**Department of Statistics & Operations Research**

**College of Science**

**King Saud University**

**OR 441**

**Final Examination**

**Second Semester 1442**

**Question 1 (4 marks)**

Use the following random numbers U(0, 1) shown in the table and the direct transformation method to generate random numbers that follow the normal distribution with μ=20, σ=3.

Write and attach the detail solutions then fill the following table.





|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **R1** | **R2** | **Z1** | **Z2** | **X1** | **X2** |
| 0.2379 | 0.7551 | 0.054294214 | -1.69377149 | 20.16288264 | 14.91868553 |
| 0.2989 | 0.247 | 0.029292758 | 1.553845022 | 20.08787827 | 24.66153507 |

**Question 2 (10 marks)**

At a grocery store one counter, customers arrive at random from 1 to 5 minutes apart (each of inter-arrival time has the same probability of occurrence). The service times vary from 1 to 5 minutes with the probabilities 0.1, 0.2, 0.3, 0.25 and 0.15, respectively. Analyze the system by simulating the arrival and service of 5 customers.

 Use the following random numbers for simulating the arrival customers:

0.01, 0.913, 0.727, 0.015, 0.648, 0.309

(hint: Fill the following tables)

A. Create tables for simulating the arrival customers:

|  |  |  |  |
| --- | --- | --- | --- |
| Time between arrivals (min) | Probability | Cumulative Probability | Intervals for choosing Time between arrivals (min) |
| From  | to |
| 1 | 0.2 | 0.2 | 0 | 0.2 |
| 2 | 0.2 | 0.4 | 0.2 | 0.4 |
| 3 | 0.2 | 0.6 | 0.4 | 0.6 |
| 4 | 0.2 | 0.8 | 0.6 | 0.8 |
| 5 | 0.2 | 1 | 0.8 | 1 |
|  |  |  |  |  |
|  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Customer Number | 1 | 2 | 3 | 4 | 5 |
| Random Number | 0.01 | 0.913 | 0.727 | 0.015 | 0.648 |
| Arrival time (min) | 1 | 5 | 4 | 1 | 4 |

**Continue question 2…next page**

B. Use the following random numbers for simulating the service customers:

0.841, 0.102, 0.742, 0.535, 0.176,

Create a table for simulating the service time for the customers:

|  |  |  |  |
| --- | --- | --- | --- |
| Service Time (min) | Probability | Cumulative Probability | Intervals for choosing Service Time (min) |
| From | to |
| 1 | 0.1 | 0.1 | 0 | 0.1 |
| 2 | 0.2 | 0.3 | 0.1 | 0.3 |
| 3 | 0.3 | 0.6 | 0.3 | 0.6 |
| 4 | 0.25 | 0.85 | 0.6 | 0.85 |
| 5 | 0.15 | 1 | 0.85 | 1 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Customer Number | 1 | 2 | 3 | 4 | 5 |
| Random Number | 0.841 | 0.102 | 0.742 | 0.535 | 0.176 |
| Service time (min) | 4 | 2 | 4 | 3 | 2 |

Using the above information, fill the following table

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Customers Number | Time between arrivals (min) | Cum. Arrival time (min) | Service time(min) | Service start (min) | Do customers wait? | Departure time (min) | Wait time | Cashier idle time  | Time in system |
| 1 | 1 | 1 | 4 | 1 | 0 | 5 | 0 | 1 | 4 |
| 2 | 5 | 6 | 2 | 6 | 0 | 8 | 0 | 1 | 2 |
| 3 | 4 | 10 | 4 | 10 | 0 | 14 | 0 | 2 | 4 |
| 4 | 1 | 11 | 3 | 14 | 1 | 17 | 3 | 0 | 6 |
| 5 | 4 | 15 | 2 | 17 | 1 | 19 | 2 | 0 | 4 |
| sum |  |  | 15 |  | 2 |  | 5 | 4 | 20 |

**Continue question 2…next page**

Having the data computed in the previous table find the following:

1. The average waiting time for a customer =5/5=1 (min)
2. The probability that a customer has to wait in the queue =2/5=0.4
3. The fraction of idle time of the server =4/19
4. The probability of the server being busy = 15/19
5. The average service time = 15/5=3
6. The expected service time = 3.15
7. The average time between arrivals = 15/4=3.75
8. The expected time between arrivals = (1+5)/2 = 3
9. The average waiting time of those who wait = 5/2=2.5
10. The average time a customer spends in the system= 20/5=4

1. 

 =5/5=1 (min)

2. 

 =2/5=0.4

3. 

 = 4/19

4. The probability of the server being busy = 1-4/19= 15/19

5.



 =15/5=3

6. The expected service time

|  |  |  |
| --- | --- | --- |
| Service Time (min) | Probability | ST\*Prob |
|
| 1 | 0.1 | 0.1 |
| 2 | 0.2 | 0.4 |
| 3 | 0.3 | 0.9 |
| 4 | 0.25 | 1.0 |
| 5 | 0.15 | 0.75 |

 = 3.15

7. 

=15/4 =3.75

8. The expected time between arrivals:

 = (1+5)/2 = 3

9. The average waiting time of those who wait



 = 5/2=2.5 (min)

10. The average time a customer spends in the system



= 20/5=4

**Question 3**

Using the linear congruential (LCG) method check if the LCG generator has a full period conditions with *X0=1*, *m=26, a= 17, c=19*. Find the first 10 values in the period of the generator with the given values.



|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 36 | 55 | 58 | 45 | 16 | 35 | 38 | 25 |