Optical Gain and Population Inversion

CONTENTS:

- Absorption, Amplification, Loss and Gain
- Spatial Gain Coefficient
- Population Inversion

Terminology: Attenuation, Amplification, Loss and Gain

"Attenuation" processes *decrease* light intensity:

- induced absorption
- light scattering
- cavity loss (light emission from cavity)

"Amplification" processes *increase* light intensity:

- stimulated emission

Extrinsic processes:

- light scattering
  - loss coefficient α_i
  - bad!
- cavity loss (through mirrors)
  - loss coefficient α_m
  - essential!

Intrinsic radiative processes:

- (stimulated emission – induced absorption)
  - gain coefficient γ

Calculation of Optical Gain in 2-level System

- We know that *stimulated emission* gives rise to *amplification* of light ("LASER"). So how does the irradiance change with propagation (along a direction z) through an amplifying material?
- We will calculate the irradiance of light propagating in a two-level system as a result of stimulated emission and induced absorption
- We will *neglect* spontaneous emission (which radiates in all directions and doesn't enhance gain, and is usually small compared to stimulated emission under lasing conditions)

Rate equation for photon density $\phi(\nu)$:

one photon created/destroyed per radiative electronic transition

Hence the rate equation for photon density is:

$$\frac{d\phi(\nu)}{dt} = N_2\rho(\nu)B_{21} - N_1\rho(\nu)B_{12}$$

Dr. Abdallah M.Azzeer 3

Calculation of Optical Gain (2)

Starting from the rate equation:

$$\frac{d\phi(\nu)}{dt} = N_2\rho(\nu)B_{21} - N_1\rho(\nu)B_{12}$$

Using Einstein's relations:

$$\frac{d\phi(\nu)}{dt} = \left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{c^3 A_{21}}{8\pi h\nu^3}\rho(\nu)$$

Converting to intensity:

$$\phi(\nu) = \rho(\nu)/h\nu = I(\nu)/(h\nu\cdot c)$$

Converting to spatial coordinates:

$$\frac{d\phi(\nu)}{dt} = \frac{d\phi(\nu)}{dz}\cdot\frac{dz}{dt} = \frac{1}{h\nu\cdot c}\frac{dI(\nu)}{dz}\cdot c = \frac{1}{h\nu}\frac{dI(\nu)}{dz}$$

Substituting into the rate equation, and writing $I(\nu,z)$ because we are calculating how $I(\nu)$ changes on propagation along z, we have:

$$\frac{dI(\nu,z)}{dz} = \left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{c^2 A_{21}}{8\pi\nu^2}I(\nu,z)$$

Dr. Abdallah M.Azzeer 4

Calculation of Optical Gain (3)

$$\frac{dI(\nu,z)}{dz}=\left(N_2-\frac{g_2}{g_1}N_1\right)\frac{c^2A_{21}}{8\pi\nu^2}I(\nu,z)$$

has the form:

$$\frac{dI(\nu,z)}{dz}=\gamma(\nu)\cdot I(\nu,z)$$

where γ is the ***spatial gain coefficient***

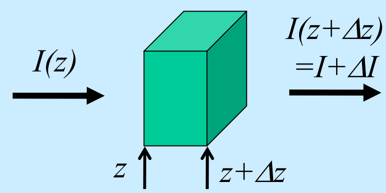
The solution is:

$$I(\nu,z)=I_0(\nu)\exp[\gamma(\nu)z]$$

i.e. the irradiance increases *exponentially* with propagation distance.

Spatial gain coefficient γ:

Uniform slice with constant amplification:



Fractional change in intensity scales with thickness

$$\frac{\Delta I}{I}=+\gamma\,\Delta z$$

$$\frac{dI}{dz}=+\gamma\,I$$

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Optical Gain and Population Inversion

Gain coefficient:

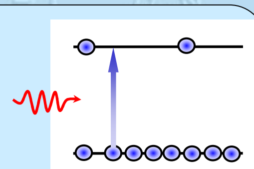
$$\gamma(\nu)=\left(N_2-\frac{g_2}{g_1}N_1\right)\left(\frac{c^2A_{21}}{8\pi\nu^2}\right)$$

"Population inversion"

Normal Population

$g_2N_1 > g_1N_2$

induced absorption
> stimulated emission

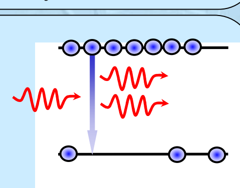


exponential decay of intensity

Inverted Population

$g_2N_1 < g_1N_2$

induced absorption
< stimulated emission



exponential growth of intensity

"Stimulated emission cross-section parameter"

$$\sigma_0=\frac{c^2A_{21}}{8\pi\nu^2}=\frac{c^2}{8\pi\nu^2\tau_{sp}}\propto\frac{1}{\nu^2}$$

UV lasers have smaller gain than IR lasers

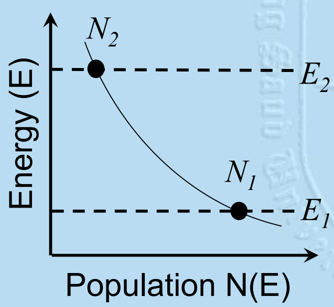
- masers easier than lasers...
- X-ray lasers hard ...

Dr. Abdallah M.Azzeer 6

Population inversion and pumping

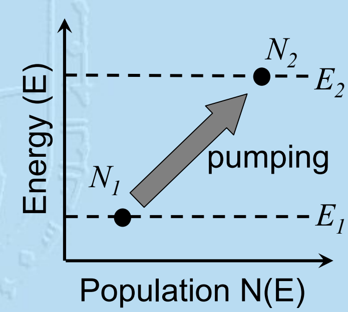
In thermal equilibrium:

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{kT}\right)$$



At all temperatures, $N_2 < N_1$
Hence no population inversion and no net gain

In non-equilibrium:



Can achieve $N_2 > N_1$
and hence population inversion by "*pumping*"

Dr. Abdallah M.Azzeer 7

What U need 2 know: Optical Gain

Definition of:
rate equation for stimulated photons

$$\frac{d\phi(\nu)}{dt} = N_2\rho(\nu)B_{21} - N_1\rho(\nu)B_{12}$$

optical gain coefficient γ

$$\frac{dI(\nu,z)}{dz} = \gamma(\nu)\cdot I(\nu,z)$$

Derivation of:
gain coefficient γ

$$\gamma(\nu) = \left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{c^2 A_{21}}{8\pi\nu^2}$$

Definition of:
Normal population

$$\left(N_2 - \frac{g_2}{g_1}N_1\right) < 0$$

induced absorption > stimulated emission

Population inversion

$$\left(N_2 - \frac{g_2}{g_1}N_1\right) > 0$$

induced absorption < stimulated emission

Definition of:
intrinsic loss coefficient α_i
cavity loss coefficient α_m

Dr. Abdallah M.Azzeer 8

What U need 2 know: Optical Gain

Definition of:
rate equation for stimulated photons

$$\frac{d\phi(\nu)}{dt} = N_2\rho(\nu)B_{21} - N_1\rho(\nu)B_{12}$$

optical gain coefficient γ
(in absence of optical loss)

$$\frac{dI(\nu,z)}{dz} = \gamma(\nu)\cdot I(\nu,z)$$

Derivation of:
gain coefficient γ

$$\gamma(\nu) = \left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{c^2 A_{21}}{8\pi\nu^2}$$

Definition of:
Normal population

$$\left(N_2 - \frac{g_2}{g_1}N_1\right) < 0$$

induced absorption > stimulated emission

Population inversion

$$\left(N_2 - \frac{g_2}{g_1}N_1\right) > 0$$

induced absorption < stimulated emission

Definition of:
intrinsic loss coefficient α_i
cavity loss coefficient α_m

$$\frac{dI(z)}{dz} = \{\gamma - \alpha_i - \alpha_m\}\cdot I(z)$$

Dr. Abdallah M.Azzeer 9