) = 0.3$ . Thus

$$q_{91} = 3q_{[91]} = 3 \left(1 - \frac{1}{1250} \frac{920}{1 - \frac{0.6}{3}}\right) = 0.24.$$

Finally

$$\ell_{[90]+1} = \frac{\ell_{[90]+2}}{1 - q_{[90]+1}} = \frac{\ell_{92}}{1 - q_{[90]+1}} = \frac{1}{1 - \frac{2}{3}q_{91}} \frac{\ell_{92}}{1 - q_{92}} = \frac{1}{1 - \frac{2}{3} \times 0.24} \times \frac{920}{1 - 0.3} \simeq 1565$$