**King Saud University**
**College of Sciences**
**Mathematics Department**

**Academic Year (G) 2020–2021**
**Academic Year (H) 1442**
**Bachelor AFM: M. Eddahbi**

# Solution of Quizzes ACTU-362-372 (10%)

**April 29, 2021**

1. Given $\mu_{40.5} = 1.35$ calculate $\mu_{40.25}$ and $\mu_{40.75}$ assuming UDD between integral ages.

2. You are given $\int_0^n {}_sp_{40}\, ds = 30.352$ and $\mu_{40+t} = \frac{0.5}{50-t}$ for all $t < 50$. Find $n$.

3. Assuming UDD between integral ages you are given: $x$ is an integer and $0 < s < 1$ such that ${}_{0.25}p_{x+0.3} = 0.8$ and ${}_sp_{x+0.5} = 0.8$. Find $s$.

4. A life, age 65, is subject to mortality as described in the following excerpt from a 3–year select–and–ultimate table:

| $x$ | $\ell_{[x]}$ | $\ell_{[x]+1}$ | $\ell_{[x]+2}$ | $\ell_{x+3}$ |
|---|---|---|---|---|
| 65 | 5,000 | 4,750 | 4,500 | 4,200 |
| 66 | 4,800 | 4,550 | 4,250 | 3,800 |

Complete the following table

| $x$ | $q_{[x]}$ | $q_{[x]+1}$ | $q_{[x]+2}$ |
|---|---|---|---|
| 65 | | | |
| 66 | | | |

5. Calculate $p_{70}$ given $1000\ A_{70} = 516$, $1000\ A_{71} = 530$ and $v = 0.95$

6. Calculate $10^5 A^{1\,(2)}_{40:\overline{2}|}$ using the following information: $i = 0.04$, $p_{40} = 0.8$ and $p_{41} = 0.75$ and assuming constant force of mortality between integral ages.

7. A life annuity of 1 on (30), is payable at the beginning of each year until age 60. The annuity payments are certain for the first 10 years. Calculate the actuarial present value of this annuity using ILT with $i = 6\%$.

8. An actuary uses Woolhouse's formula with three terms to approximates values of $\ddot{a}^{(2)}_{60} = 10.25$ and $\ddot{a}^{(4)}_{60} = 10.05$. Use the same formula, same mortality and interest rate assumptions as the actuary to calculate $\ddot{a}^{(12)}_{60}$.

9. For a special fully discrete whole life insurance on (45): (i) The death benefit is 2000 if death occurs before age 65, otherwise 1000. (ii) Mortality follows the Illustrative Life Table and $i = 0.06$. (iii) Expenses are 80% of first year premium and 10% of renewal premium. (iv) Gross premiums, payable annually until death, are determined using the equivalence principle.

   (a) Calculate the gross premium.

   (b) Calculate the gross premium reserve at time $t = 20$.

10. For a whole life policy on (40) with benefits payable at the moment of death: (i) The face amount is 1000. (ii) Premiums are payable annually for 20 years. (iii) First year expenses are 60% of first year premium plus 100, paid at issue. (iv) Renewal expenses are 5% of premium plus 5, payable at the beginning of every year. (v) $\bar{A}_{45} = 0.305$ (vi) $\ddot{a}_{45} = 14$ (vii) $\ddot{a}_{45:\overline{15}|} = 10.2$ (viii) $d = 0.05$ (ix) The gross premium reserve at time 5 is 200. Determine the gross premium.

11. For a fully discrete 20–year endowment insurance of 10,000 on (45) with a net premium equals to 297.89: (i) Mortality follows the Illustrative Life Table. (ii) $i = 0.06$ (iii) Percent of premium expenses are 30% in the first year, 5% in years 2–10, and 2% in years 11–20. (iv) Per policy expenses are 100 in the first year and 10 in renewal years. Calculate the expense premium.

12. For a fully discrete 20–year deferred whole life insurance of 1000 on (50), such that Premiums are payable for 20 years and Deaths are Uniformly Distributed between integral ages. Given $i = 0.045$, $q_{59} = 0.5$ and ${}_9V = 60$, ${}_{9.5}V = 250$.
Calculate the level net premium for this policy.

13. You are given: (i) $i = 0.04$ (ii) $1000\ q_{46} = 4.24$ (iii) $A_{47:\overline{18}|} = 0.382$. Calculate the FPT reserve at the end of year 2 for a fully discrete 20–year endowment insurance of 10,000 on (45).

14. You are given: (i) $A_{55} = 0.305$ (ii) $A_{65} = 0.440$ (iii) $1000\ q_{55} = 6.5$ (iv) $i = 0.05$. Consider a fully discrete whole life policy of 100,000 on (55) Calculate the **difference** $10^5\left({}_{10}V_{55} - {}_{10}V_{55}^{\text{FPT}}\right)$.

15. For a fully discrete whole life policy of 1000 issued to (65): Mortality follows the Illustrative Life Table and $i = 0.06$.

    (a) Calculate the **first year modified premium**, **renewal modified premium** under the **full preliminary term method**

    (b) Calculate **the reserve at the end of year** 5.

16. For a fully continuous 20–year deferred whole life insurance of 10,000 on (45), you are given: (i) $\bar{A}_{65} = 0.25821$ (ii) The annual net premium is 71.25, and is payable for the first 20 years. (iii) $\mu_x = 0.00015(1.06)^x$ and $\delta = 0.05$. Use Euler's method

$$\frac{{}_{t+h}V^g - {}_tV^g}{h} = P + \delta\ {}_tV - (b_t - {}_tV)\,\mu_{x+t}$$

    with step 0.5 to calculate ${}_{19}V$.

17. $G$ is the gross annual premium for a fully discrete whole life insurance. You are given: (i) No deaths or withdrawals are expected during the first two policy years, (ii) $i = 5\%$, (iii) Expenses are incurred at the beginning of each policy year, (iv) Percent of premium expenses are 7% of $G$ each year. (v) Per policy expenses are 10 for year 1 and 2 for year 2. (vi) the level gross annual premium $G$ equals 100. Calculate ${}_2\text{AS}$.

**Solution**:

1. Under UDD, we have $\mu_{x+r} = \frac{q_x}{1-rq_x}$ for all $0 < r < 1$, so $\mu_{40.5} = 1.35 = \frac{q_{40}}{1-0.5q_{40}}$ hence $q_{40} = 0.80597$, Thus

$$\mu_{40.25} = \frac{0.80597}{1 - 0.25 \times 0.80597} = \mathbf{1.0093} \quad \text{and} \quad \mu_{40.75} = \frac{0.80597}{1 - 0.75 \times 0.80597} = \mathbf{2.0377}$$

2. We have

$$ {}_sp_{40} = e^{-\int_0^s \frac{0.5}{50-u}du} = e^{0.5\ln\left(\frac{50-s}{50}\right)} = \sqrt{1-\frac{s}{50}}, $$

thus

$$ \int_0^n \sqrt{1-\frac{s}{50}}ds = \frac{100}{3}\left(1-\left(1-\frac{n}{50}\right)^{\frac{3}{2}}\right) = 30.352 $$

which gives $n = \mathbf{40}$.

3. Under UDD, we know

$$ {}_{0.25}q_{x+0.3} = \frac{0.25q_x}{1-0.3q_x} = 0.2 $$

which gives $q_x = 0.64516$ and

$$ {}_sp_{x+0.5} = \frac{1}{5} = \frac{sq_x}{1-0.5q_x} = s\frac{0.64516}{1-0.5\times 0.64516} = 0.2, $$

hence $s = \mathbf{0.21}$.

4. We shall use $q_{[x]+k} = 1 - \frac{\ell_{[x]+k+1}}{\ell_{[x]+k}}$ for $k = 0,\ 1,\ 2.$

| $x$ | $q_{[x]}$ | $q_{[x]+1}$ | $q_{[x]+2}$ |
|---|---|---|---|
| 65 | $1-\frac{475}{500} = 1-\frac{19}{20} = \mathbf{0.05}$ | $1-\frac{450}{475} = \frac{1}{19} = \mathbf{0.05263}$ | $1-\frac{42}{45} = \frac{1}{15} = \mathbf{0.06667}$ |
| 66 | $1-\frac{455}{480} = \frac{5}{96} = \mathbf{0.05208}$ | $1-\frac{425}{455} = \frac{6}{91} = \mathbf{0.06593}$ | $1-\frac{380}{425} = \frac{9}{85} = \mathbf{0.10588}$ |

5. From whole life insurance recursion we have

$$ A_{70} = v\ q_{70} + v\ p_{70}\ A_{71} = v\,(1-\ p_{70}) + v\ p_{70}\ \ A_{71} = v - p_{70}\ v\,(1-A_{71}) $$

thus

$$ p_{70} = \frac{v - A_{70}}{v\,(1-A_{71})} = \frac{0.95-0.516}{0.95\,(1-0.530)} = \mathbf{0.972} $$

6. The actuarial present value of future benefits is given by

$$ A^{1\,(2)}_{45:\overline{2}|} = v^{\frac{0}{2}+\frac{1}{2}}\ {}_{\frac{0}{2}|\frac{1}{2}}q_{40} + v^{\frac{1}{2}+\frac{1}{2}}\ {}_{\frac{1}{2}|\frac{1}{2}}q_{40} + v^{\frac{2}{2}+\frac{1}{2}}\ {}_{\frac{2}{2}|\frac{1}{2}}q_{40} + v^{\frac{3}{2}+\frac{1}{2}}\ {}_{\frac{3}{2}|\frac{1}{2}}q_{40} $$

$$ \begin{aligned} {}_{\frac{0}{2}|\frac{1}{2}}q_{40} &= {}_{0.5}q_{45} = 1 - {}_{0.5}p_{40} = 1 - p_{40}^{0.5} = 1 - 0.8^{0.5} = 0.10557, \\ {}_{0.5|0.5}q_{40} &= {}_{0.5}p_{40} - p_{40} = p_{40}^{0.5} - p_{40} = 0.8^{0.5} - 0.8 = 0.094427, \\ {}_{1|0.5}q_{40} &= p_{40}\,(1 - {}_{0.5}p_{41}) = p_{40}\left(1-p_{41}^{0.5}\right) = 0.8\left(1-0.75^{0.5}\right) = 0.10718, \\ {}_{\frac{3}{2}|\frac{1}{2}}q_{40} &= {}_{1.5}p_{40} - {}_2p_{40} = p_{40}\ {}_{0.5}p_{41} - p_{40}p_{41} \\ &= p_{40}\left(p_{41}^{0.5} - p_{41}\right) = 0.8\left(0.75^{0.5} - 0.75\right) = 0.09282 \end{aligned} $$

hence

$$ 10^5 A^{1\,(2)}_{45:\overline{2}|} = 10^5\left(\frac{0.10557}{1.04^{0.5}} + \frac{0.094427}{1.04} + \frac{0.10718}{1.04^{1.5}} + \frac{0.09282}{1.04^2}\right) = \mathbf{38119}. $$

7. This annuity is the sum of a 10–year annuity–certain and a 10–year deferred 20–year temporary life annuity on (30). So APV(of the Annuity) is

$$
\begin{aligned}
\ddot{a}_{\overline{10}|} + \ddot{a}_{[10|]30:\overline{20}|} &= \ddot{a}_{\overline{10}|} + {}_{10}E_{30}\ \ddot{a}_{40:\overline{20}|} \\
&= \ddot{a}_{\overline{10}|} + {}_{10}E_{30}\left(\ddot{a}_{40} - {}_{20}E_{40}\ \ddot{a}_{60}\right) \\
&= \frac{1-v^{10}}{d} + {}_{10}E_{30}\left(\ddot{a}_{40} - {}_{20}E_{40}\ \ddot{a}_{60}\right) \\
&= \frac{1-(0.9434)^{10}}{1-0.9434} + 0.54733\left(14.8166 - 0.27414 \times 11.1454\right) = \mathbf{14.239}.
\end{aligned}
$$

8. Remember that the Woolhouse's formula with three terms for a $m$–thly whole life annuity is

$$
\ddot{a}_x^{(m)} \simeq \ddot{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2}\left(\mu_x + \delta\right).
$$

By assumption we have

$$
\begin{aligned}
\ddot{a}_{60}^{(2)} &= \ddot{a}_{60} - \frac{2-1}{4} - \frac{2^2-1}{12 \times 2^2}\left(\mu_{60} + \delta\right) = \ddot{a}_{60} - 0.250 - 0.0625\left(\mu_{60} + \delta\right) = 10.25 \\
\ddot{a}_{60}^{(4)} &= \ddot{a}_{60} - \frac{4-1}{8} - \frac{4^2-1}{12 \times 4^2}\left(\mu_{60} + \delta\right) = \ddot{a}_{60} - 0.375 - 0.0781\left(\mu_{60} + \delta\right) = 10.05
\end{aligned}
$$

which leads to $\ddot{a}_{60} = 10.8005$ and $\mu_{60} + \delta = 4.8077$, therefore

$$
\begin{aligned}
\ddot{a}_{60}^{(12)} &= \ddot{a}_{60} - \frac{12-1}{24} - \frac{12^2-1}{12^3}\left(\mu_{60} + \delta\right) \\
&= 10.8005 - \frac{12-1}{24} - \frac{12^2-1}{12^3}(4.8077) = \mathbf{9.9443}
\end{aligned}
$$

9.

(a) We have

$$
\mathbf{APV(FB)}_0 = 1000\ A_{45} + 1000\bar{A}^{\,1}_{45:\overline{20}|} = 1000\ A_{45} + 1000\left(A_{45} - {}_{20}E_{45}A_{65}\right)
$$

and

$$
\mathbf{APV(FE)}_0 = 0.8G + 0.1G\left(\ddot{a}_{45} - 1\right).
$$

The $\mathbf{APV(FP)}_0 = G\ddot{a}_{45}$ hence by E.P. we get

$$
0.9G\left(\ddot{a}_{45} - 1\right) = 1000\ A_{45} + 1000\left(A_{45} - {}_{20}E_{45}A_{65}\right) = 2000\ A_{45} - 1000_{20}E_{45}A_{65}.
$$

we obtain then

$$
G = \frac{2 \times 201.20 - 0.25634(439.80)}{0.9\left(14.1121 - 1\right)} = \frac{289.66167}{11.80089} = \mathbf{24.545748}.
$$

(b) Now, we calculate the APV of benefits and expenses.

$$
1000\ A_{65} + 0.1G\ddot{a}_{65} = 439.80 + 0.1(43.652)(9.8969) = 483.
$$

The gross premium reserve at time 20 is

$$
\begin{aligned}
{}_{20}V^g &= 1000\ A_{65} + 0.1G\ddot{a}_{65} - G\ddot{a}_{65} = 1000\ A_{65} - 0.9G\ddot{a}_{65} \\
&= 439.80 - 0.9(24.545748)(9.8969) = \mathbf{221.16587}.
\end{aligned}
$$

10. Note that renewal expenses are payable even past the premium payment period, although the percent of premium expenses are 0 after the premium payment period. The gross premium reserve in terms of the gross premium is

$$\begin{aligned} {}_5V^g &= 1000\bar{A}_{45} + 5\ddot{a}_{45} - 0.95G\ddot{a}_{45:\overline{15}|} \\ 200 &= 305 + 5(14) - 0.95(10.2)G = 375.0 - 9.69G \\ G &= \frac{375-200}{9.69} = \mathbf{18.06} \end{aligned}$$

11. The gross premium is determined by E.P.

$$5\times 10^4 A_{45:\overline{20}|} + 100 + 0.3G + 0.05Ga_{45:\overline{9}|} + 0.02G\ddot{a}_{[}10|]45{:}\overline{10}| + 10a_{45:\overline{19}|} = G\ddot{a}_{45:\overline{20}|}$$

which can written also as

$$\begin{aligned} G\ddot{a}_{45:\overline{20}|} &= 5\times 10^4 A_{45:\overline{20}|} + 100 + 0.3G + (0.02G+10)\,a_{45:\overline{19}|} + 0.03Ga_{45:\overline{9}|} \\ &= 5\times 10^4 A_{45:\overline{20}|} + 90 + 0.25G + (0.02G+10)\,\ddot{a}_{45:\overline{20}|} + 0.03G\ddot{a}_{45:\overline{10}|} \end{aligned}$$

Now, calculate the needed insurances and annuities.

$$\begin{aligned} A_{45:\overline{20}|} &= 1 - d\ddot{a}_{45:\overline{20}|} = 1 - d\left(\ddot{a}_{45} - {}_{20}E_{45}\ddot{a}_{65}\right) \\ &= 1 - \frac{0.06}{1.06}(14.1121 - 0.25634\times 9.8969) = 1 - \frac{0.06}{1.06}(11.5751) = 0.34481 \\ \ddot{a}_{45:\overline{10}|} &= \left(\ddot{a}_{45} - {}_{10}E_{45}\ddot{a}_{55}\right) = 14.1121 - (0.52652)(12.2758) = 7.6486. \end{aligned}$$

Consequently

$$\begin{aligned} G &= \frac{10^4 A_{45:\overline{20}|} + 90 + 10\ddot{a}_{45:\overline{20}|}}{0.98\ddot{a}_{45:\overline{20}|} - 0.03\ddot{a}_{45:\overline{10}|} - 0.25} \\ &= \frac{10^4(0.34481) + 10(11.5751) + 90}{0.98(11.5751) - 0.03(7.6486) - 0.25} = \frac{3653.9}{10.864} = 336.33. \end{aligned}$$

The expense premium $P^e$ is $336.33 - 297.89 = \mathbf{38.44}$.

12. From recursion formula we have

$$\begin{aligned} ({}_9V + P)(1+i)^{0.5} &= v^{1-s}\ {}_{0.5}q_{59}\times 0 + {}_{9.5}V\ {}_{0.5}p_{59} \\ &= (1 - {}_{0.5}q_{59})\ {}_{9.5}V = \left(1 - \frac{1}{2}\times\frac{1}{2}\right)250 = 187.5 \end{aligned}$$

then

$$P = \frac{187.5}{\sqrt{1.045}} - 60 = \mathbf{123.418}.$$

13. The FPT reserve at time 2 is the time 1 level net premium reserve for a 19–year endowment insurance on (46),

$${}_2V^{\text{FPT}}_{45:\overline{20}|} = {}_1V_{46:\overline{19}|} = 1 - \frac{\ddot{a}_{47:\overline{18}|}}{\ddot{a}_{46:\overline{19}|}}$$

so we need $\ddot{a}_{47:\overline{18}|}$ and $\ddot{a}_{46:\overline{19}|}$. But $\ddot{a}_{47:\overline{18}|} = \frac{1-A_{47:\overline{18}|}}{d} = \frac{1-0.382}{0.04}(1.04) = 16.068$ and $\ddot{a}_{46:\overline{19}|}$ can be backed by recursion on annuities.

$$\ddot{a}_{46:\overline{19}|} = 1 + v\ p_{46}\ \ddot{a}_{47:\overline{18}|} = 1 + \frac{1-0.00424}{1.04}(16.068) = 16.384.$$

Then FPT reserve at time 2 is

$$10^4 \ {}_2V_{20}^{\text{FPT}} = 10000\left(1 - \frac{16.068}{16.384}\right) = \mathbf{192.87}.$$

14. The level net premium reserve for the benefit 1 is $A_{65}$

$${}_{10}V_{55} = A_{65} - P_{55}\ddot{a}_{65} = \frac{A_{65} - A_{55}}{1 - A_{55}} = \frac{0.440 - 0.305}{1 - 0.305} = 0.19424.$$

By the relationship between the FPT reserve and the net premium reserve, we have

$${}_{10}V_{55}^{\text{FPT}} = \ {}_9V_{56} = \frac{A_{65} - A_{56}}{1 - A_{56}}.$$

So, we need $A_{56,}$ which can be backed by insurance recursion $A_{55} = v\ q_{55} + v\ p_{55}\ A_{56}$. Hence

$$A_{56} = \frac{1.05A_{55} - 0.0065}{1 - 0.0065} = 0.315803,$$

therefore ${}_{10}V_{55}^{\text{FPT}} = \frac{0.440 - 0.315803}{1 - 0.315803} = 0.181\,52$. The difference is

$$10^5\left({}_{10}V_{55} - \ {}_{10}V_{55}^{\text{FPT}}\right) = 100000\,(0.19424 - 0.181\,52) = \mathbf{1272}.$$

15.

(a) The modified premium in the first year is $1000\alpha = 1000vq_{65} = \frac{21.32}{1.06} = \mathbf{20.11321}$. The modified premium in renewal years is the net premium for (66), or

$$1000\beta = 1000\ P_{66} = 1000\frac{A_{66}}{\ddot{a}_{66}} = \frac{454.56}{9.6362} = \mathbf{47.17212}$$

(b) The reserve at the end of 5 years is the net premium reserve at time 4 for a whole life issued on (66), which is given by

$${}_5V^{\text{FPT}} = 1000\ {}_4V_{66} = 1000\left(1 - \frac{\ddot{a}_{70}}{\ddot{a}_{66}}\right) = 1000\left(1 - \frac{8.5693}{9.6362}\right) = \mathbf{110.72}$$

16. There is no death benefit in year 19, so the benefit $b_t = 0$ in that period. Let us calculate the two $\mu_x$'s that we need.

$$\mu_{64.5} = 0.00015(1.06)^{64.5} = 0.00643161 \quad \text{and} \quad \mu_{64} = 0.00015(1.06)^{64} = 0.00624693$$

The net premium reserve at time 20, since the policy is paid up then, is $10000\bar{A}_{65} = 2582.10$. We shall apply the discritization

$${}_tV \simeq \frac{{}_{t+h}V - h\left(P - b\ \mu_{x+t}\right)}{1 + h\left(\delta + \mu_{x+t}\right)}.$$

Since ${}_{20}V = 2582.10$

$$\begin{aligned}
{}_{19.5}V &= \frac{2582.10 - 0.5(71.25)}{1 + 0.5(0.05 + 0.00643161)} = \mathbf{2476.60}.\\
{}_{19}V &= \frac{2476.60 - 0.5(71.25)}{1 + 0.5(0.05 + 0.00624693)} = \mathbf{2374.20}.
\end{aligned}$$

17. Since there are no death or withdrawal during the first two policy years,

$$({}_0\text{AS} + G - 0.07G - 10) \times 1.05 = 0q_{40} + {}_1\text{AS}\ \ p_{40} = {}_1\text{AS}$$

thus

$${}_1\text{AS} = (0.93G - 10) \times 1.05 = (93 - 10) \times 1.05 = 87.15$$

Since $b^{(d)} = b^{(w)} = 0$, we have

$${}_2\text{AS} = ({}_1\text{AS} + 0.93G - 2) \times 1.05 = (87.15 + 93 - 2) \times 1.05 = \mathbf{187.06}.$$