King Saud University
College of Sciences
Mathematics Department

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Projects for ACTU 362 +ACTU 372, project period five days

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Project Group 1

Abdullah aged 22 purchases a 30–year deferred whole life annuity due of 1000 per year as soon as the Abdullah is alive. Assume that Annual premiums of P and $2P$, determined using equivalence principle, are paid at the beginning of each year during the first 15 years and the second 15 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the first 15 years
2. Calculate the net premium reserve at ages 40, 50, 60 using prospective approach
3. Calculate the net premium reserve at ages 40, 50, 60 using retrospective approach

$$\begin{aligned}\mathbf{APV}(\mathbf{FA})_0 &= a\ddot{a}_{22:\overline{15}|} + b\ {}_{15|}\ddot{a}_{22:\overline{15}|}\\ &= a\left(\ddot{a}_{22} - {}_5E_{22}\ {}_{10}E_{27}\ \ddot{a}_{37}\right) + b\ {}_5E_{22}\ {}_{10}E_{27}\left(\ddot{a}_{37} - {}_5E_{37}\ {}_{10}E_{42}\ddot{a}_{52}\right)\end{aligned}$$

$$\mathbf{APV}(\mathbf{FB})_0 = 1000\ {}_{10}E_{22}\ {}_{20}E_{32}\ \ddot{a}_{52} \quad \text{and} \quad P = \frac{1000\ {}_{10}E_{22}\ {}_{20}E_{32}\ \ddot{a}_{52}}{\mathbf{APV}(\mathbf{FA})_0}$$

Prospective: $${}_{18}V = 1000\ \ddot{a}_{40},\quad {}_{40}V = 1000\ \ddot{a}_{50},\quad {}_{40}V = 1000\ \ddot{a}_{60}$$

$${}_{40}V = \frac{P\mathbf{APV}(\mathbf{FA})_0 - 1000\ {}_{30}E_{22}\left(\ddot{a}_{52} - {}_{10}E_{52}\ \ddot{a}_{62}\right)}{{}_{20}E_{22}\ {}_{20}E_{42}}$$

Retrospective: $${}_{50}V = \frac{P\mathbf{APV}(\mathbf{FA})_0 - 1000\ {}_{30}E_{22}\left(\ddot{a}_{52} - {}_{20}E_{52}\ \ddot{a}_{72}\right)}{{}_{20}E_{22}\ {}_{20}E_{42}\ {}_{10}E_{62}}$$

$${}_{60}V = \frac{P\mathbf{APV}(\mathbf{FA})_0 - 1000\ {}_{30}E_{22}\left(\ddot{a}_{52} - {}_{30}E_{52}\ \ddot{a}_{82}\right)}{{}_{20}E_{22}\ {}_{20}E_{42}\ {}_{20}E_{62}}$$

Project group 2

Abdullah aged 22 purchases a 30–year deferred whole life annuity due of 2000 per year as soon as the Abdullah is alive. Assume that Annual premiums of P and $3P$, determined using equivalence principle, are paid at the beginning of each year during the first 15 years and the second 15 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the second 15 years
2. Calculate the net premium reserve at ages 40, 50, 60 using prospective approach
3. Calculate the net premium reserve at ages 40, 50, 60 using retrospective approach

Project Group 3

Abdullah aged 22 purchases a 30–year deferred whole life annuity due of 3000 per year as soon as the Abdullah is alive. Assume that Annual premiums of P and $4P$, determined using equivalence principle, are paid at the beginning of each year during the first 15 years and the second 15 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the first 15 years
2. Calculate the net premium reserve at ages 40, 50, 60 using prospective approach
3. Calculate the net premium reserve at ages 40, 50, 60 using retrospective approach

Project Group 4

Abdullah aged 22 purchases a 30–year deferred whole life annuity due of 4000 per year as soon as the Abdullah is alive. Assume that Annual premiums of P and $5P$, determined using equivalence principle, are paid at the beginning of each year during the first 15 years and the second 15 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the second 15 years
2. Calculate the net premium reserve at ages 40, 50, 60 using prospective approach
3. Calculate the net premium reserve at ages 40, 50, 60 using retrospective approach

Project group 5

Abdullah aged 22 purchases a 40–year deferred whole life annuity due of 5000 per year as soon as the Abdullah is alive. Assume that Annual premiums of $2P$ and P, determined using equivalence principle, are paid at the beginning of each year during the first 20 years and the second 20 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the first 20 years
2. Calculate the net premium reserve at ages 50, 60, 70 using prospective approach
3. Calculate the net premium reserve at ages 50, 60, 70 using retrospective approach

Solution:

$$\begin{aligned} \mathbf{APV(FA)}_0 &= a\ddot{a}_{22:\overline{20|}} + b\ {}_{20|}\ddot{a}_{22:\overline{20|}} \\ &= a\left(\ddot{a}_{22} - {}_{20}E_{22}\ \ddot{a}_{42}\right) + b\ \ {}_{20}E_{22}\ \left(\ddot{a}_{42} - {}_{20}E_{42}\ddot{a}_{62}\right) \end{aligned}$$

$$\mathbf{APV(FB)}_0 = 1000\ {}_{40}E_{22}\ \ddot{a}_{62} \quad \text{and} \quad P = \frac{1000\ \ {}_{20}E_{22}\ {}_{20}E_{42}\ \ddot{a}_{62}}{\mathbf{APV(FA)}_0}$$

Prospective: $\quad {}_{50}V = 1000\ \ddot{a}_{72},\quad {}_{60}V = 1000\ \ddot{a}_{82},\quad {}_{70}V = 1000\ \ddot{a}_{92}$

$${}_{50}V = \frac{P\mathbf{APV(FA)}_0 - 1000\ {}_{20}E_{22}\ {}_{20}E_{42}\left(\ddot{a}_{62} - \ {}_{10}E_{62}\ \ddot{a}_{72}\right)}{{}_{20}E_{22}\ {}_{20}E_{42}\ {}_{10}E_{62}}$$

Retrospective: $${}_{60}V = \frac{P\mathbf{APV(FA)}_0 - 1000\ {}_{20}E_{22}\ {}_{20}E_{42}\left(\ddot{a}_{62} - \ {}_{20}E_{62}\ \ddot{a}_{82}\right)}{{}_{20}E_{22}\ {}_{20}E_{42}\ {}_{20}E_{62}}$$

$${}_{70}V = \frac{P\mathbf{APV(FA)}_0 - 1000\ {}_{20}E_{22}\ {}_{20}E_{42}\left(\ddot{a}_{62} - \ {}_{20}E_{62}\ {}_{10}E_{82}\ \ddot{a}_{92}\right)}{{}_{20}E_{22}\ {}_{20}E_{42}\ {}_{20}E_{62}\ {}_{10}E_{82}}$$

Project group 6

Abdullah aged 22 purchases a 40–year deferred whole life annuity due of 6000 per year as soon as the Abdullah is alive. Assume that Annual premiums of $3P$ and P, determined using equivalence principle, are paid at the beginning of each year during the first 20 years and the second 20 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the second 20 years
2. Calculate the net premium reserve at ages 50, 60, 70 using prospective approach
3. Calculate the net premium reserve at ages 50, 60, 70 using retrospective approach

Project group G7

Abdullah aged 22 purchases a 40–year deferred whole life annuity due of 7000 per year as soon as the Abdullah is alive. Assume that Annual premiums of $4P$ and P, determined using equivalence principle, are paid at the beginning of each year during the first 20 years and the second 20 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the first 20 years
2. Calculate the net premium reserve at ages 50, 60, 70 using prospective approach
3. Calculate the net premium reserve at ages 50, 60, 70 using retrospective approach

Project group G8

Abdullah aged 22 purchases a 40–year deferred whole life annuity due of 8000 per year as soon as the Abdullah is alive. Assume that Annual premiums of $5P$ and P, determined using equivalence principle, are paid at the beginning of each year during the first 20 years and the second 20 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the second 20 years
2. Calculate the net premium reserve at ages 50, 60, 70 using prospective approach
3. Calculate the net premium reserve at ages 50, 60, 70 using retrospective approach

Project Group 9

Abdullah aged 22 purchases a 30–year deferred whole life annuity due of 9000 per year as soon as the Abdullah is alive. Assume that Annual premiums of P and $2P$, determined using equivalence principle, are paid at the beginning of each year during the first 15 years and the second 15 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the first 15 years
2. Calculate the net premium reserve at ages 40, 50, 60 using prospective approach
3. Calculate the net premium reserve at ages 40, 50, 60 using retrospective approach

Project group 10

Abdullah aged 22 purchases a 30–year deferred whole life annuity due of 10000 per year as soon as the Abdullah is alive. Assume that Annual premiums of P and $3P$, determined using equivalence principle, are paid at the beginning of each year during the first 15 years and the second 15 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the second 15 years
2. Calculate the net premium reserve at ages 40, 50, 60 using prospective approach
3. Calculate the net premium reserve at ages 40, 50, 60 using retrospective approach

Project Group 11

Abdullah aged 22 purchases a 30–year deferred whole life annuity due of 11000 per year as soon as the Abdullah is alive. Assume that Annual premiums of P and $4P$, determined using equivalence principle, are paid at the beginning of each year during the first 15 years and the second 15 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the first 15 years
2. Calculate the net premium reserve at ages 40, 50, 60 using prospective approach
3. Calculate the net premium reserve at ages 40, 50, 60 using retrospective approach

Project Group 12

Abdullah aged 22 purchases a 30–year deferred whole life annuity due of 12000 per year as soon as the Abdullah is alive. Assume that Annual premiums of P and $5P$, determined using equivalence principle, are paid at the beginning of each year during the first 15 years and the second 15 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the second 15 years
2. Calculate the net premium reserve at ages 40, 50, 60 using prospective approach
3. Calculate the net premium reserve at ages 40, 50, 60 using retrospective approach

Project group 13

Abdullah aged 22 purchases a 40–year deferred whole life annuity due of 13000 per year as soon as the Abdullah is alive. Assume that Annual premiums of $2P$ and P, determined using equivalence principle, are paid at the beginning of each year during the first 20 years and the second 20 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the first 20 years
2. Calculate the net premium reserve at ages 50, 60, 70 using prospective approach
3. Calculate the net premium reserve at ages 50, 60, 70 using retrospective approach

Project group 14

Abdullah aged 22 purchases a 40–year deferred whole life annuity due of 14000 per year as soon as the Abdullah is alive. Assume that Annual premiums of $3P$ and P, determined using equivalence principle, are paid at the beginning of each year during the first 20 years and the second 20 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the second 20 years
2. Calculate the net premium reserve at ages 50, 60, 70 using prospective approach
3. Calculate the net premium reserve at ages 50, 60, 70 using retrospective approach

Project group G15

Abdullah aged 22 purchases a 40–year deferred whole life annuity due of 15000 per year as soon as the Abdullah is alive. Assume that Annual premiums of $4P$ and P, determined using equivalence principle, are paid at the beginning of each year during the first 20 years and the second 20 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the first 20 years
2. Calculate the net premium reserve at ages 50, 60, 70 using prospective approach
3. Calculate the net premium reserve at ages 50, 60, 70 using retrospective approach

Project group G16

Abdullah aged 22 purchases a 40–year deferred whole life annuity due of 16000 per year as soon as the Abdullah is alive. Assume that Annual premiums of $5P$ and P, determined using equivalence principle, are paid at the beginning of each year during the first 20 years and the second 20 years in the deferred period. Mortality follows ILT for $i = 6\%$.

1. Calculate the level premium of the second 20 years
2. Calculate the net premium reserve at ages 50, 60, 70 using prospective approach
3. Calculate the net premium reserve at ages 50, 60, 70 using retrospective approach

Model Answer to the project Spring 2020 Substitute of MID2

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$	x
0	10,000,000.000	20.420	16.801	49.000	25.920	728.540	541.950	299.890	0
5	9,749,503.000	0.980	17.038	35.590	8.450	743.890	553.480	305.900	5
10	9,705,588.000	0.850	16.912	42.720	9.370	744.040	553.340	305.240	10
15	9,663,731.000	0.910	16.738	52.550	11.330	743.710	552.690	303.960	15
20	9,617,802.000	1.030	16.513	65.280	14.300	743.160	551.640	301.930	20
21	9,607,896.000	1.060	16.461	68.240	15.060	743.010	551.360	301.400	21
22	9,597,695.000	1.100	16.406	71.350	15.870	742.860	551.060	300.820	22
23	9,587,169.000	1.130	16.348	74.620	16.760	742.680	550.730	300.190	23
24	9,576,288.000	1.180	16.288	78.050	17.710	742.490	550.360	299.490	24
25	9,565,017.000	1.220	16.224	81.650	18.750	742.290	549.970	298.730	25
26	9,553,319.000	1.270	16.157	85.430	19.870	742.060	549.530	297.900	26
27	9,541,153.000	1.330	16.087	89.400	21.070	741.810	549.050	297.000	27
28	9,528,475.000	1.390	16.014	93.560	22.380	741.540	548.530	296.010	28
29	9,515,235.000	1.460	15.937	97.920	23.790	741.240	547.960	294.920	29
30	9,501,381.000	1.530	15.856	102.480	25.310	740.910	547.330	293.740	30
31	9,486,854.000	1.610	15.772	107.270	26.950	740.550	546.650	292.450	31
32	9,471,591.000	1.700	15.683	112.280	28.720	740.160	645.900	291.040	32
33	9,455,522.000	1.790	15.591	117.510	30.630	739.720	545.070	289.500	33
34	9,438,571.000	1.900	15.494	122.990	32.680	739.250	544.170	287.820	34
35	9,420,657.000	2.010	15.393	128.720	34.880	738.730	543.180	286.000	35
36	9,401,688.000	2.140	15.287	134.700	37.260	738.160	542.110	284.000	36
37	9,381,566.000	2.280	15.177	140.940	39.810	737.540	540.920	281.840	37
38	9,360,184.000	2.430	15.062	147.460	42.550	736.860	539.630	279.480	38
39	9,337,427.000	2.600	14.942	154.250	45.480	736.110	538.220	276.920	39
40	9,313,166.000	2.780	14.817	161.320	48.630	735.290	536.670	274.140	40
41	9,287,264.000	2.980	14.686	168.690	52.010	734.400	534.990	271.120	41
42	9,259,571.000	3.200	14.551	176.360	55.620	733.420	533.140	267.850	42
43	9,229,925.000	3.440	14.410	184.330	59.480	732.340	531.120	264.310	43
44	9,198,149.000	3.710	14.264	192.610	63.610	731.170	528.920	260.480	44
45	9,164,051.000	4.000	14.112	201.200	68.020	729.880	526.520	256.340	45
46	9,127,426.000	4.310	13.955	210.120	72.720	728.470	523.890	251.880	46
47	9,088,049.000	4.660	13.791	219.360	77.730	726.930	521.030	247.080	47
48	9,045,679.000	5.040	13.622	228.920	83.060	725.240	517.910	241.930	48

49	9,000,057.000	5.460	13.448	238.820	88.730	723.390	514.510	236.390	49
50	8,950,901.000	5.920	13.267	249.050	94.760	721.370	510.810	230.470	50
51	8,897,913.000	6.420	13.080	259.610	101.150	719.170	506.780	224.150	51
52	8,840,770.000	6.970	12.888	270.500	107.920	716.760	502.400	217.420	52
53	8,779,128.000	7.580	12.690	281.720	115.090	714.120	497.640	210.270	53
54	8,712,621.000	8.240	12.486	293.270	122.670	711.240	492.470	202.700	54
55	8,640,861.000	8.960	12.276	305.140	130.670	708.100	486.860	194.720	55
56	8,563,435.000	9.750	12.060	317.330	139.110	704.670	480.790	186.320	56
57	8,479,908.000	10.620	11.840	329.840	147.990	700.930	474.220	177.530	57
58	8,389,826.000	11.580	11.613	342.650	157.330	696.850	467.120	168.370	58
59	8,292,713.000	12.620	11.382	355.750	167.130	692.410	459.460	158.870	59
60	8,188,074.000	13.760	11.145	369.130	177.410	687.560	451.200	149.060	60
61	8,075,403.000	15.010	10.904	382.190	188.170	682.290	442.310	139.000	61
62	7,954,179.000	16.380	10.658	396.700	199.410	676.560	432.770	128.750	62
63	7,823,879.000	17.880	10.408	410.850	211.130	670.330	422.540	118.380	63
64	7,683,979.000	19.520	10.154	425.220	223.340	663.560	411.610	107.970	64
65	7,533,964.000	21.320	9.897	439.800	236.030	656.230	399.940	97.600	65
66	7,373,338	23.29	9.6362	454.56	249.20	648.27	387.53	87.37	66
67	7,201,635	25.44	9.3726	469.47	262.83	639.66	374.36	77.38	67
68	7,018,432	27.79	9.1066	484.53	276.92	630.35	360.44	67.74	68
69	6,823,367	30.37	8.8387	499.70	291.46	620.30	345.77	58.54	69
70	6,616,155	33.18	8.5693	514.95	306.42	609.46	330.37	49.88	70
71	6,396,609	36.26	8.2988	530.26	321.78	597.79	314.27	41.86	71
72	6,164,663	39.62	8.0278	545.60	337.54	585.25	297.51	34.53	72
73	5,920,394	43.30	7.7568	560.93	353.64	571.81	280.17	27.96	73
74	5,664,051	47.31	7.4864	576.24	370.08	557.43	262.31	22.19	74
75	5,396,081	51.69	7.2170	591.49	386.81	542.07	244.03	17.22	75
76	5,117,152	56.47	6.9493	606.65	403.80	525.71	225.46	13.04	76
77	4,828,182	61.68	6.6836	621.68	421.02	508.35	206.71	9.61	77
78	4,530,360	67.37	6.4207	636.56	438.42	489.97	187.94	6.88	78
79	4,225,163	73.56	6.1610	651.26	455.95	470.57	169.31	4.77	79
80	3,914,365	80.30	5.9050	665.75	473.59	450.19	151.00	3.19	80
81	3,600,038	87.64	5.6533	680.00	491.27	428.86	133.19	2.05	81
82	3,284,542	95.61	5.4063	693.98	508.96	406.62	116.06	1.27	82
83	2,970,496	104.28	5.1645	707.67	526.60	383.57	99.81	0.75	83
84	2,660,734	113.69	4.9282	721.04	544.15	359.79	84.59	0.42	84
85	2,358,246	123.89	4.6980	734.07	561.57	335.40	70.56	0.22	85
86	2,066,090	134.94	4.4742	746.74	578.80	310.56	57.83	0.11	86
87	1,787,299	146.89	4.2571	759.03	595.79	285.44	46.50	0.05	87
88	1,524,758	159.81	4.0470	770.92	612.51	260.21	36.61	0.02	88
89	1,281,083	173.75	3.8442	782.41	628.92	235.11	28.17	0.01	89
90	1,058,491	188.77	3.6488	793.46	644.96	210.36	21.13	0.00	90

91	858,676	204.93	3.4611	804.09	660.61	186.21	15.41	0.00	91
92	682,707	222.27	3.2812	814.27	675.83	162.90	10.91	0.00	92
93	530,959	240.86	3.1091	824.01	690.59	140.69	7.47	0.00	93
94	403,072	260.73	2.9450	833.30	704.86	119.79	4.93	0.00	94
95	297,981	281.91	2.7888	842.14	718.61	100.43	3.13	0.00	95
96	213,977	304.45	2.6406	850.53	731.83	82.78	1.90	0.00	96
97	148,832	328.34	2.5002	858.48	744.50	66.97	1.10	0.00	97
98	99,965	353.60	2.3676	865.99	756.60	53.09	0.60	0.00	98
99	64,617	380.20	2.2426	873.06	768.13	41.14	0.31	0.00	99
100	40,049	408.12	2.1252	879.70	779.08	31.12	0.15	0.00	100
101	23,705	437.28	2.0152	885.93	789.44	22.91	0.07	0.00	101
102	13,339	467.61	1.9123	891.76	799.21	16.37	0.03	0.00	102
103	7,101	498.99	1.8164	897.19	808.41	11.33	0.01	0.00	103
104	3,558	531.28	1.7273	902.23	817.02	7.56	0.00	0.00	104
105	1,668	564.29	1.6447	906.90	825.06	4.86	0.00	0.00	105
106	727	597.83	1.5685	911.22	832.53	2.99	0.00	0.00	106
107	292	631.64	1.4984	915.19	839.46	1.76	0.00	0.00	107
108	108	665.45	1.4341	918.82	845.84	0.98	0.00	0.00	108
109	36	698.97	1.3755	922.14	851.69	0.52	0.00	0.00	109
110	11	731.87	1.3223	925.15	857.04	0.26	0.00	0.00	110

v	Age	Benefit	CFP	PFP	CSP	PSP	Defer. perio	Prospective Reserve			
0.94								18	28	38	48
G1	22	1000	1	111.956	2	223.912	30	4123.73	10666.89	11145.40	8569.30
G2	22	2000	1	183.035	3	549.106	30	7362.59	21118.62	22290.80	17138.60
G3	22	3000	1	232.169	4	928.676	30	10126.39	31454.85	33436.20	25707.90
G4	22	4000	1	268.161	5	1340.806	30	12605.73	41721.92	44581.60	34277.20
G5	22	5000	2	311.427	1	155.714	40		20134.44	45774.25	42846.50
G6	22	6000	3	390.312	1	130.104	40		24526.71	55038.66	51415.80
G7	22	7000	4	465.708	1	116.427	40		28842.17	64280.03	59985.10
G8	22	8000	5	539.591	1	107.918	40		33124.34	73511.42	68554.40
G9	22	9000	1	1007.605	2	2015.211	30	37113.5	96001.98	100308.6	77123.7
G10	22	10000	1	915.177	3	2745.532	30	36812.9	105593.1	111454.0	85693.0
G11	22	11000	1	851.286	4	3405.145	30	37130.1	115334.5	122599.4	94262.3
G12	22	12000	1	804.484	5	4022.419	30	37817.2	125165.7	133744.8	102831.6
G13	22	13000	2	809.710	1	404.855	40		52349.54	119013.1	111400.9
G14	22	14000	3	910.729	1	303.576	40		57229.00	128423.5	119970.2
G15	22	15000	4	997.945	1	249.486	40		61804.64	137742.9	128539.5
G16	22	16000	5	1079.182	1	215.836	40		66248.68	147022.8	137108.8

$$\begin{aligned}\mathbf{APV}(\mathbf{FA})_0 &= a\ddot{a}_{22:\overline{15}|} + b\ {}_{15|}\ddot{a}_{22:\overline{15}|} \\ &= a\left(\ddot{a}_{22} - {}_5E_{22}\ {}_{10}E_{27}\ \ddot{a}_{37}\right) + b\ {}_5E_{22}\ {}_{10}E_{27}\left(\ddot{a}_{37} - {}_5E_{37}\ \ {}_{10}E_{42}\ddot{a}_{52}\right)\end{aligned}$$

$$\mathbf{APV}(\mathbf{FB})_0 = 1000\ {}_{10}E_{22}\ {}_{20}E_{32}\ \ddot{a}_{52} \quad \text{and} \quad P = \frac{1000\ {}_{10}E_{22}\ {}_{20}E_{32}\ \ddot{a}_{52}}{\mathbf{APV}(\mathbf{FA})_0}$$

Prospective: $\quad {}_{40}V = 1000\ \ddot{a}_{62}, \quad {}_{40}V = 1000\ \ddot{a}_{72}, \quad {}_{40}V = 1000\ \ddot{a}_{82}$

$${}_{40}V = \frac{P\,\mathbf{APV}(\mathbf{FA})_0 - 1000\ {}_{30}E_{22}\left(\ddot{a}_{52} - \ {}_{10}E_{52}\ \ddot{a}_{62}\right)}{{}_{20}E_{22}\ {}_{20}E_{42}}$$

Retrospective: $\quad {}_{50}V = \dfrac{P\,\mathbf{APV}(\mathbf{FA})_0 - 1000\ {}_{30}E_{22}\left(\ddot{a}_{52} - \ {}_{20}E_{52}\ \ddot{a}_{72}\right)}{{}_{20}E_{22}\ {}_{20}E_{42}\ {}_{10}E_{62}}$

$${}_{60}V = \frac{P\,\mathbf{APV}(\mathbf{FA})_0 - 1000\ {}_{30}E_{22}\left(\ddot{a}_{52} - \ {}_{30}E_{52}\ \ddot{a}_{82}\right)}{{}_{20}E_{22}\ {}_{20}E_{42}\ {}_{20}E_{62}}$$

$$\mathbf{APV}(\mathbf{FA})_0 = a\ddot{a}_{22:\overline{20}|} + b\ {}_{20|}\ddot{a}_{22:\overline{20}|}$$

$$= \quad a\,(\ddot{a}_{22} - \ {}_{20}E_{22}\ \ddot{a}_{42}) + b\ \ {}_{20}E_{22}\ \ (\ddot{a}_{42} - \ {}_{20}E_{42}\ddot{a}_{62})$$

$$\mathbf{APV(FB)}_0 = 1000\ {}_{40}E_{22}\ \ddot{a}_{62} \quad \text{and} \quad P = \frac{1000\ \ {}_{20}E_{22}\ {}_{20}E_{42}\ \ddot{a}_{62}}{\mathbf{APV(FA)}_0}$$

Prospective: $\quad {}_{50}V = 1000\ \ddot{a}_{72}, \quad {}_{60}V = 1000\ \ddot{a}_{82}, \quad {}_{70}V = 1000\ \ddot{a}_{92}$

$${}_{50}V = \frac{P\mathbf{APV(FA)}_0 - 1000\ {}_{20}E_{22}\ {}_{20}E_{42}\,(\ddot{a}_{62} - \ {}_{10}E_{62}\ \ddot{a}_{72})}{{}_{20}E_{22}\ {}_{20}E_{42}\ {}_{10}E_{62}}$$

Retrospective: $\quad {}_{60}V = \dfrac{P\mathbf{APV(FA)}_0 - 1000\ {}_{20}E_{22}\ {}_{20}E_{42}\,(\ddot{a}_{62} - \ {}_{20}E_{62}\ \ddot{a}_{82})}{{}_{20}E_{22}\ {}_{20}E_{42}\ {}_{20}E_{62}}$

$${}_{70}V = \frac{P\mathbf{APV(FA)}_0 - 1000\ {}_{20}E_{22}\ {}_{20}E_{42}\,(\ddot{a}_{62} - \ {}_{20}E_{62}\ {}_{10}E_{82}\ \ddot{a}_{92})}{{}_{20}E_{22}\ {}_{20}E_{42}\ {}_{20}E_{62}\ {}_{10}E_{82}}$$