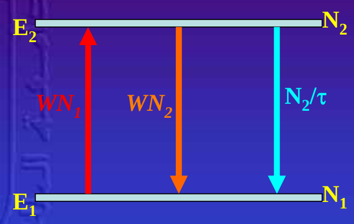
SATURATION

When a two level system is excited by radiation, the absorption coefficient actually decreases as radiation intensity increases. Likewise, the gain for an inverted system also show saturation.

Consider a two-level system



$$N_t = N_1 + N_2$$

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt}$$

$$\frac{dN_2}{dt} = -W(N_2 - N_1) - \frac{N_2}{\tau}$$

Let $\Delta N = N_1 - N_2$

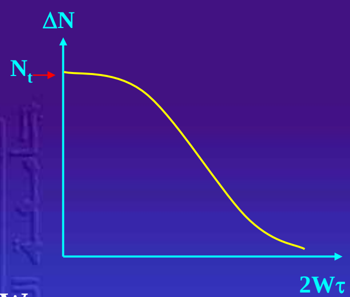
$$\frac{d\Delta N}{dt} = \frac{d(N_1 - N_2)}{dt} = -2\frac{dN_2}{dt} = -2W(N_1 - N_2) + \frac{2N_2}{\tau}$$

$$= -2W\Delta N + \frac{N_2 + N_1}{\tau} - \frac{N_1 - N_2}{\tau} = -\Delta N(2W + \frac{1}{\tau}) + \frac{N_t}{\tau}$$

Dr. Abdallah M.Azzeer

In steady-state ⇨ $\frac{d(\Delta N)}{dt} = 0$

$$\Delta N = (\frac{N_t}{\tau})/(2W + \frac{1}{\tau}) = \frac{N_t}{1 + 2W\tau}$$



At saturation;

Wτ >> 1 , ΔN = 0 , $N_1 = N_2 = N_t/2$

One can achieve this by either letting W→ ∞ , or τ → ∞

To maintain at a given population difference ΔN, the material absorb from the incident radiation a power per unit volume (dP/dV) given by;

$$\frac{dP}{dV} = \hbar\omega W\Delta N = \hbar\omega\frac{N_t W}{1 + 2W\tau}$$

Dr. Abdallah M.Azzeer

At saturation;

$$\left.\frac{dP}{dV}\right)_s = \frac{\hbar\omega N_t}{2\tau} = \textit{power per unit vol. lost from the decaying of } N_2 = \frac{N_t}{2}$$

Re-write;

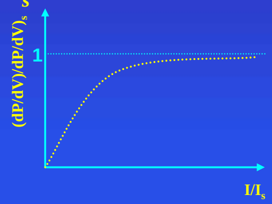
$$\frac{\Delta N}{N_t} = \frac{1}{1+2W\tau} = \frac{1}{1+\frac{I}{I_s}}$$

Where

$$W \equiv \sigma F = \frac{\sigma I}{\hbar\omega} \quad , \quad I_s = \frac{\hbar\omega}{2\sigma\tau} \quad , \quad \frac{I}{I_s} = 2W\tau$$

And

$$\frac{dP/dV}{(dP/dV)_s} = \frac{2W\tau}{1+2W\tau} = \frac{I/I_s}{1+\frac{I}{I_s}}$$



Of course, the above quantities σ , $W \propto \sigma$ and $I_s \propto 1/\sigma$ are all dependent on frequency ω. We shall therefore use

$\sigma = \sigma(\omega) = \sigma_0\, g(\omega - \omega_0)$

$$I_s = \frac{\hbar\omega}{2\sigma_0\tau\, g(\omega-\omega_0)} \equiv \frac{I_{so}}{g(\omega-\omega_0)}$$