College of Science.

> كثـية الـعوم
> قَسم الإحصاء ويـعوث الالعثليات

Second Midterm Exam
Academic Year 1443-1444 Hijri- First Semester


Student Information Aعta


## General Instructions:

- Your Exam consists of $\square$ PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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(المؤكة)


This section is ONLY for instructor

| $\#$ | Course Learning Outcomes (CLOs) | Related <br> Question (s) | Points | Final <br> Score |
| :--- | :--- | :--- | :--- | :--- |
| 1 | To know the basics of pseudo random generation and apply <br> different methods of random generation techniques |  |  |  |
| 2 | Chose and fit theoretical distribution on collected data |  |  |  |
| 3 | Define and compute performance measures from simulation <br> models |  |  |  |
| 4 | Recognize and analyze simple models and its main elements <br> for simulation |  |  |  |
| 5 | Understanding how to use computer software (ECXEL) for <br> simulation models |  |  |  |
| 6 | use appropriate statistical techniques to analyze and evaluate <br> outputs of simulation models |  |  |  |
| 7 | Generate random variates from different probability functions <br> and directly from collected data |  |  |  |
| 88 | build simple simulation models of real-life problems |  |  |  |

## Question \#1:

A continuous random variable ( X ) ranges from -3 to 4 is defined by the following CDF:

$$
F(x)= \begin{cases}0, & x \leq-3 \\ \frac{1}{2}+\frac{x}{6}, & -3<x \leq 0 \\ \frac{1}{2}+\frac{x^{2}}{32}, & 0<x \leq 4 \\ 1, & x>4\end{cases}
$$

(1) Write the inverse transform for this random variable.
(2) Let ( X ) be the percentage of change in a week for a given share. Given that the closing price of this week for this share is 123 SR write the closing price for this share for the next 8 weeks. (use the table below)
(3) From the simulated date, what is the probability of closing with increase.

| Week \# | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| U1 | 0.353 | 0.034 | 0.672 | 0.622 | 0.408 | 0.218 | 0.889 | 0.643 |  |
| U2 | 0.962 | 0.981 | 0.781 | 0.313 | 0.600 | 0.910 | 0.808 | 0.632 |  |
| \%-change |  |  |  |  |  |  |  |  |  |
| Closing <br> Price |  |  |  |  |  |  |  |  |  |

## Question \#2:

Consider a continuous random variable X with the following pdf

$$
f(x)= \begin{cases}e^{2 x}, & -\infty<x \leq 0 \\ e^{-2 x}, & 0<x<\infty\end{cases}
$$

(1) Derive the inverse transform to generate random values for X and apply it for Table(1)
(2) Use acceptance/rejection method to generate random values for X and apply it for Table(2) assuming that $\mathrm{x} \in[-7,7]$
Table (1)

| Week \# | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| U1 | 0.570 | 0.462 | 0.055 | 0.571 | 0.062 | 0.760 | 0.701 | 0.493 | 0.082 | 0.261 |
| U2 | 0.826 | 0.127 | 0.318 | 0.106 | 0.850 | 0.830 | 0.714 | 0.429 | 0.079 | 0.816 |
| X |  |  |  |  |  |  |  |  |  |  |

Table (2)

| Week \# | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| U1 | 0.570 | 0.462 | 0.055 | 0.571 | 0.062 | 0.760 | 0.701 | 0.493 | 0.082 | 0.261 |
| U2 | 0.826 | 0.127 | 0.318 | 0.106 | 0.850 | 0.830 | 0.714 | 0.429 | 0.079 | 0.816 |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| X |  |  |  |  |  |  |  |  |  |  |

## Question \#3:

A machine is taken out of production (turn off the machine) either if it fails or after 5 hours shift of continuous operation, whichever comes first. From past data, it has found that the machine can operate without failure for a random amount of time (X) following a Weibull distribution with parameters $\alpha=0.75$ and $\beta=8$ :

$$
f(x)=\frac{\alpha}{\beta^{\alpha}} x^{\alpha-1} e^{-(x / \beta)^{\alpha}}
$$

Thus, the time until the machine is taken out of production can be represented as $Y=\min (X, 5)$.
(1) Write a step-by-step procedure for generating Y.
(2) Use your answer in (1) to apply on the following uniform numbers.
(3) What is the probability that the machine operates the shift without failure.
(4) What is the average machine operation time per shift.

| Week \# | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| U 1 | 0.201 | 0.417 | 0.797 | 0.316 | 0.042 | 0.190 | 0.697 | 0.083 | 0.867 | 0.354 |
| X |  |  |  |  |  |  |  |  |  |  |
| Y |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

## Question \#4:

Students arrive at a self-service cafeteria at the rate of one every $30 \pm 20$ seconds. It is estimated that $40 \%$ of students go to the sandwich bar, where every student prepares has own sandwich in $60 \pm 30$ seconds. The rest go to the main counter, where one server spoons the prepared meal onto a plate in $45 \pm 30$ seconds. All students take their seats in the cafeteria and spend $20 \pm 10$ minutes eating. After eating, $10 \%$ of the students go back for dessert, and return to their table to spend an additional $10 \pm 2$ minutes in the cafeteria.

1. Simulate until 10 people have left the cafeteria using the following table of $\mathrm{U}[0,1]$ numbers.
2. At the final simulation time, estimate the following from the simulation data:
a. How many students are still in the cafeteria
b. What percentage of students at the sandwich bar.
c. What percentage of students at the main course counter.
d. What percentage of students on tables
e. What percentage of students take dessert.
3. From the simulation data, what is the average time that a student who gets a sandwich spends in the cafeteria until he leaves after finishing his entire meal.

|  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |
| Std 1 | 0.454 | 0.516 | 0.922 | 0.405 | 0.965 | 0.686 | 0.623 | 0.327 |
|  |  |  |  |  |  |  |  |  |
| Std 2 | 0.046 | 0.239 | 0.356 | 0.686 | 0.577 | 0.234 | 0.439 | 0.588 |
|  |  |  |  |  |  |  |  |  |
| Std 3 | 0.024 | 0.034 | 0.134 | 0.534 | 0.648 | 0.244 | 0.525 | 0.340 |
|  |  |  |  |  |  |  |  |  |
| Std 4 | 0.162 | 0.032 | 0.224 | 0.209 | 0.441 | 0.493 | 0.850 | 0.607 |
|  |  |  |  |  |  |  |  |  |
| Std 5 | 0.359 | 0.946 | 0.607 | 0.420 | 0.058 | 0.197 | 0.336 | 0.353 |
|  |  |  |  |  |  |  |  |  |
| Std 6 | 0.908 | 0.385 | 0.181 | 0.683 | 0.067 | 0.856 | 0.736 | 0.328 |
|  |  |  |  |  |  |  |  |  |
| Std 7 | 0.287 | 0.537 | 0.196 | 0.087 | 0.297 | 0.772 | 0.564 | 0.633 |
|  |  |  |  |  |  |  |  |  |
| Std 8 | 0.980 | 0.383 | 0.485 | 0.909 | 0.061 | 0.201 | 0.356 | 0.361 |
|  |  |  |  |  |  |  |  |  |
| Std 9 | 0.253 | 0.671 | 0.545 | 0.765 | 0.651 | 0.030 | 0.839 | 0.546 |
|  |  |  |  |  |  |  |  |  |
| Std 10 | 0.160 | 0.498 | 0.090 | 0.432 | 0.187 | 0.588 | 0.248 | 0.954 |
|  |  |  |  |  |  |  |  |  |

## Question \#5:

A traffic department of a small city wants to simulate the occurrence time for the accident in an intersection using some different choices of distributions to estimate the number of accidents per month ( 30 days). Use the following $U(0,1)$ numbers to compute the simulation under the following assumptions.

1. Estimate the number of accidents in the $1^{\text {st }}$ month assuming the time between accident is a truncated exponential distribution with between ( 2 and 10) and mean 4 days
2. Estimate the number of accidents in the $1^{\text {st }}$ month assuming the time between accident is Erlang distribution with parameters $k=3$ and $\lambda=0.75$.
3. Estimate the number of accidents in the $1^{\text {st }}$ month assuming the time between accident is negative binomial (R.V. X be number of trials until success) with parameters $k=3$ and $p$ (success) $=0.65$.

Use uniform numbers (as needed) by columns until you finish all numbers in the column then move to the next.
$\downarrow$ start

| 0.737 | 0.454 | 0.516 |
| :--- | :--- | :--- |
| 0.293 | 0.346 | 0.239 |
| 0.136 | 0.024 | 0.034 |
| 0.848 | 0.162 | 0.132 |
| 0.692 | 0.359 | 0.946 |
| 0.727 | 0.908 | 0.385 |
| 0.116 | 0.287 | 0.537 |
| 0.074 | 0.980 | 0.383 |
| 0.262 | 0.253 | 0.671 |
| 0.385 | 0.160 | 0.498 |
| 0.317 | 0.815 | 0.728 |
| 0.923 | 0.500 | 0.336 |
| 0.057 | 0.872 | 0.600 |
| 0.441 | 0.993 | 0.965 |

