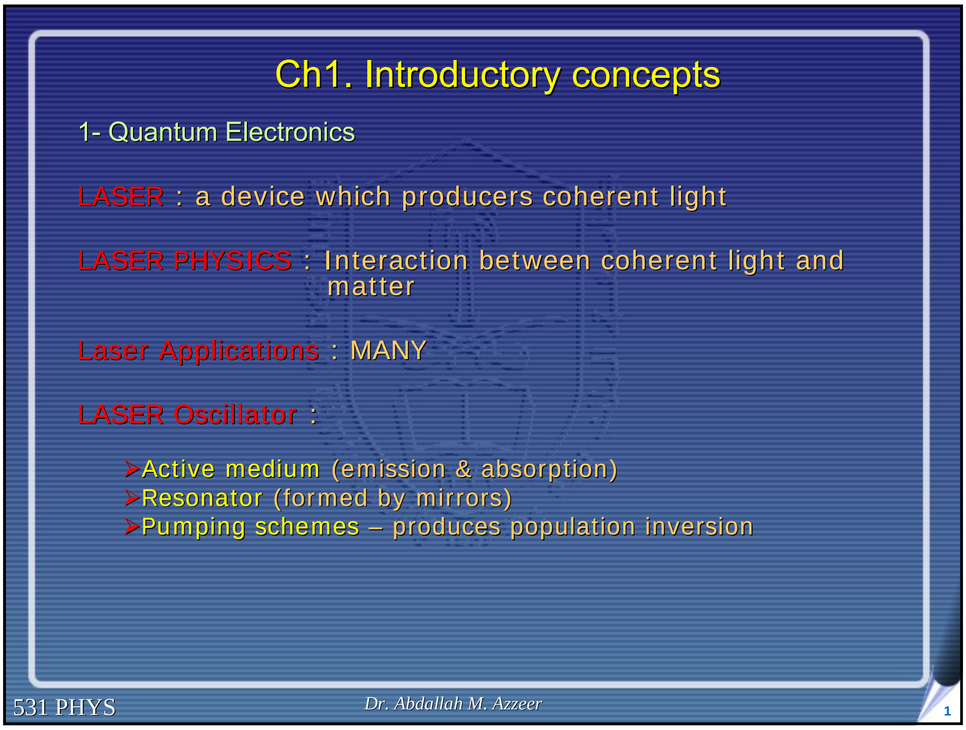


Ch1. Introductory concepts

1- Quantum Electronics

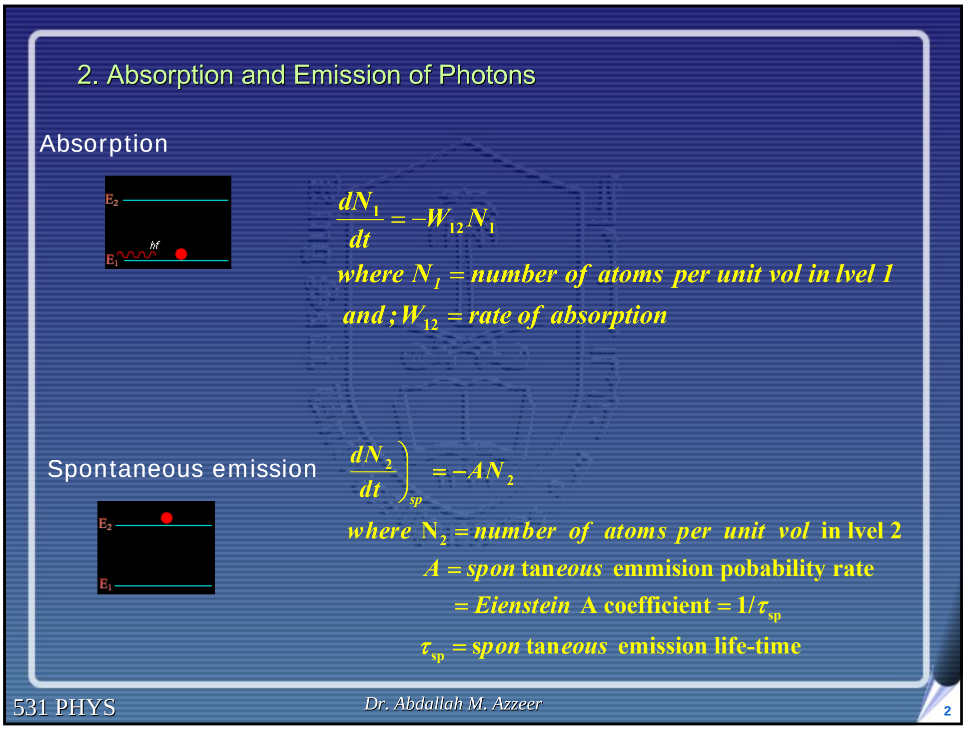
LASER : a device which producers coherent light

LASER PHYSICS : Interaction between coherent light and matter

Laser Applications : MANY

LASER Oscillator :

- Active medium (emission & absorption)
- Resonator (formed by mirrors)
- Pumping schemes – produces population inversion



2. Absorption and Emission of Photons

Absorption

$$\frac{dN_1}{dt} = -W_{12}N_1$$

$where\ N_1 = number\ of\ atoms\ per\ unit\ vol\ in\ lvel\ 1$

$and\,;W_{12} = rate\ of\ absorption$

Spontaneous emission

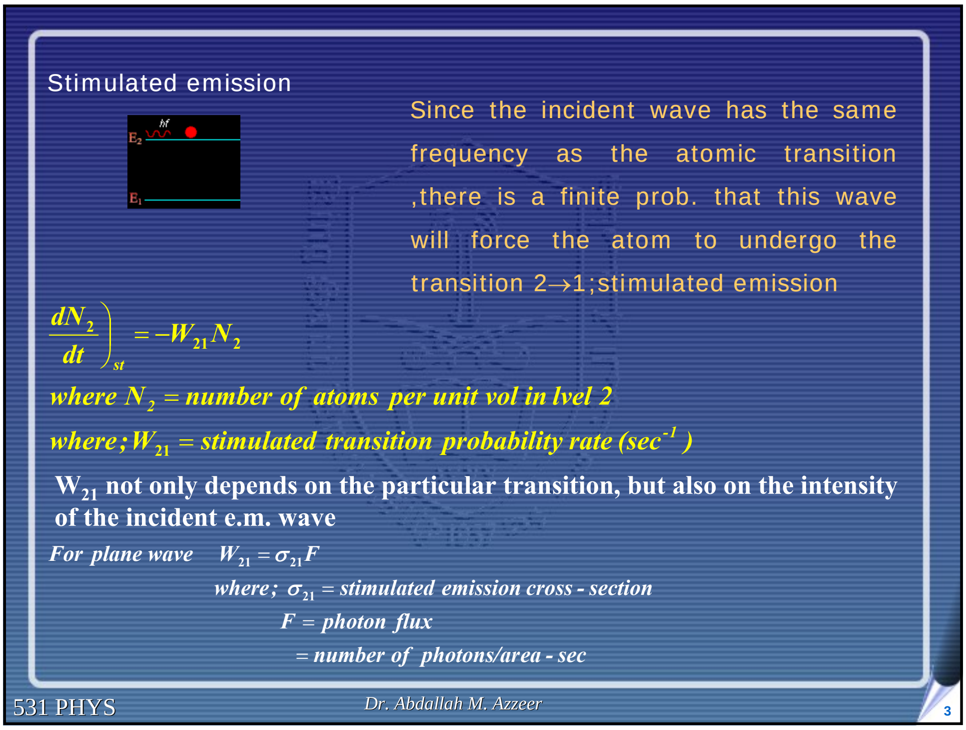
$$\left.\frac{dN_2}{dt}\right)_{sp} = -AN_2$$

$where\ N_2 = number\ of\ atoms\ per\ unit\ vol\ in\ lvel\ 2$

$A = spontaneous\ emmision\ pobability\ rate$

$= Eienstein\ A\ coefficient = 1/\tau_{sp}$

$\tau_{sp} = spontaneous\ emission\ life\text{-}time$



Stimulated emission

Since the incident wave has the same frequency as the atomic transition ,there is a finite prob. that this wave will force the atom to undergo the transition 2→1;stimulated emission

$$\left.\frac{dN_2}{dt}\right)_{st} = -W_{21}N_2$$

$where\ N_2 = number\ of\ atoms\ per\ unit\ vol\ in\ lvel\ 2$

$where; W_{21} = stimulated\ transition\ probability\ rate\ (sec^{-1})$

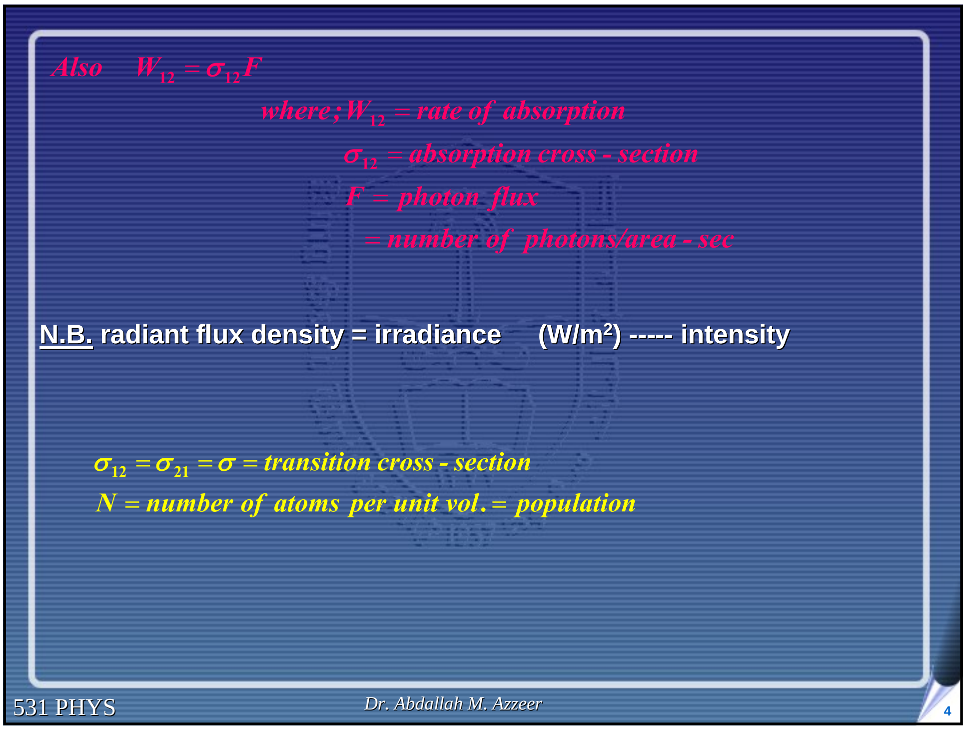
W_{21} not only depends on the particular transition, but also on the intensity of the incident e.m. wave

$For\ plane\ wave\quad W_{21} = \sigma_{21}F$

$where;\ \sigma_{21} = stimulated\ emission\ cross-section$

$F = photon\ flux$

$= number\ of\ photons/area-sec$



$Also\quad W_{12} = \sigma_{12}F$

$where; W_{12} = rate\ of\ absorption$

$\sigma_{12} = absorption\ cross-section$

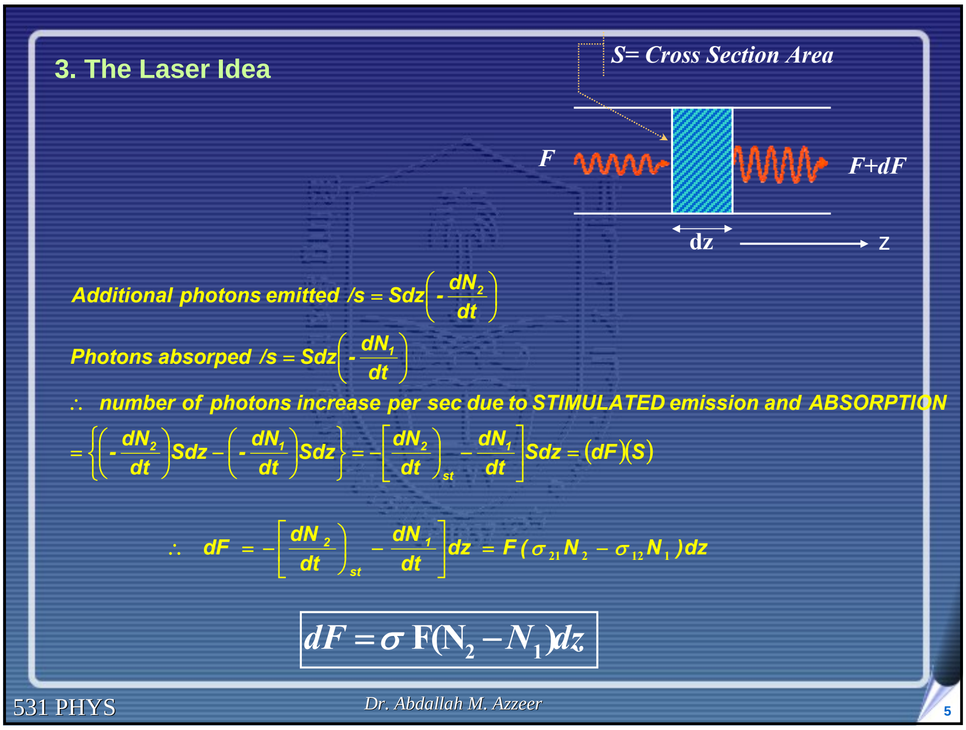
$F = photon\ flux$

$= number\ of\ photons/area-sec$

N.B. radiant flux density = irradiance (W/m²) ----- intensity

$\sigma_{12} = \sigma_{21} = \sigma = transition\ cross-section$

$N = number\ of\ atoms\ per\ unit\ vol. = population$



3. The Laser Idea

$$\text{Additional photons emitted /s} = Sdz\left(-\frac{dN_2}{dt}\right)$$

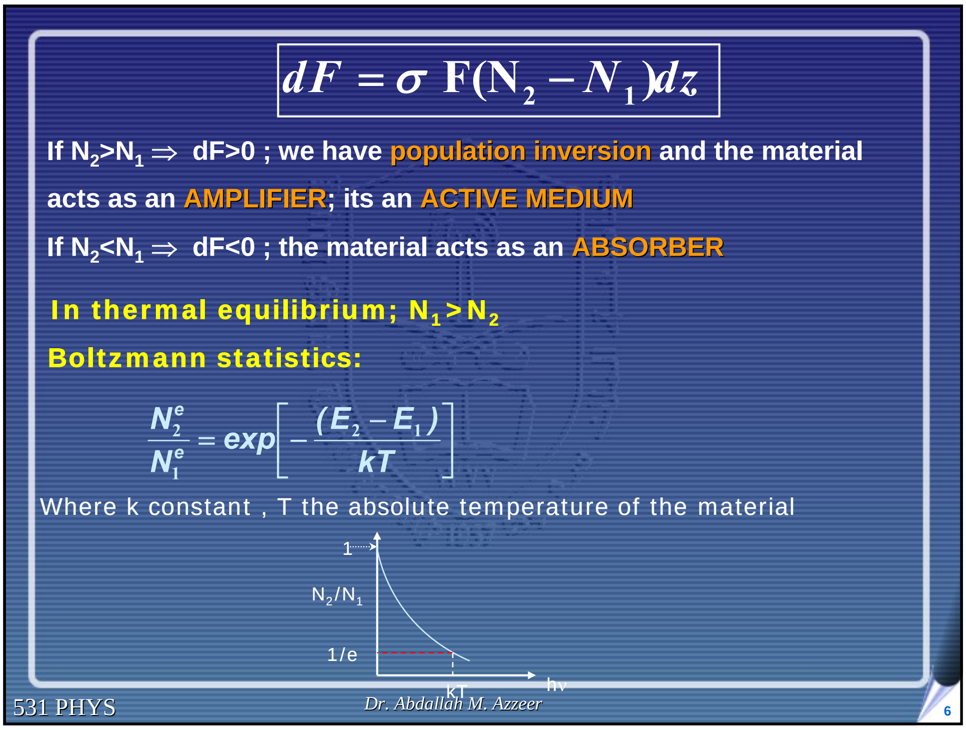
$$\text{Photons absorped /s} = Sdz\left(-\frac{dN_1}{dt}\right)$$

$\therefore$ number of photons increase per sec due to STIMULATED emission and ABSORPTION

$$= \left\{\left(-\frac{dN_2}{dt}\right)Sdz - \left(-\frac{dN_1}{dt}\right)Sdz\right\} = -\left[\left(\frac{dN_2}{dt}\right)_{st} - \frac{dN_1}{dt}\right]Sdz = (dF)(S)$$

$$\therefore \quad dF = -\left[\left(\frac{dN_2}{dt}\right)_{st} - \frac{dN_1}{dt}\right]dz = F(\sigma_{21}N_2 - \sigma_{12}N_1)dz$$

$$\boxed{dF = \sigma\,F(N_2 - N_1)dz}$$



$$\boxed{dF = \sigma\,F(N_2 - N_1)dz}$$

If $N_2 > N_1 \Rightarrow$ dF>0 ; we have population inversion and the material acts as an AMPLIFIER; its an ACTIVE MEDIUM

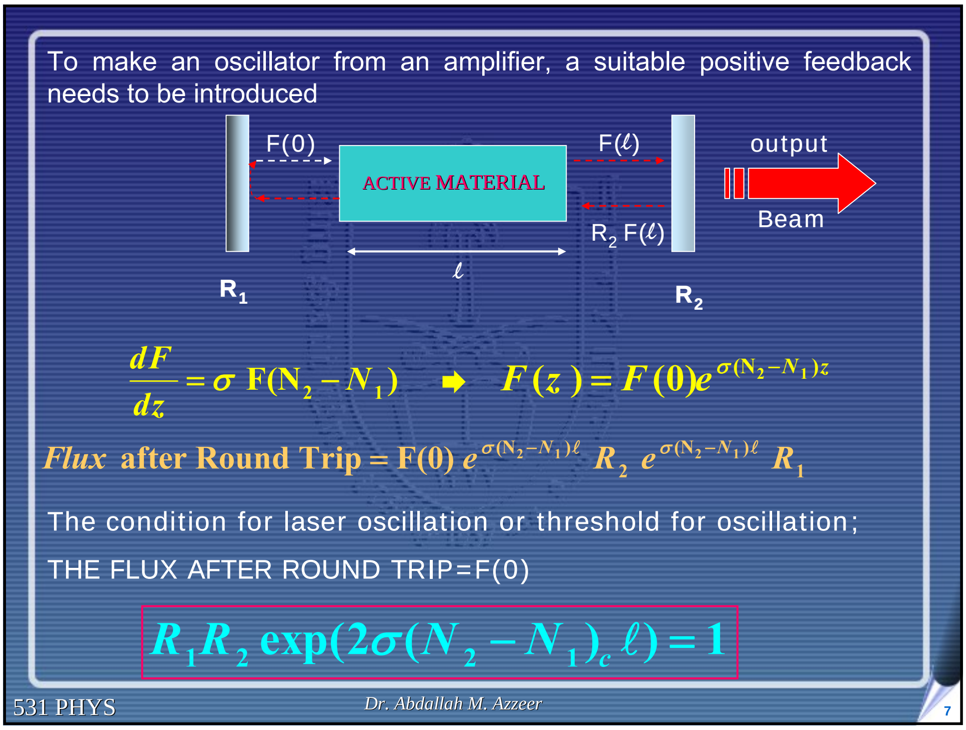
If $N_2 < N_1 \Rightarrow$ dF<0 ; the material acts as an ABSORBER

In thermal equilibrium; $N_1 > N_2$

Boltzmann statistics:

$$\frac{N_2^e}{N_1^e} = \exp\left[-\frac{(E_2 - E_1)}{kT}\right]$$

Where k constant , T the absolute temperature of the material



To make an oscillator from an amplifier, a suitable positive feedback needs to be introduced

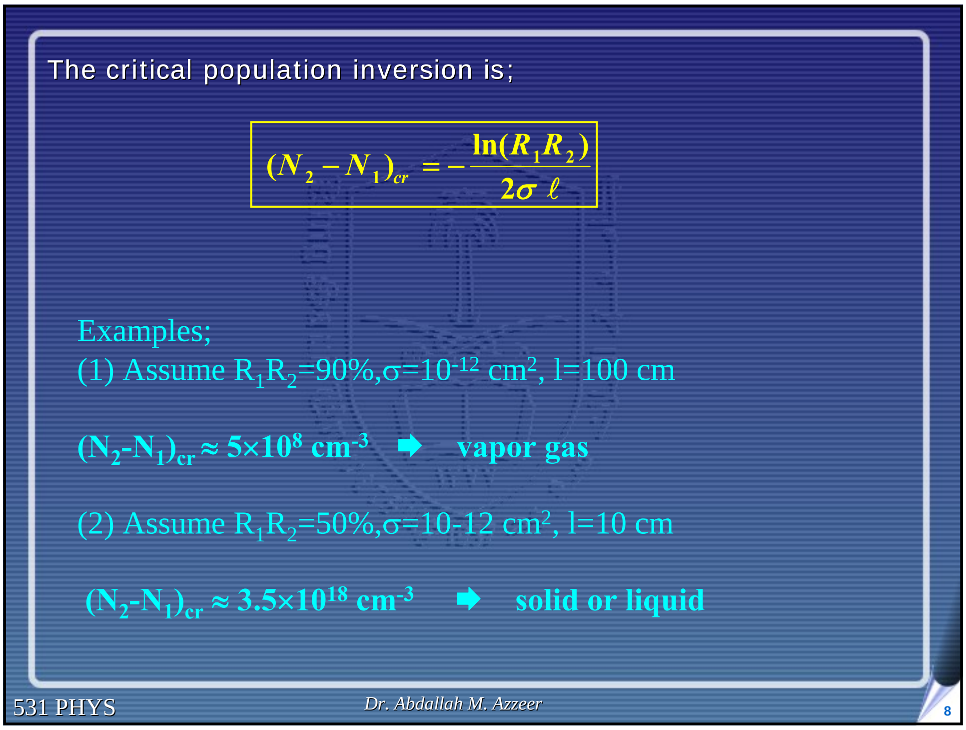
$$\frac{dF}{dz} = \sigma\, F(N_2 - N_1) \quad ➡ \quad F(z) = F(0)e^{\sigma(N_2 - N_1)z}$$

$$\textit{Flux}\ \text{after Round Trip} = F(0)\, e^{\sigma(N_2 - N_1)\ell}\, R_2\, e^{\sigma(N_2 - N_1)\ell}\, R_1$$

The condition for laser oscillation or threshold for oscillation;

THE FLUX AFTER ROUND TRIP=F(0)

$$\boxed{R_1 R_2 \exp(2\sigma(N_2 - N_1)_c\, \ell) = 1}$$



The critical population inversion is;

$$\boxed{(N_2 - N_1)_{cr} = -\frac{\ln(R_1 R_2)}{2\sigma\ \ell}}$$

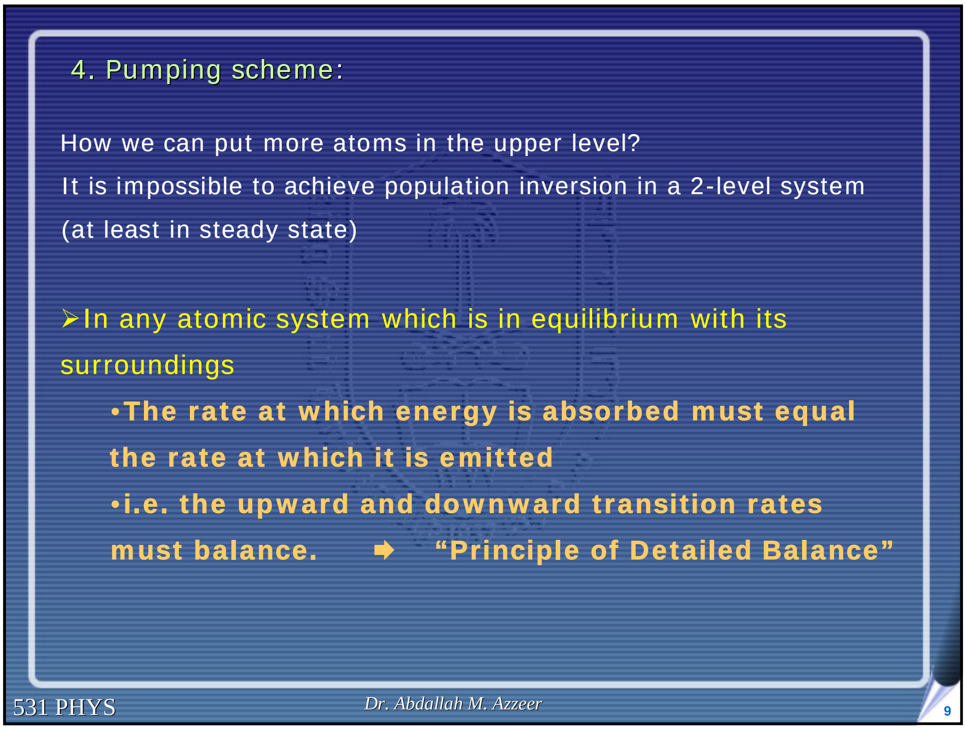
Examples;

(1) Assume R_1R_2=90%,$\sigma=10^{-12}$ cm^2, l=100 cm

$(N_2\text{-}N_1)_{cr} \approx 5\times10^8$ cm^{-3} ➡ vapor gas

(2) Assume R_1R_2=50%,σ=10-12 cm^2, l=10 cm

$(N_2\text{-}N_1)_{cr} \approx 3.5\times10^{18}$ cm^{-3} ➡ solid or liquid



4. Pumping scheme:

How we can put more atoms in the upper level?

It is impossible to achieve population inversion in a 2-level system (at least in steady state)

➢In any atomic system which is in equilibrium with its surroundings

- **The rate at which energy is absorbed must equal the rate at which it is emitted**
- **i.e. the upward and downward transition rates must balance. ➡ "Principle of Detailed Balance"**

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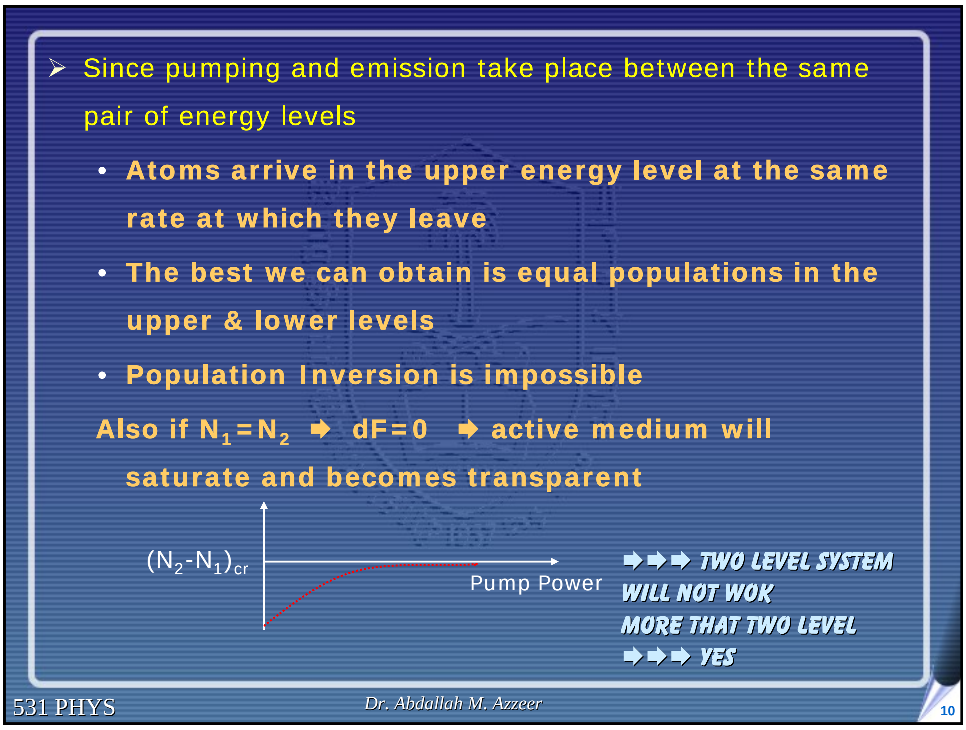

➢ Since pumping and emission take place between the same pair of energy levels

- **Atoms arrive in the upper energy level at the same rate at which they leave**
- **The best we can obtain is equal populations in the upper & lower levels**
- **Population Inversion is impossible**

Also if $N_1=N_2$ ➡ dF=0 ➡ active medium will saturate and becomes transparent

➡➡➡ TWO LEVEL SYSTEM WILL NOT WOK

MORE THAT TWO LEVEL ➡➡➡ YES

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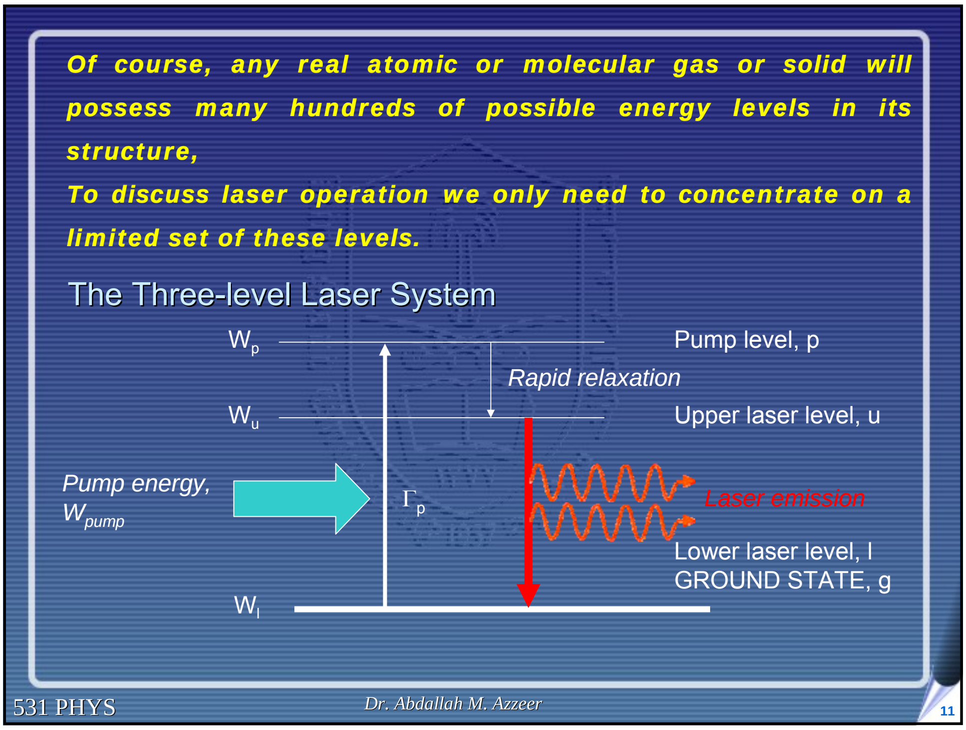


Of course, any real atomic or molecular gas or solid will possess many hundreds of possible energy levels in its structure,

To discuss laser operation we only need to concentrate on a limited set of these levels.

The Three-level Laser System

W_p Pump level, p

Rapid relaxation

W_u Upper laser level, u

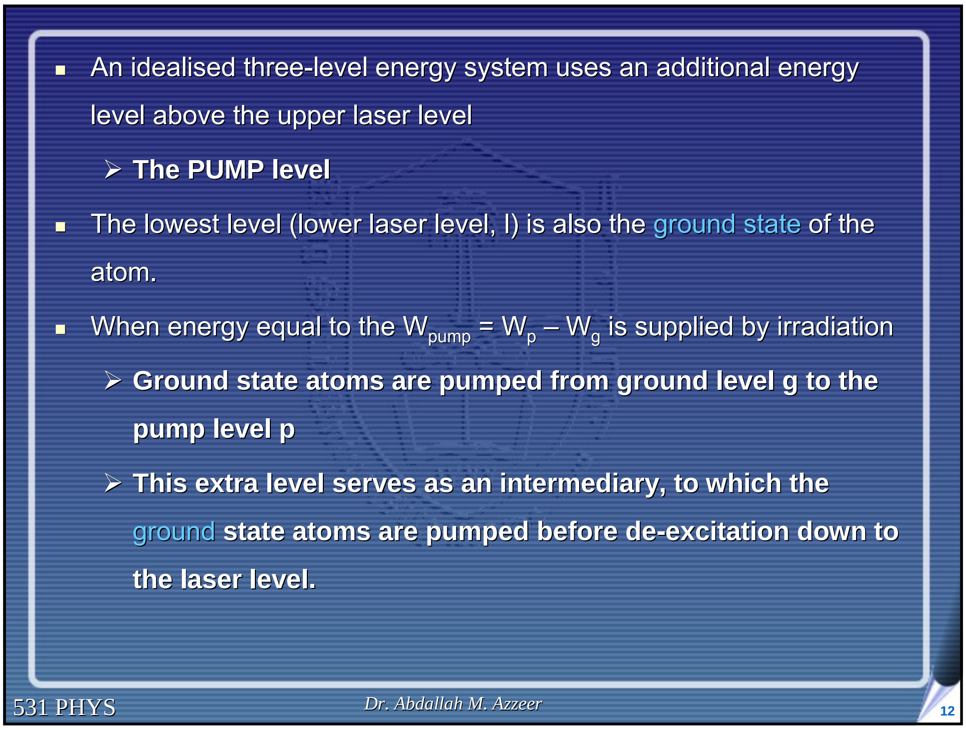
Pump energy, W_{pump}

Γ_p

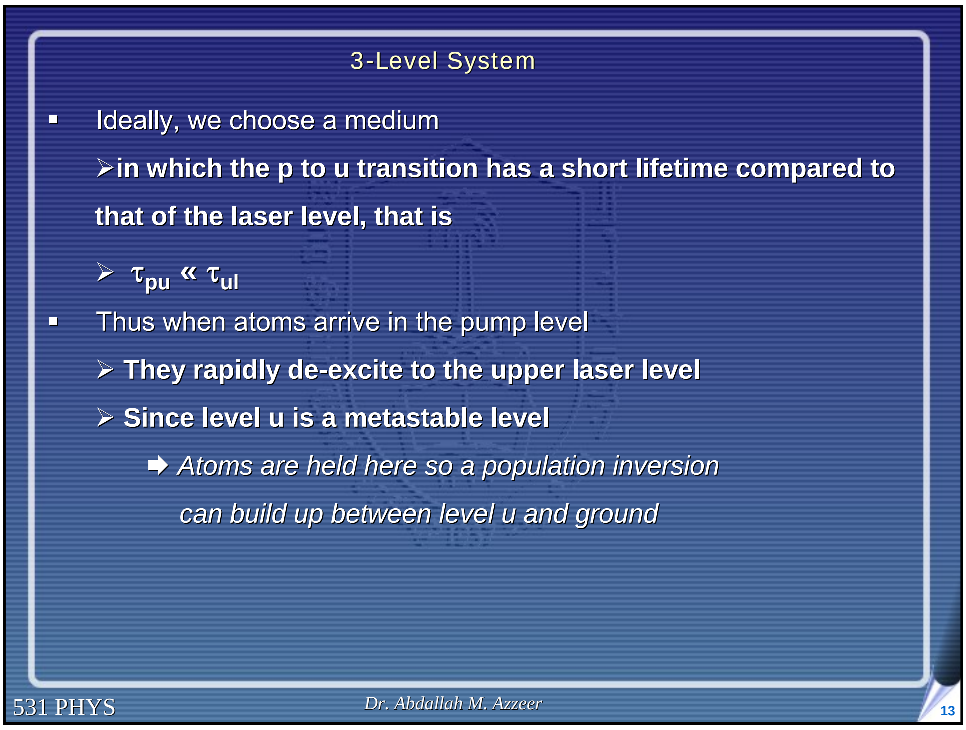
Laser emission

Lower laser level, l
GROUND STATE, g

W_l

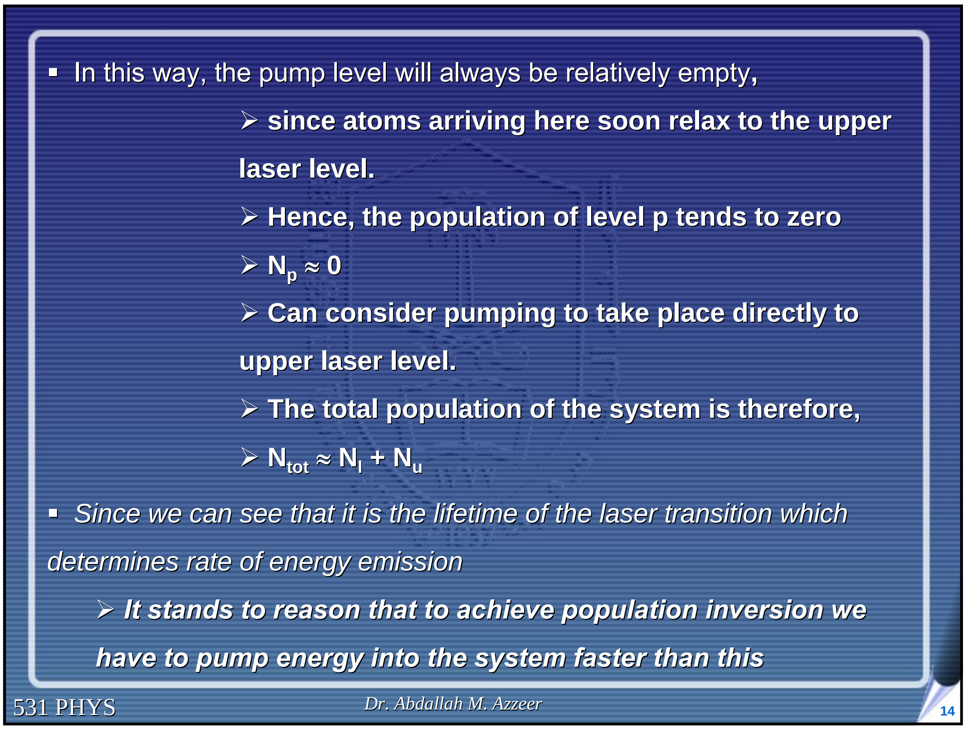


- An idealised three-level energy system uses an additional energy level above the upper laser level
  - ➢ **The PUMP level**
- The lowest level (lower laser level, l) is also the ground state of the atom.
- When energy equal to the $W_{pump} = W_p - W_g$ is supplied by irradiation
  - ➢ **Ground state atoms are pumped from ground level g to the pump level p**
  - ➢ **This extra level serves as an intermediary, to which the** ground **state atoms are pumped before de-excitation down to the laser level.**

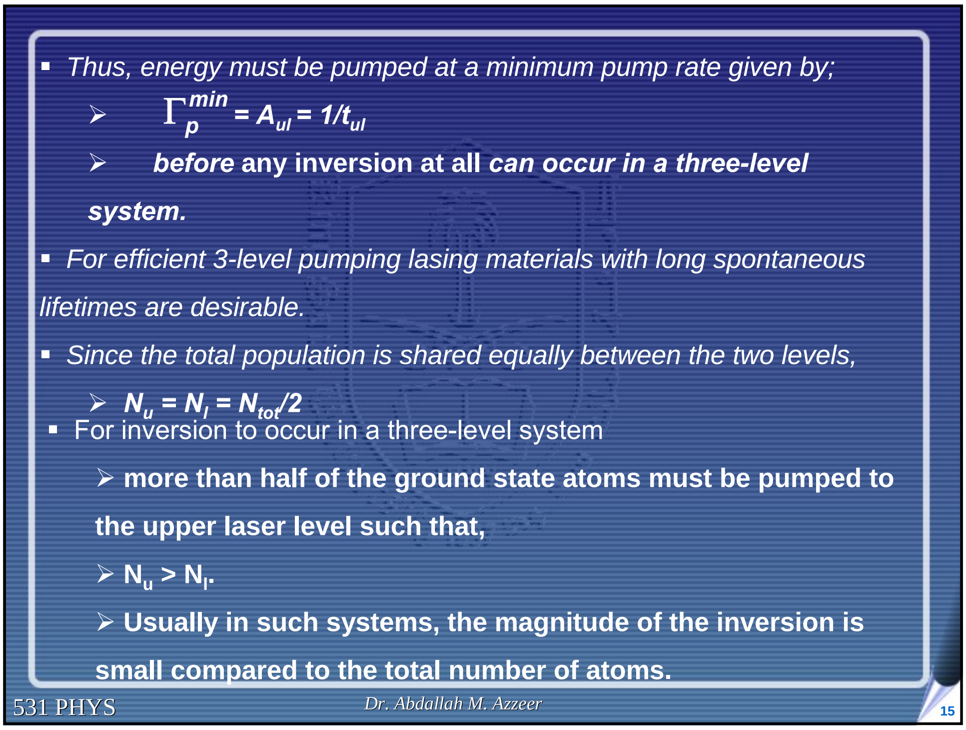


3-Level System

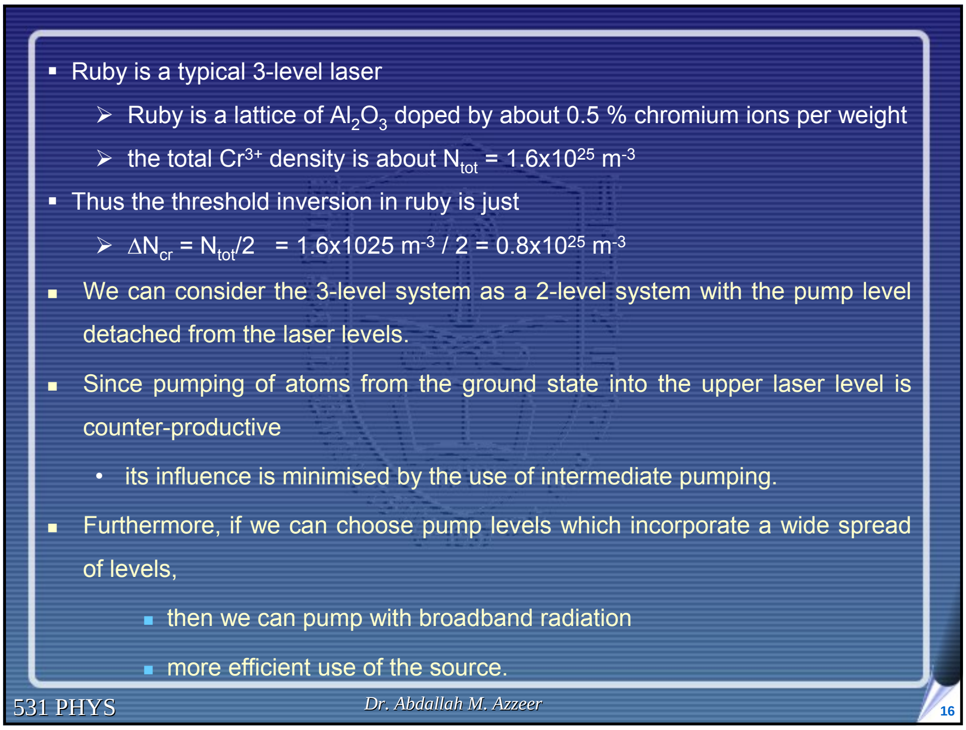
- Ideally, we choose a medium
  - **in which the p to u transition has a short lifetime compared to that of the laser level, that is**
  - $\tau_{pu} \ll \tau_{ul}$
- Thus when atoms arrive in the pump level
  - **They rapidly de-excite to the upper laser level**
  - **Since level u is a metastable level**
    - ➡ *Atoms are held here so a population inversion can build up between level u and ground*



- In this way, the pump level will always be relatively empty,
  - **since atoms arriving here soon relax to the upper laser level.**
  - **Hence, the population of level p tends to zero**
  - $N_p \approx 0$
  - **Can consider pumping to take place directly to upper laser level.**
  - **The total population of the system is therefore,**
  - $N_{tot} \approx N_l + N_u$
- *Since we can see that it is the lifetime of the laser transition which determines rate of energy emission*
  - ***It stands to reason that to achieve population inversion we have to pump energy into the system faster than this***



- *Thus, energy must be pumped at a minimum pump rate given by;*
  - $\Gamma_p^{min} = A_{ul} = 1/t_{ul}$
  - ***before* any inversion at all *can occur in a three-level system.***
- *For efficient 3-level pumping lasing materials with long spontaneous lifetimes are desirable.*
- *Since the total population is shared equally between the two levels,*
  - $N_u = N_l = N_{tot}/2$
- For inversion to occur in a three-level system
  - **more than half of the ground state atoms must be pumped to the upper laser level such that,**
  - $N_u > N_l$.
  - **Usually in such systems, the magnitude of the inversion is small compared to the total number of atoms.**

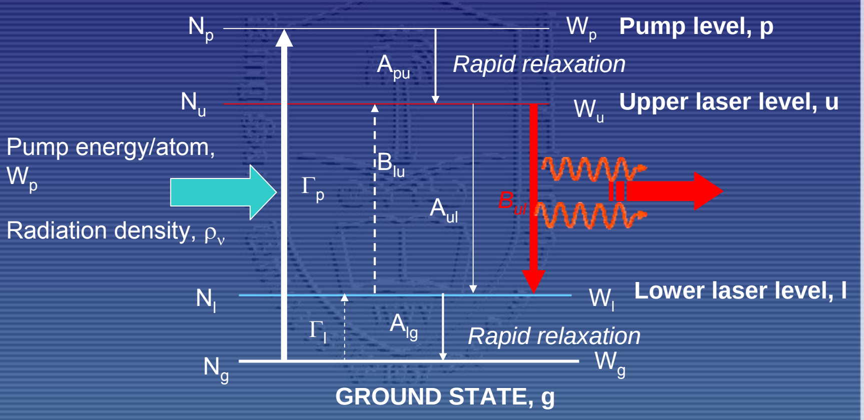


- Ruby is a typical 3-level laser
  - Ruby is a lattice of Al_2O_3 doped by about 0.5 % chromium ions per weight
  - the total Cr^{3+} density is about $N_{tot} = 1.6 \times 10^{25}\ m^{-3}$
- Thus the threshold inversion in ruby is just
  - $\Delta N_{cr} = N_{tot}/2 \ = 1.6x1025\ m^{-3} / 2 = 0.8 \times 10^{25}\ m^{-3}$
- We can consider the 3-level system as a 2-level system with the pump level detached from the laser levels.
- Since pumping of atoms from the ground state into the upper laser level is counter-productive
  - its influence is minimised by the use of intermediate pumping.
- Furthermore, if we can choose pump levels which incorporate a wide spread of levels,
  - then we can pump with broadband radiation
  - more efficient use of the source.

The Four-level Laser System

- Our analysis of the three-level system has shown that although population inversion at a suitable pumping rate
  - **its magnitude is small compared to the total population.**
  - **the pumping rate is still too dependent on material parameters.**
- Clearly further improvement is required for efficient laser operation.
  - **The idealised 4-level atomic energy system differs from the 3-level case in possessing an available level *below* the lower laser level.**
  - **This additional level is usually the ground state or very close to it.**
- The function of this level is to allow rapid depopulation of the lower laser level such that it is effectively empty at all times.
- In this way, a significant population inversion can be sustained between the upper & lower laser levels.

4-Level Transitions



N_p W_p **Pump level, p**

A_{pu} *Rapid relaxation*

N_u W_u **Upper laser level, u**

Pump energy/atom, W_p

Radiation density, ρ_ν

Γ_p B_{lu} A_{ul} B_{ul}

N_l W_l **Lower laser level, l**

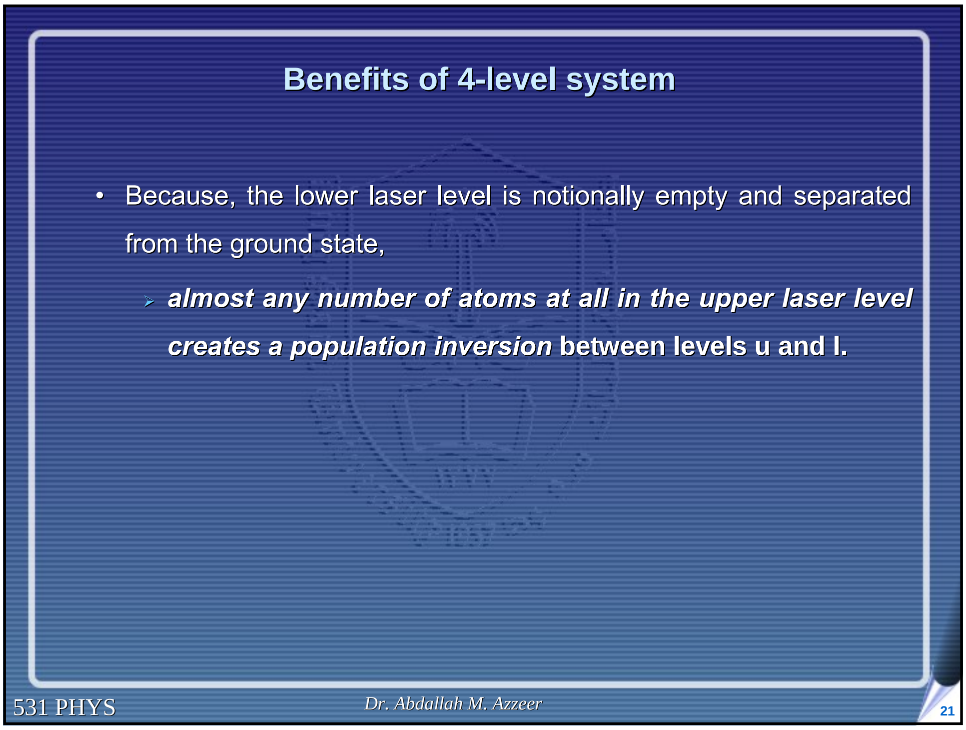
Γ_l A_{lg} *Rapid relaxation*

N_g W_g

GROUND STATE, g

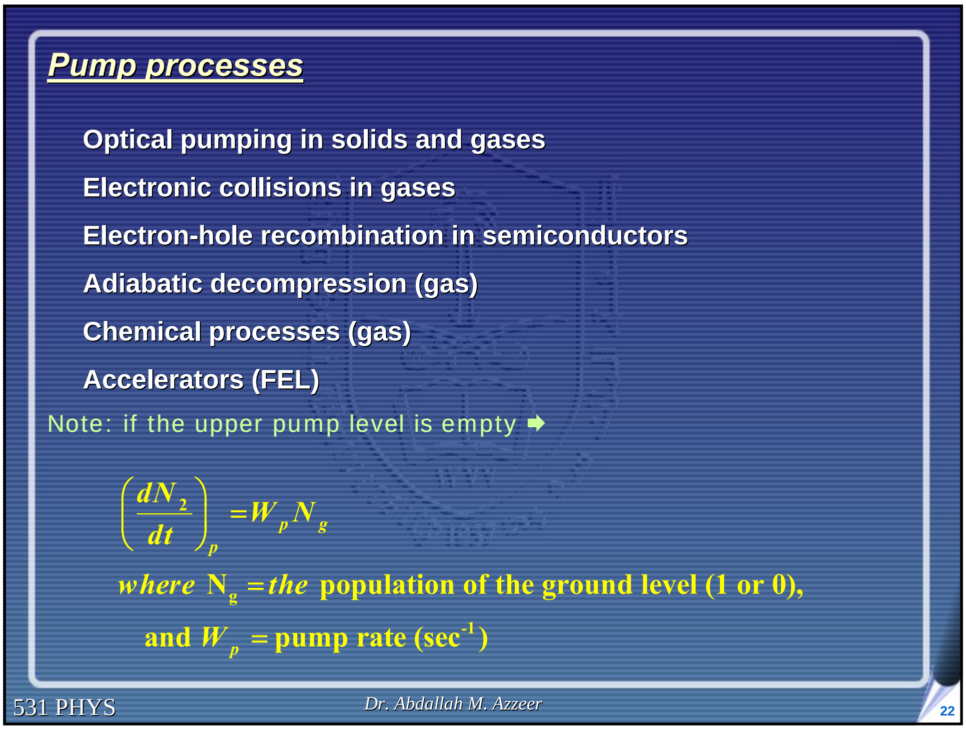
- We choose an atomic medium where the lifetime of the lower laser level is very much shorter than that of the upper laser level.
- Population of the upper laser level u is effectively achieved by
  - **pumping of atoms from the ground level g to the pump level p at a rate $\Gamma_p N_g$ atoms per unit volume per unit time**
  - **followed by rapid relaxation of atoms from pump level to level u.**
- Both stimulated emission at a rate $\rho_\nu B_{ul} N_u$ and spontaneous emission at a rate $A_{ul} N_u$
  - **can take place between the levels u and the lower laser level l,**
  - **whereupon atoms in level l relax to the ground g at a rate $A_{lg} N_l$.**
- Also, parasitic effects can put atoms into unwanted transitions
  - **atoms in level l can be excited into level u at a rate $\rho\nu B_{lu} N_l$**
  - **atoms in ground g can be excited to lower laser level at a rate $\Gamma_l N_g$.**
    - **Want these to be small**

- Thus for the spontaneous lifetimes of the levels, we need
  - **$t_{pu} \ll t_{ul}$ and $t_{lg} \ll t_{ul}$**
  - **which in turn implies that,**
  - **$N_p \approx 0$ and $N_l \approx 0$**
  - **Thus the pump & lower laser levels are effectively empty**
- The total population is thus shared amongst the two remaining levels, thus
  - **$N_{tot} \approx N_g + N_u$**
  - **and, the resulting population inversion is given by**
  - **$\Delta N_{cr} = N_u - N_l \approx N_u$**
- **Furthermore, pumping of atoms from level g to level l is parasitic**
  - **since it effectively pushes atoms into a level we need empty.**
  - **we assume the rate, $G_l N_g$, at which atoms are pumped to level l from level g so small as to be negligible.**
- **Pumping of atoms from l to u is also parasitic**
  - **But since N_l is zero so is $\rho\nu B_{lu} N_l$**



Benefits of 4-level system

- Because, the lower laser level is notionally empty and separated from the ground state,
  - ***almost any number of atoms at all in the upper laser level creates a population inversion*** **between levels u and l.**



Pump processes

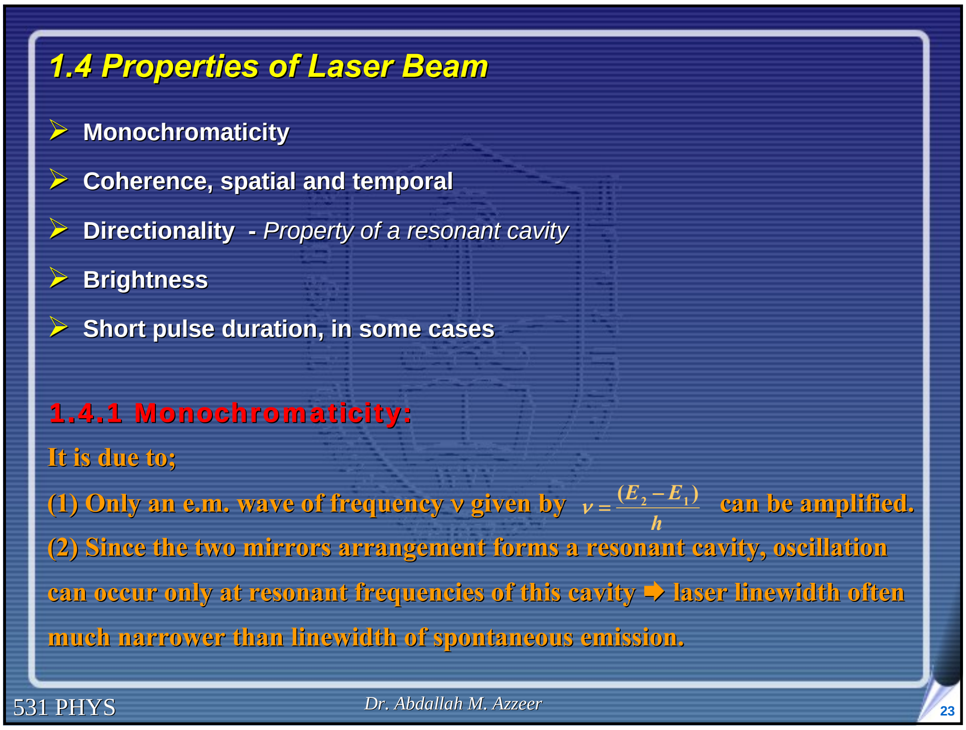
Optical pumping in solids and gases
Electronic collisions in gases
Electron-hole recombination in semiconductors
Adiabatic decompression (gas)
Chemical processes (gas)
Accelerators (FEL)

Note: if the upper pump level is empty ➡

$$\left(\frac{dN_2}{dt}\right)_p = W_p N_g$$

where N_g = the population of the ground level (1 or 0),

and W_p = pump rate ($\sec^{-1}$)



1.4 Properties of Laser Beam

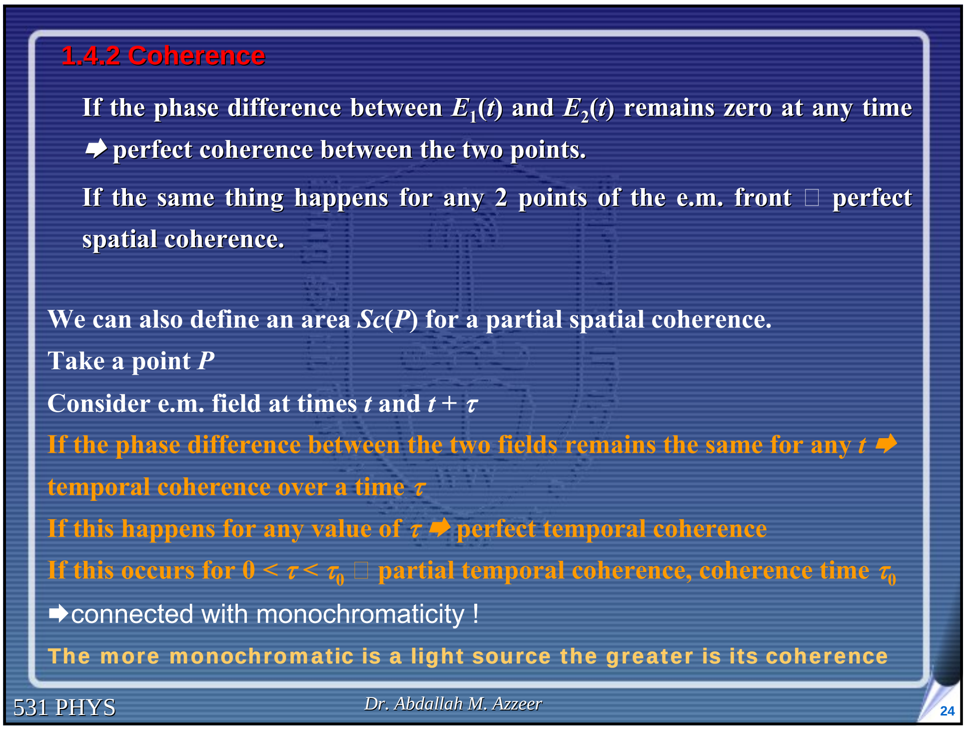
- Monochromaticity
- Coherence, spatial and temporal
- Directionality - *Property of a resonant cavity*
- Brightness
- Short pulse duration, in some cases

1.4.1 Monochromaticity:

It is due to;

(1) Only an e.m. wave of frequency ν given by $\nu = \frac{(E_2 - E_1)}{h}$ can be amplified.

(2) Since the two mirrors arrangement forms a resonant cavity, oscillation can occur only at resonant frequencies of this cavity ➡ laser linewidth often much narrower than linewidth of spontaneous emission.



1.4.2 Coherence

If the phase difference between $E_1(t)$ and $E_2(t)$ remains zero at any time ➡ perfect coherence between the two points.

If the same thing happens for any 2 points of the e.m. front □ perfect spatial coherence.

We can also define an area $Sc(P)$ for a partial spatial coherence.

Take a point P

Consider e.m. field at times t and $t + \tau$

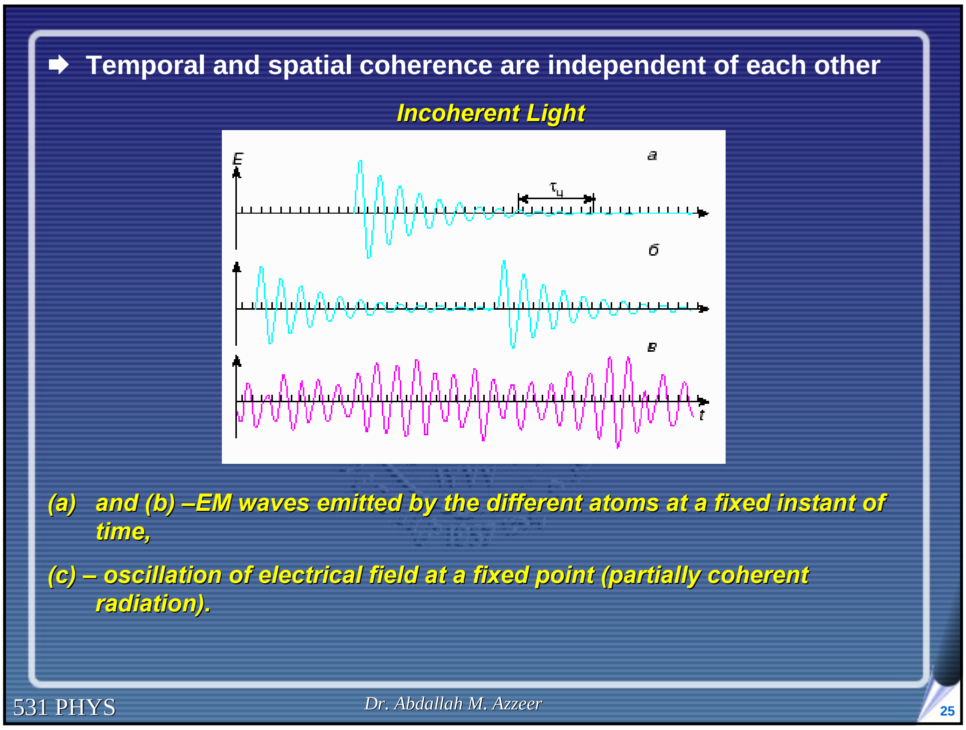
If the phase difference between the two fields remains the same for any t ➡ temporal coherence over a time τ

If this happens for any value of τ ➡ perfect temporal coherence

If this occurs for $0 < \tau < \tau_0$ □ partial temporal coherence, coherence time τ_0

➡connected with monochromaticity !

The more monochromatic is a light source the greater is its coherence

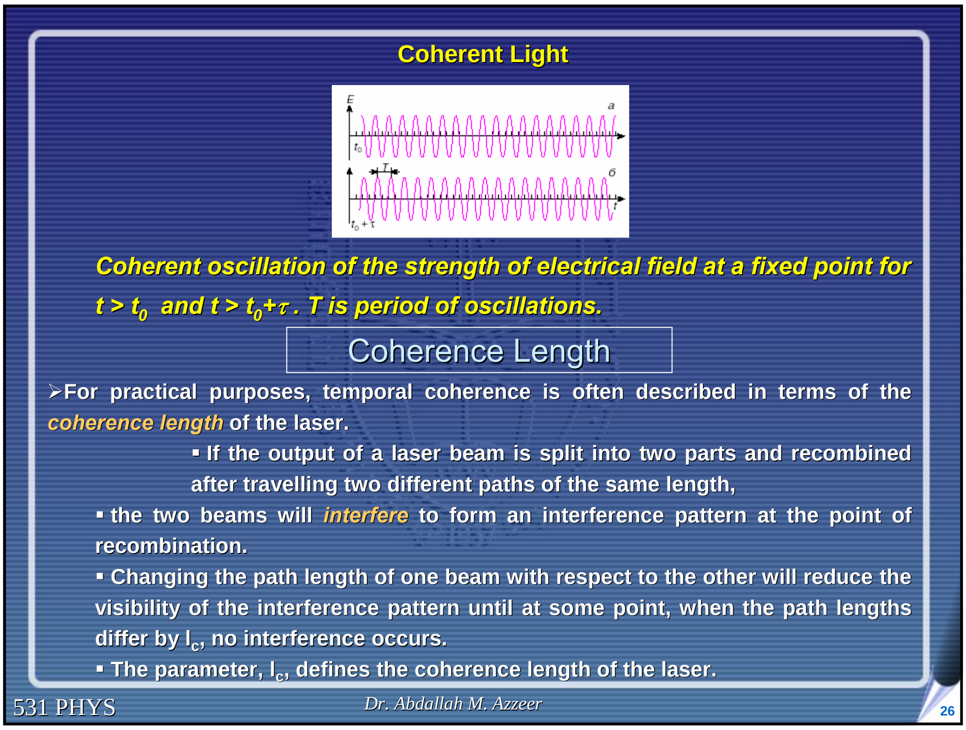


➡ Temporal and spatial coherence are independent of each other

Incoherent Light

(a) and (b) –EM waves emitted by the different atoms at a fixed instant of time,

(c) – oscillation of electrical field at a fixed point (partially coherent radiation).



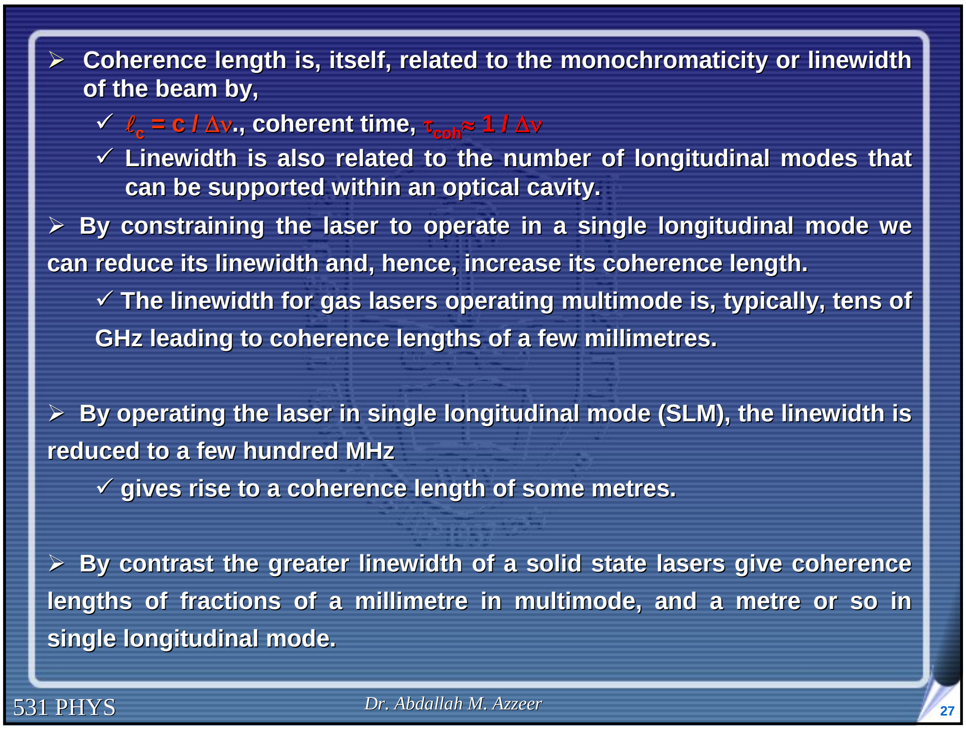
Coherent Light

Coherent oscillation of the strength of electrical field at a fixed point for $t > t_0$ and $t > t_0+\tau$. T is period of oscillations.

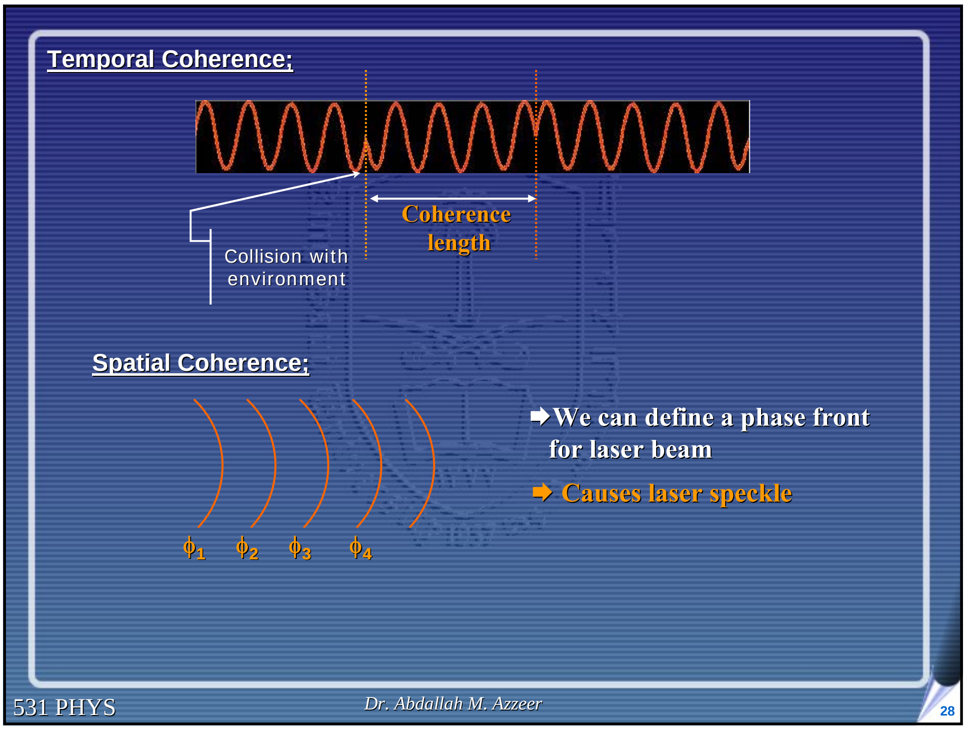
Coherence Length

➢For practical purposes, temporal coherence is often described in terms of the *coherence length* of the laser.

- If the output of a laser beam is split into two parts and recombined after travelling two different paths of the same length,
- the two beams will *interfere* to form an interference pattern at the point of recombination.
- Changing the path length of one beam with respect to the other will reduce the visibility of the interference pattern until at some point, when the path lengths differ by l_c, no interference occurs.
- The parameter, l_c, defines the coherence length of the laser.



- Coherence length is, itself, related to the monochromaticity or linewidth of the beam by,
  - ✓ $\ell_c = c / \Delta\nu$., coherent time, $\tau_{coh} \approx 1 / \Delta\nu$
  - ✓ Linewidth is also related to the number of longitudinal modes that can be supported within an optical cavity.
- By constraining the laser to operate in a single longitudinal mode we can reduce its linewidth and, hence, increase its coherence length.
  - ✓ The linewidth for gas lasers operating multimode is, typically, tens of GHz leading to coherence lengths of a few millimetres.
- By operating the laser in single longitudinal mode (SLM), the linewidth is reduced to a few hundred MHz
  - ✓ gives rise to a coherence length of some metres.
- By contrast the greater linewidth of a solid state lasers give coherence lengths of fractions of a millimetre in multimode, and a metre or so in single longitudinal mode.



Temporal Coherence;

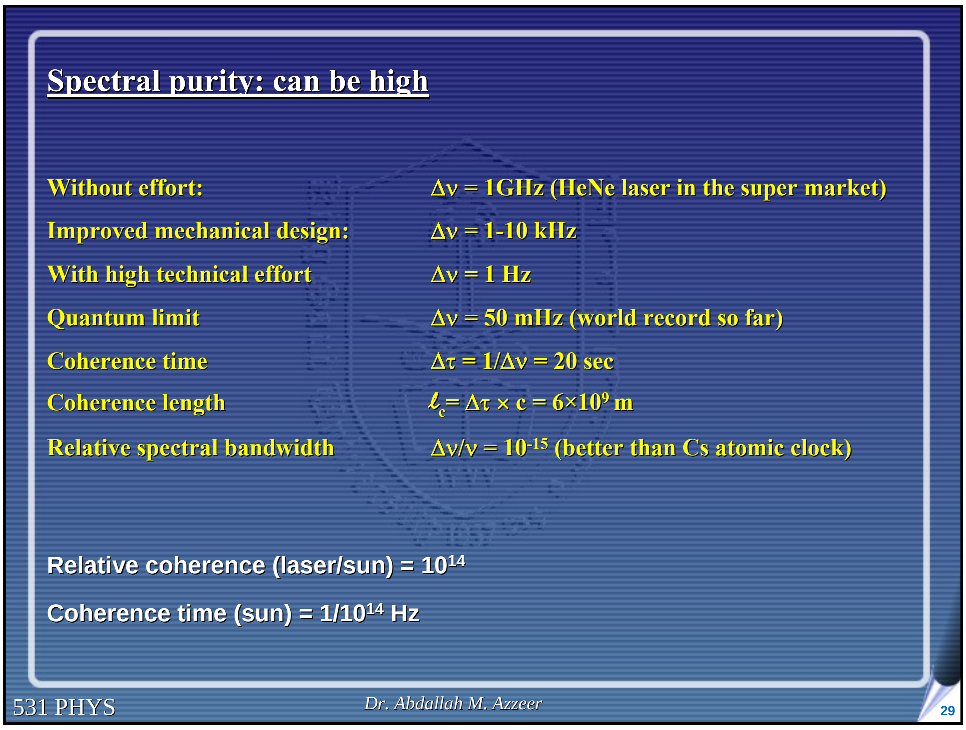
Coherence length

Collision with environment

Spatial Coherence;

➡ We can define a phase front for laser beam

➡ Causes laser speckle

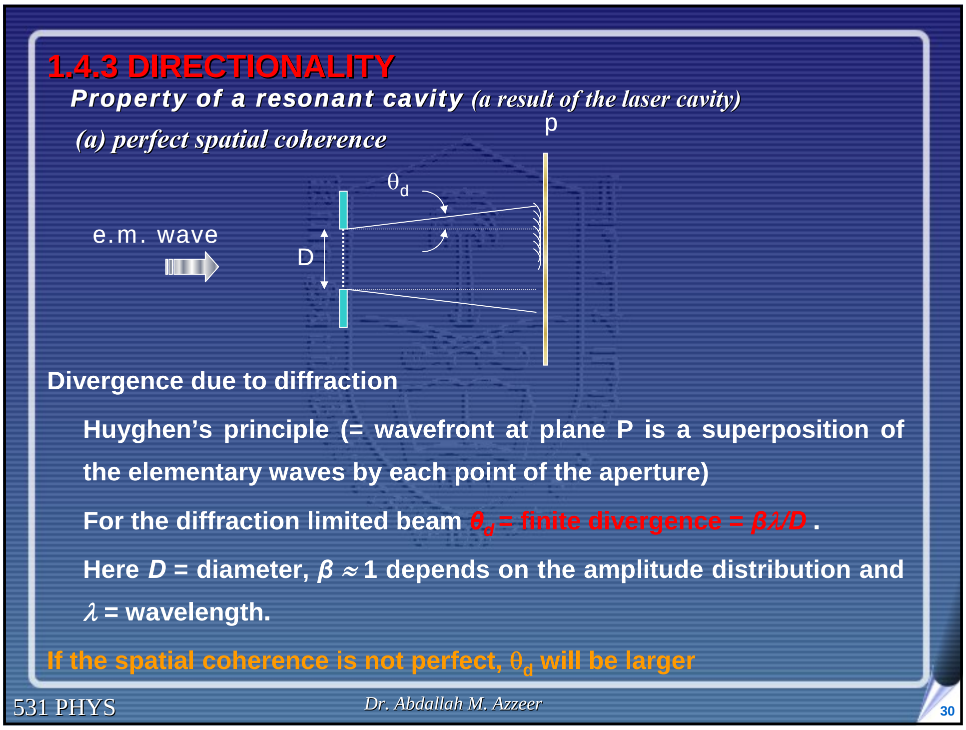


Spectral purity: can be high

Without effort:	$\Delta\nu$ = 1GHz (HeNe laser in the super market)
Improved mechanical design:	$\Delta\nu$ = 1-10 kHz
With high technical effort	$\Delta\nu$ = 1 Hz
Quantum limit	$\Delta\nu$ = 50 mHz (world record so far)
Coherence time	$\Delta\tau = 1/\Delta\nu$ = 20 sec
Coherence length	$\ell_c = \Delta\tau \times c = 6\times10^9$ m
Relative spectral bandwidth	$\Delta\nu/\nu = 10^{-15}$ (better than Cs atomic clock)

Relative coherence (laser/sun) = 10^{14}

Coherence time (sun) = $1/10^{14}$ Hz



1.4.3 DIRECTIONALITY

***Property of a resonant cavity** (a result of the laser cavity)*

(a) perfect spatial coherence

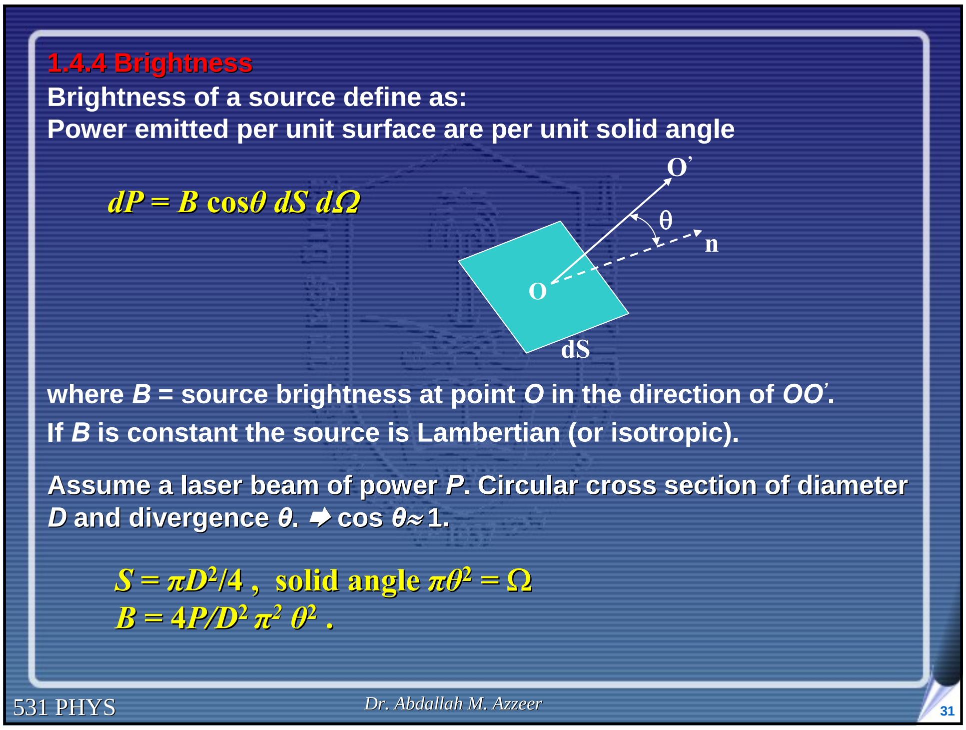
Divergence due to diffraction

Huyghen's principle (= wavefront at plane P is a superposition of the elementary waves by each point of the aperture)

For the diffraction limited beam θ_d = finite divergence = $\beta\lambda/D$.

Here D = diameter, $\beta \approx 1$ depends on the amplitude distribution and λ = wavelength.

If the spatial coherence is not perfect, θ_d will be larger



1.4.4 Brightness

Brightness of a source define as:
Power emitted per unit surface are per unit solid angle

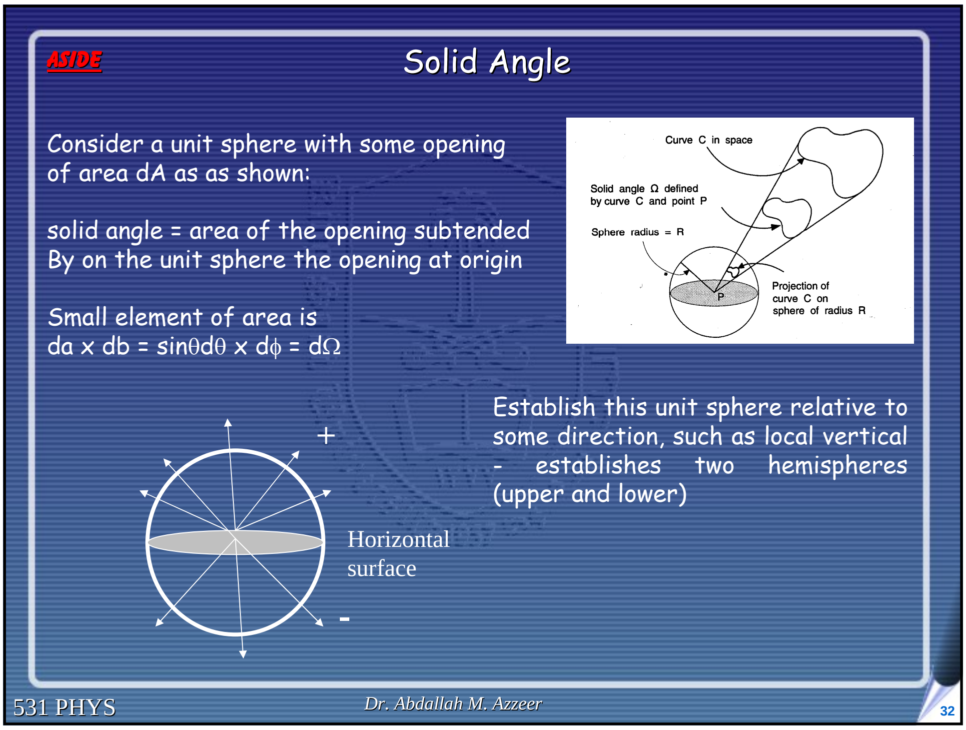
$$dP = B\cos\theta\, dS\, d\Omega$$

where *B* = source brightness at point *O* in the direction of *OO′*.

If *B* is constant the source is Lambertian (or isotropic).

Assume a laser beam of power *P*. Circular cross section of diameter *D* and divergence *θ*. ➡ cos *θ*≈1.

$$S = \pi D^2/4\ ,\ \text{solid angle } \pi\theta^2 = \Omega$$
$$B = 4P/D^2\,\pi^2\,\theta^2\ .$$



ASIDE

Solid Angle

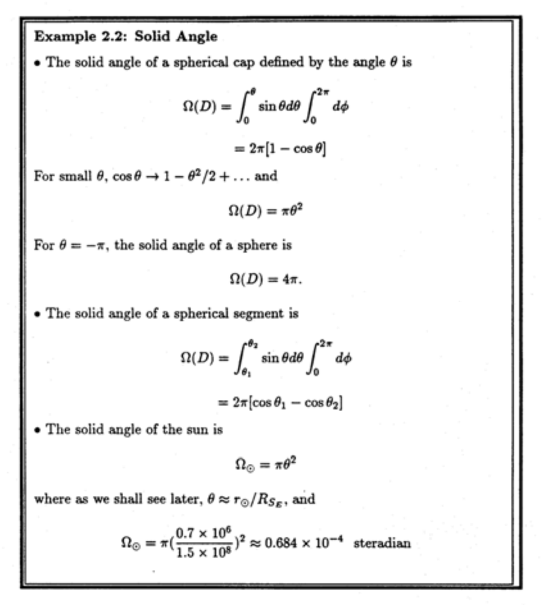
Consider a unit sphere with some opening of area dA as as shown:

solid angle = area of the opening subtended By on the unit sphere the opening at origin

Small element of area is
da x db = sinθdθ x dϕ = dΩ

Establish this unit sphere relative to some direction, such as local vertical - establishes two hemispheres (upper and lower)

Solid Angle Examples



Example 2.2: Solid Angle

- The solid angle of a spherical cap defined by the angle θ is

$$\Omega(D) = \int_0^{\theta} \sin\theta d\theta \int_0^{2\pi} d\phi$$

$$= 2\pi[1 - \cos\theta]$$

For small θ, $\cos\theta \rightarrow 1 - \theta^2/2 + \ldots$ and

$$\Omega(D) = \pi\theta^2$$

For $\theta = -\pi$, the solid angle of a sphere is

$$\Omega(D) = 4\pi.$$

- The solid angle of a spherical segment is

$$\Omega(D) = \int_{\theta_1}^{\theta_2} \sin\theta d\theta \int_0^{2\pi} d\phi$$

$$= 2\pi[\cos\theta_1 - \cos\theta_2]$$

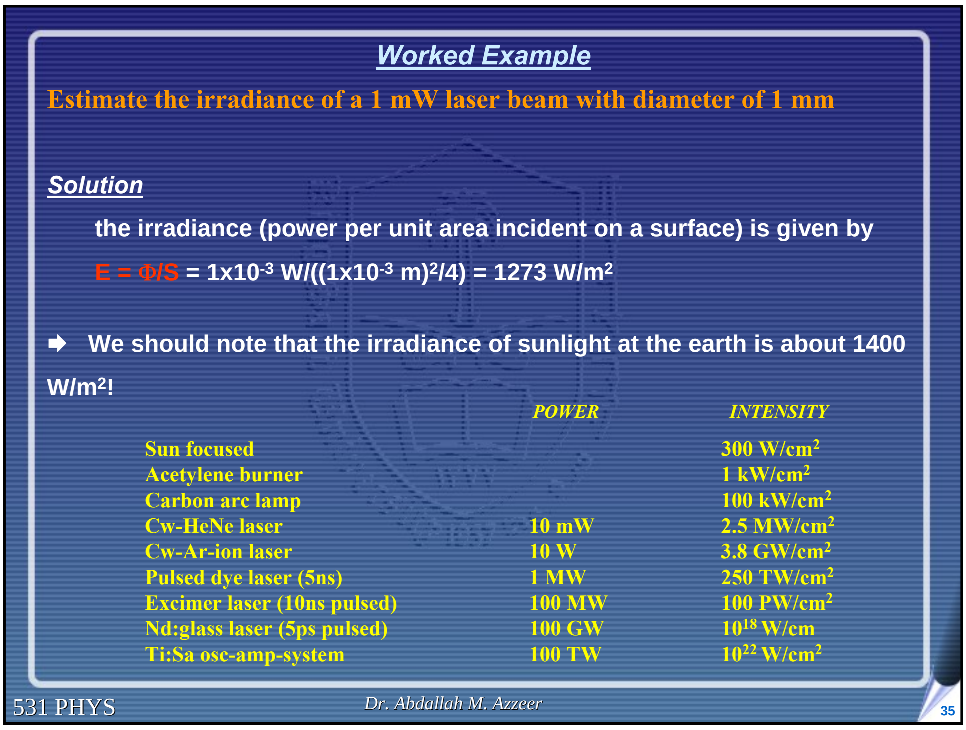
- The solid angle of the sun is

$$\Omega_{\odot} = \pi\theta^2$$

where as we shall see later, $\theta \approx r_{\odot}/R_{S_E}$, and

$$\Omega_{\odot} = \pi(\frac{0.7 \times 10^6}{1.5 \times 10^8})^2 \approx 0.684 \times 10^{-4} \text{ steradian}$$

- We should by now appreciate that lasers are amplifiers of light.
- One of their great virtues is that they can deliver high radiant power whether this is in pulsed or continuous mode.
  - Optical power is often called (*Radiant*) *Flux*.
  - Pulsed powers range from a few watts produced by semiconductor lasers to around 10^{18} W delivered by the solid state lasers used in laser fusion systems.
  - Continuous optical powers range from a few milliwatts in He-Ne lasers to several kilowatts delivered by CO_2 lasers.
- Because laser light is usually concentrated into a narrow beam (a few millimetres)
  - light possesses high radiant power (flux) per unit area
  - i.e. high irradiance



Worked Example

Estimate the irradiance of a 1 mW laser beam with diameter of 1 mm

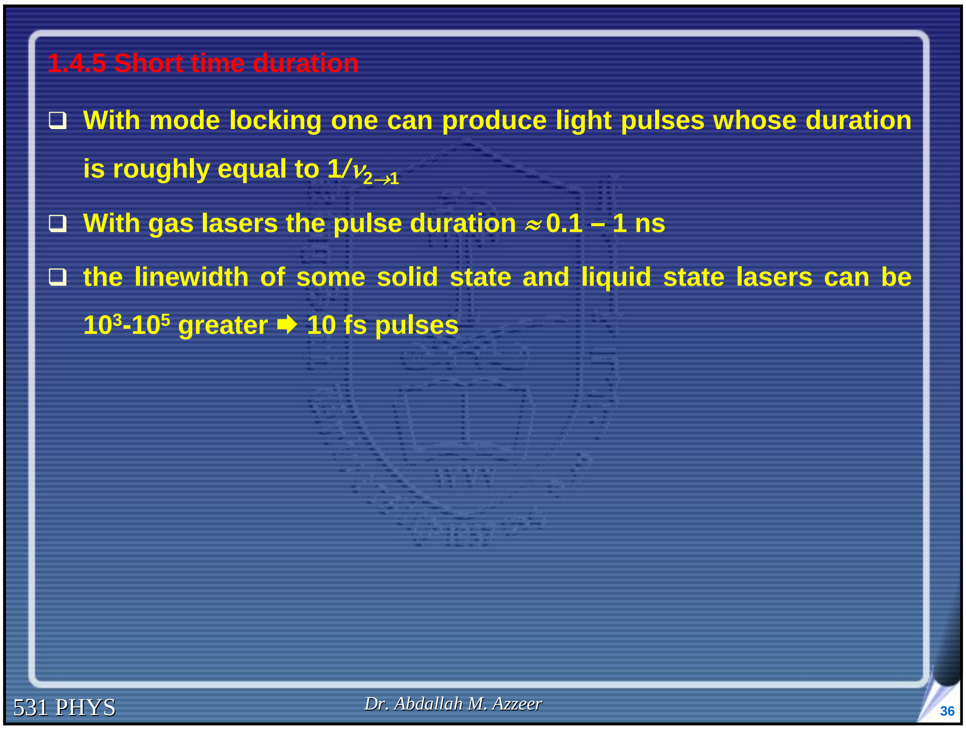
Solution

the irradiance (power per unit area incident on a surface) is given by

$E = \Phi/S = 1x10^{-3}\ W/((1x10^{-3}\ m)^2/4) = 1273\ W/m^2$

➨ We should note that the irradiance of sunlight at the earth is about 1400 W/m^2!

	POWER	*INTENSITY*
Sun focused		300 W/cm^2
Acetylene burner		1 kW/cm^2
Carbon arc lamp		100 kW/cm^2
Cw-HeNe laser	10 mW	2.5 MW/cm^2
Cw-Ar-ion laser	10 W	3.8 GW/cm^2
Pulsed dye laser (5ns)	1 MW	250 TW/cm^2
Excimer laser (10ns pulsed)	100 MW	100 PW/cm^2
Nd:glass laser (5ps pulsed)	100 GW	10^{18} W/cm
Ti:Sa osc-amp-system	100 TW	10^{22} W/cm^2



1.4.5 Short time duration

- With mode locking one can produce light pulses whose duration is roughly equal to $1/\nu_{2\rightarrow1}$
- With gas lasers the pulse duration $\approx$ 0.1 – 1 ns
- the linewidth of some solid state and liquid state lasers can be 10^3-10^5 greater ➨ 10 fs pulses

WHAT IS UNIQUE ABOUT LASERS?

It is a source of coherent, super-high-frequency EM radiation which can be concentrated in:

- spectral interval $\delta\omega/\omega \sim 10^{-15}$ (monochromatic);
- solid angle $\delta\Omega$ (directed radiation) $\delta\Omega \sim (\lambda/D)^2 \sim 10^{-15}$;
- spot $A \sim (\lambda)^2$ and volume $V \sim (\lambda)^3 \sim 10^{-15}$ cm^3 (focusing);
- time interval $\tau \sim 2\pi/\omega \sim 10^{-15}$ s
- $E \propto (I)^{1/2} = (\Delta Q/\tau A)^{1/2}$

Laser Types:

- Gas laser (e.g. He-Ne, Ar+, CO_2, N_2, …), Liquid (Organic Dye), Solid-State laser (e.g. Ruby, Nd:YAG, Nd:Glass, Alexandrite, Ti:Sapphire,..), Fiber (a special case of solid-state lasers), Semiconductor laser (e.g. AlGaAs, AlGaInP, InGaN, InGaAsP, VCSEL,…), Chemical (HF) laser, Excimer laser, Free-Electron laser, X-Ray laser.
- X-Ray (λ ~1 *nm*) to far infrared (λ ~1 *mm*)
- CW power → ~ 1-mW (communications, data storage, laser pointers)
  to ~ 100-kW (machining)
  to ~ 5MW (military)
- Pulsed Power → to ~ 1015 W
- Pulse Length → as short as ~ 5-fsec
- Cavity Length → ~ 1µm (VCSEL) to 6.5-km