Beam Optics:

- ♦ Light can take the form of beams that comes as close as possible to spatially localized and nondiverging waves.
- ♦ Two extremes: (a) Plane wave: no angular spread; (b) spherical wave: diverge in all directions;
- ♦ Paraxial waves satisfy the paraxial Helmholtz equation! An important solution of this equation that exhibits the characteristics of an optical beam is the a wave called the Gaussian beam.
- The Gaussian beam:

The complex amplitude of a paraxial waves is

$U(\mathbf{r}) = A(\mathbf{r})\exp(-ikz)$

$A(\mathbf{r})$ is a slow varying function of position => the envelope is assumed to be approximately constant locally (within λ)

$U(\mathbf{r})$: satisfy the Helmholtz equation $(\nabla^2 + k^2)U(\mathbf{r}) = 0$

=> $A(\mathbf{r})$: satisfy the paraxial Helmholtz equation

$$\nabla_T^2 A(\mathbf{r}) - i2k\frac{\partial A(\mathbf{r})}{\partial z} = 0$$

One simple solution to the Paraxial Helmholtz Equation

$$A(\mathbf{r}) = \frac{A_1}{z}\exp\left(-ik\frac{x^2+y^2}{2z}\right)$$ Paraboloidal wave

Another solution of the Paraxial Helmholtz Equation

$$A(\mathbf{r}) = \frac{A_1}{q(z)}\exp\left(-ik\frac{x^2+y^2}{2q(z)}\right)\ ;\ q(z) = z - \xi$$ Gaussian beam

A paraboloidal wave centered about the point $z = \xi$.
ξ could be complex value; dramatically different properties acquired when ξ is real or complex.

When ξ is purely imaginary, i.e. $\xi = -iz_0$; z_0: real =>
the complex envelope of the Gaussian beam

$$A(\mathbf{r}) = \frac{A_1}{q(z)} \exp\left(-ik\frac{x^2+y^2}{2q(z)}\right) \; ; \; q(z) = z + iz_0$$

z_0: Rayleigh range

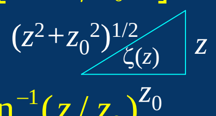
Separate amplitude and phase of this complex envelope

$$\frac{1}{q(z)} = \frac{1}{z+iz_0} = \frac{1}{R(z)} - i\frac{\lambda}{\pi W^2(z)}$$

$$\frac{1}{z+iz_0} = \frac{z-iz_0}{z^2+z_0^2} = \frac{z}{z^2+z_0^2} - \frac{iz_0}{z^2+z_0^2} = \frac{1}{R(z)} - i\frac{\lambda}{\pi W^2(z)}$$

$$R(z) = z[1+(z_0/z)^2]; W(z) = \underbrace{(\lambda z_0/\pi)^{1/2}}_{=W_0}\left[1+(z/z_0)^2\right]^{1/2}$$

$$R(z)/z = (W(z)/W_0)^2$$



$$\frac{1}{q(z)} = \frac{1}{z+iz_0} = |q|\exp[i\varsigma(z)]; \varsigma(z) = \tan^{-1}(z/z_0)$$

Put all this equation into the complex envelope of the Gaussian beam

$$\Rightarrow A(\mathbf{r}) = \frac{A_1}{iz_0}\frac{W_0}{W(z)}\exp\left[-\frac{x^2+y^2}{W^2(z)}\right]\exp\left[-ik\frac{x^2+y^2}{2R(z)} + i\varsigma(z)\right]$$

$$\Rightarrow U(\mathbf{r}) = A_0\frac{W_0}{W(z)}\exp\left[-\frac{\rho^2}{W^2(z)}\right]\exp\left[-ikz - ik\frac{\rho^2}{2R(z)} + i\varsigma(z)\right]$$

Gaussian-Beam Complex Amplitude $\quad A_0 = A_1/iz_0$

$R(z) = z[1+(z_0/z)^2]$ — wavefronts radius of curvature

$W(z) = W_0\left[1+(z/z_0)^2\right]^{1/2}$ → Beam width

$W_0 = (\lambda z_0/\pi)^{1/2}$

$\varsigma(z) = \tan^{-1}(z/z_0)$

Define beam parameters

A_0 and z_0 are determined from the boundary conditions

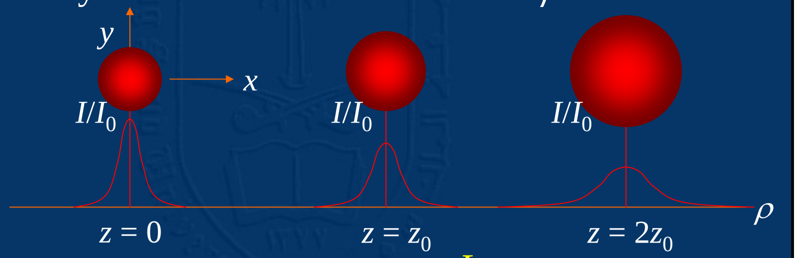
* Properties:

Intensity $I(\mathbf{r}) = |U(\mathbf{r})|^2$

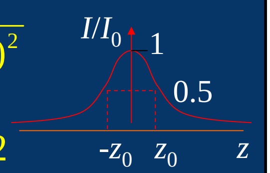
a function of axial (z) and radial (ρ)distance

$$I(\rho, z) = I_0 [W_0 / W(z)]^2 \exp\left[-2\rho^2 / W^2(z)\right] \quad ; I_0 = |A_0|^2$$

Intensity is a Gaussian function of the ρ.



$$I(0, z) = I_0 [W_0 / W(z)]^2 = \frac{I_0}{1 + (z/z_0)^2}$$



For $\rho = 0$

Power (at a given z; transverse)

$$P = \int_0^\infty I(\rho, z) 2\pi\rho d\rho = I_0 (\pi W_0^2)/2$$

Independent of z!

$$I(\rho, z) = \frac{2P}{\pi W^2(z)} \exp\left[-\frac{2\rho^2}{W^2(z)}\right]$$

Express I_0 in terms of P.

The ratio of the power carried within a circle of radius ρ_0 in the transverse plane at position z to the total power

$$\frac{\int_0^{\rho_0} I(\rho, z) 2\pi\rho d\rho}{\int_0^\infty I(\rho, z) 2\pi\rho d\rho} = 1 - \exp\left[-\frac{2\rho_0^2}{W^2(z)}\right]$$

For $\rho_0 = W(z)$; the ratio is ~ 86%; for $\rho_0 = 1.5W(z)$ the ratio is ~ 99%.

Beam Radius (width):

∵ 86% of the power is carried within a circle of $W(z)$; $W(z)$; is regarded as the beam radius. The rms width of the intensity distribution is $\sigma = W(z)/2$.

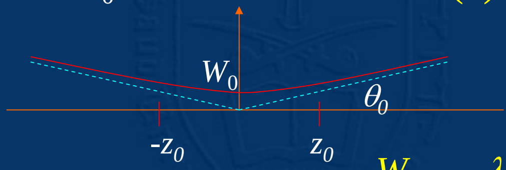
The beam width is governed by

$$W(z) = W_0\left[1 + (z/z_0)^2\right]^{1/2}$$

$W(z)$ has the minimum value of W_0 at $z = 0$, called the beam waist. W_0 is the waist radius; $2W_0$ is called the spot size. The beam radius increases gradually with z.

At $z = z_0$, the beam radius = $2^{1/2}W_0$.

At $z >> z_0$, the beam radius $W(z) = W_0\, z/z_0 = \theta_0 z$



$$\theta_0 = \frac{W_0}{z_0}$$

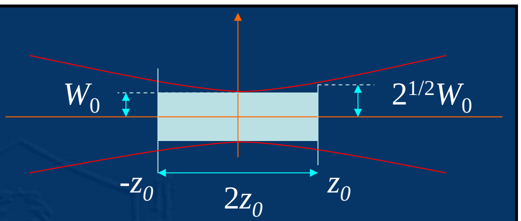
Beam Divergence : $\theta_0 = \dfrac{W_0}{z_0} = \dfrac{\lambda}{\pi W_0}$

If the waist is squeezed => the beam diverges;
A short wavelength and fat beam waist => a highly directional beam!

Depth of Focus :

Defined as twice the Rayleigh range $2z_0$.

$$2z_0 = 2\pi W_0^2/\lambda$$



A small spot size ($2W_0$)and a long depth of focus ($2z_0$)can not be obtained simultaneously unless λ is short.

Phase : of the Gaussian beam

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-ikz - ik\frac{\rho^2}{2R(z)} + i\varsigma(z)\right]$$

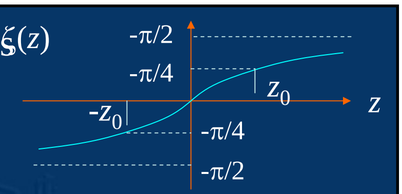
$$\varphi(\rho, z) = kz - \varsigma(z) + \frac{k\rho^2}{2R(z)} \qquad = -i\varphi(\rho, z)$$

On the beam axis $\rho = 0$ => $\varphi(0, z) = kz - \varsigma(z)$

kz: phase of a plane wave; $\zeta(z)$: retardation

$$\varsigma(z) = \tan^{-1}(z/z_0)$$

The total accumulated excess retardation as the wave travels from $z = -\infty$ to $z = \infty$ is π. **Guoy effect**

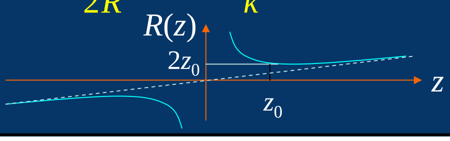


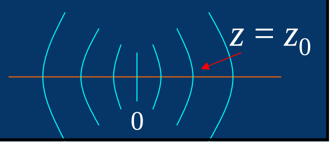
Wavefronts: the third component in the above equation is responsible for wavefront bending. The surface of constant phase satisfy

$$\varphi(\rho, z) = kz - \varsigma(z) + \frac{k\rho^2}{2R(z)} = 2\pi q$$

$\because \zeta(z)$ and $R(z)$ are relatively slowly varying => ~ constant within the beam radius on each wavefront

$$z + \frac{\rho^2}{2R} = 2\pi q + \frac{\varsigma}{k}$$ equation of paraboloidal wave



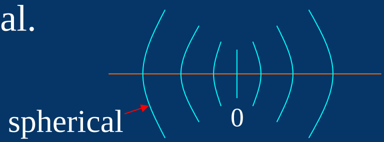


Properties of the Gaussian Beam at Special Points

- At the plane $z = z_0$, the wave has the following properties
  - (i) the beam radius is $2^{1/2}$ times greater than the radius at the beam waist, and the area is larger by a factor of 2.
  - (ii) The intensity on the beam axis is 1/2 the peak intensity
  - (iii) The phase on the beam axis is retarded by an angle $\pi/4$ relative to the phase of a plane wave
  - (iv) The radius of the curvature of the wavefront is the smallest, so that the wavefront has the greatest curvature ($R = 2z_0$)
- Near the beam center. At points for which $|z| << z_0$ and $\rho << W_0$, $\exp\left[-\rho^2/W^2(z)\right] \approx \exp\left[-\rho^2/W_0^2\right] \approx 1$

so that the beam intensity is approximately constant. Also, $R(z) \approx z_0^2/z$ and $\zeta(z) \approx 0$, so that the phase $\Rightarrow k[z+\rho^2/2R(z)] = kz[1+\rho^2/2z_0^2] \approx kz$
As a result, the wavefronts are approximately planar. => Gaussian beam ~ a plane wave near its center.

- Far from the beam waist. At points within the beam-waist radius ($\rho << W_0$), but far from the beam waist ($z >> z_0$) the wave ~ like a spherical wave. Since $W(z) \approx W_0 z/z_0 >> W_0$ and $\rho < W_0$ => $\exp[-\rho^2/W^2(z)] \approx 1$ so that the beam intensity is ~ uniform. Since $R(z) \sim z$ the wavefronts are approximately spherical.



Parameters of a Gaussian Laser Beam:

* A 1-mW He-Ne laser produces a Gaussian beam of wavelength λ = 633 nm and a spot size $2W_0$ = 0.1 mm.

- Angular divergence: $\theta_0 = W_0/z_0 = \lambda/\pi W_0 = 633\times10^{-9}/3.1416/0.05\times10^{-3} \approx 4.03\times10^{-3}$ rad.
- Depth of focus: $2z_0 = 2\pi W_0^2/\lambda = 2\times3.1416\times(0.05\times10^{-3})^2/633\times10^{-9} = 2.48\times10^{-2}$ (m)=2.48 (cm)
- At $z = 3.5\times10^5$ km (~ distance to moon), the diameter of the beam $2W(z) \sim 2\theta_0 z = 2\times4.03\times10^{-3}\times3.5\times10^5$ km = 2.821×10^3 km = 2.821×10^6 m
- The radius of curvature $R(z) = z[1+(z_0/z)^2]$ (z_0=1.24 cm); at $z = 0$ is $R(z) = 0$;
at $z = z_0$ is $R(z) = 2z_0$ = 2.48 cm;
at $z = 2z_0$ is $R(z) = 2.5z_0$ = 3.1 cm;

- Optical intensity at the beam center: $z = 0$, $\rho = 0$

$$I(\rho, z) = \frac{2P}{\pi W^2(z)} \exp\left[-\frac{2\rho^2}{W^2(z)}\right]$$

$I(0, 0) = 2P/\pi W_0^2 = 2\times1/\pi/(0.005)^2$ mW/cm^2
= 25465 mW/cm^2

$$\because W(z_0) = \sqrt{2} W_0$$

$I(0, z_0) = 2P/\pi W(z_0)^2 = P/\pi W_0^2 = 1/\pi/(0.005)^2$ mW/cm^2 = 12732 mW/cm^2

Point source of 100W at $z = 0$. At $z = z_0$, 100W is distributed over $4\pi z_0^2$.

=> $I(z_0)$=100x1000/4/π/(1.24)^2 = 5175 mW/cm^2.

Parameters required to characterize a Gaussian Beam:

* Peak amplitude, direction (beam axis), location of its waist, and the waist radius (W_0) or the Rayleigh range (z_0).

* q-parameter: $q(z) = z + iz_0$. If $q(z) = 3 + i4$ cm at some points on the beam axis => beam waist lies at a distance z = 3 cm to the point and that the depth of focus is $2z_0$ = 8 cm. $q(z)$ is linear on z, $q(z)=q_1$ and $q(z+d) = q_2$ => $q_2 = q_1 + d$. The q-parameter is sufficient for characterizing a Gaussian beam.
* Determination of q-parameter: measure the beam width, $W(z)$, and the radius of curvature, $R(z)$, at an arbitrary point on the axis => using equation

$$\frac{1}{q(z)} = \frac{1}{z + iz_0} = \frac{1}{R(z)} - i\frac{\lambda}{\pi W^2(z)}$$

or solve z, z_0, and W_0 using the following equations;

$$R(z) = z[1 + (z_0/z)^2] \quad W(z) = W_0\left[1 + (z/z_0)^2\right]^{1/2}$$

$$W_0 = (\lambda z_0/\pi)^{1/2}$$

$$\because R = z[1+(z_0/z)^2] \quad W = (\lambda z_0/\pi)^{1/2}\left[1+(z/z_0)^2\right]^{1/2}$$

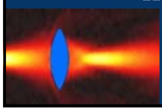
$$\Rightarrow Rz = z^2 + z_0^2 \qquad z_0(\pi W^2/\lambda) = z_0^2 + z^2$$

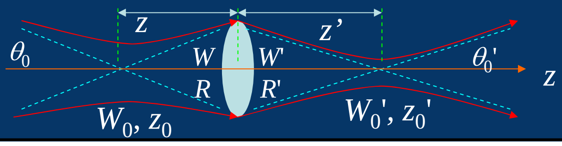
$$\Rightarrow z_0(\pi W^2/\lambda) = Rz \qquad \because z_0 = \sqrt{Rz - z^2}$$

$$\Rightarrow \sqrt{Rz - z^2}\,(\pi W^2/\lambda) = Rz$$

$$\Rightarrow z = \frac{R}{1+(\lambda R/\pi W^2)^2} \quad ; W_0 = \frac{W}{\left[1+(\pi W^2/\lambda R)\right]^{1/2}}$$

- Transmission through optical components:
  - **Transmission through a thin lens**: the complex transmittance of a thin lens of focal length $f \propto \exp(ik\rho^2/2f)$; when a Gaussian beam crosses the lens its complex amplitude is multiplied by this phase factor => wavefront is bent, but the beam radius is not altered





* A Gaussian beam centered at $z = 0$ with waist radius W_0 is transmitted through a thin lens located at z;
The phase at the plane of the lens is $kz + k(\rho^2/2R) - \varsigma$
The phase of the transmitted wave is altered to

$$kz + k(\rho^2/2R) - \varsigma - k(\rho^2/2f) = kz + k(\rho^2/2R') - \varsigma$$

where $1/R' = (1/R) - (1/f)$

R: + beam diverging;
R': - beam converging

The transmitted wave is itself a Gaussian beam with width W' (=W) and radius of curvature R'.
The waist radius of the new beam W_0' centered at z';

$$W_0' = \frac{W}{\left[1+(\pi W^2/\lambda R')\right]^{1/2}} \qquad -z' = \frac{R'}{1+(\lambda R'/\pi W^2)^2}$$

\- representing waist lies to the right of the lens

Substituting $R(z)$ and $W(z)$ into above equation!

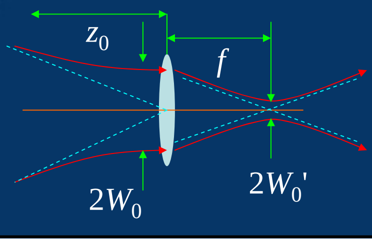
Waist radius	$W_0' = MW_0$	
Waist location	$(z' - f) = M^2(z - f)$	
Depth of focus	$2z_0' = M^2(2z_0)$	Parameter Transformed by a Lens
Divergence	$2\theta_0' = 2\theta_0 / M$	
Magnification	$M = M_r/(1+r^2)^{1/2}$	
$r = z_0/(z-f)$	$M_r = \lvert f/(z-f) \rvert$	

Consider the limiting case $(z\text{-}f) >> z_0$;

$\Rightarrow r << 1 \Rightarrow M \approx M_r \Rightarrow$ equations of ray optics

* **Beam shaping**

Beam focusing: a lens is placed at the waist of a Gaussian beam



The transmitted beam is then focused to a waist radius W_0' at a distance z' given by

$$r = -z_0/f \qquad M_r = 1 \qquad M = 1/(1+(z_0/f)^2)^{1/2}$$

$$\Rightarrow W_0' = \frac{W_0}{\left[1+(z_0/f)^2\right]^{1/2}} \quad ; \quad z' = \frac{f}{1+(f/z_0)^2}$$

If the $2z_0$ (depth of focus) >> f (focal length)

$$\Rightarrow W_0' \approx (f/z_0)W_0 = \lambda f/(\pi W_0) = \theta_0 f; \; z' \approx f$$

The incident Gaussian beam is well approximated by a plane wave at its waist => focused at the focal plane!

- Smallest possible spot size is desired in many applications (laser scanning, laser printing, and laser fusion).

$\because W_0' \approx \lambda f/(\pi W_0)$ Smallest λ and f; thickest incident beam

The lens should intercept the incident beam, its diameter D must be at least $2W_0$; assume $D = 2W_0$. $W_0' \approx \lambda f/(\pi D/2) \Rightarrow 2W_0' \approx 4\lambda F_\# / \pi$

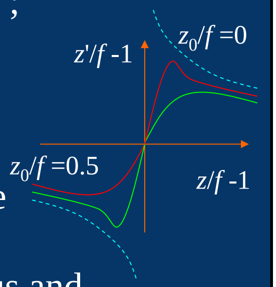
$F_\# = f/D$ F-number of the lens

A microscope objective with small F-number is often used!

Beam collimation: a Gaussian beam is transmitted through a thin lens of focal length f;

From $(z' - f) = M^2(z - f)$

$$\Rightarrow \frac{z'}{f} - 1 = \frac{z/f - 1}{(z/f - 1)^2 + (z_0/f)^2}$$



For beam collimation, z' as distance as possible from the lens; achieved by smallest z_0/f (short depth of focus and long focal length)

For a given ratio of z_0/f => the optimal value of z is maximum of z'/f; assume z/f-1 = a and $z_0/f = b$.

$$\frac{\partial}{\partial a}\left(\frac{a}{a^2 + b^2}\right) = 0 \quad \Rightarrow a^2 - b^2 = 0 \Rightarrow z = z_0 + f$$

* **Reflection from a spherical mirror**

Incident Gaussian beam: width W_1, roc R_1;

Reflected Gaussian beam: width W_2, roc R_2;

The phase of the incident beam is modified by a phase factor $\exp(-ik\rho^2/R)$

The relations between W_1, R_1, W_2, and R_2 are

$W_2 = W_1; (1/R_2) = (1/R_1) + (2/R)$

· If the mirror is planar => $R = \infty$ $R_2 = R_1$

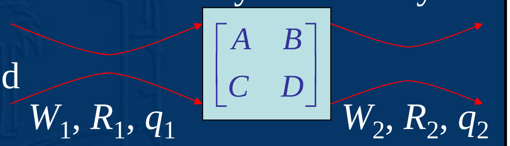
· If $R_1 = \infty$, i.e. the beam waist lies on the mirror

$R_2 = R/2$

· If $R_1 = -R$, i.e. the incident beam has the same curvature as the mirror => $R_2=R$.

* **Transmission through an arbitrary optical system**

Modification of a Gaussian beam by an arbitrary paraxial optical system characterized by a matrix ***M***.



The ABCD Law:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$

Transmission through free space: distance *d* of free space; $q_2 = q_1 + d$ => $A=1, B=d, C=0, D=1$

$$\boldsymbol{M} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Transmission through a thin optical component; ray position is the same, angle is altered

$y_2 = y_1; \theta_2 = Cy_1 + D\theta_1$

$D = n_1/n_2;$

In terms of beam parameters

The optical component is thin => beam width does not changed => $W_1 = W_2$;

$\theta_2 \approx y_2 / R_2; \theta_1 \approx y_1 / R_1 \Rightarrow 1/R_2 = C + D/R_1$

Paraxial approximation

=> $1/q_2 = C + D/q_1$ => $q_2 = q_1/(Cq_1 + D)$

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 \\ C & D \end{bmatrix}$$

The matrix used in chapter could be used in Gaussian beam!

- **Hermite-Gaussian Beams:**
  - Beam of paraboloidal wavefronts are of importance; The curvature of the wavefronts could match the curvature of spherical mirrors that form a resonator Reflection inside the resonator won't change the curvature of the wavefront.
  - Consider a Gaussian beam of complex envelope

$$A_G(x,y,z) = \frac{A_1}{q(z)} \exp\left[-ik\frac{x^2+y^2}{2q(z)}\right] \quad ; q(z) = z + iz_0$$

  - Consider a second wave whose complex envelope

$$A(x,y,z) = \mathcal{X}\left[\tfrac{\sqrt{2}x}{W(z)}\right]\mathcal{Y}\left[\tfrac{\sqrt{2}y}{W(z)}\right]\exp[i\mathcal{Z}(z)]A_G(x,y,z)$$

$\mathcal{X}(\cdot); \mathcal{Y}(\cdot); \mathcal{Z}(\cdot)$ are real functions

· Except for an excess phase of $\mathcal{Z}(z)$, the phase of the of the wave have the same phase as that of the underlying Gaussian wave!

· The magnitude

$$(A_1/iz_0)\mathcal{X}\left[\frac{\sqrt{2}x}{W(z)}\right]\mathcal{Y}\left[\frac{\sqrt{2}y}{W(z)}\right]\left[\frac{W_0}{W(z)}\right]\exp\left[-\frac{x^2+y^2}{W^2(z)}\right]$$

Function of $x/W(z)$ and $y/W(z)$ whose width in the x and y directions vary with z in accordance with the same scaling factor $W(z)$. As z increase, the intensity distribution in the transverse plane remains fixed (except for a magnification factor $W(z)$. => Gaussian function modulated in the x and y directions.

The existence of this wave is assured if three real function $\mathcal{X}(\cdot); \mathcal{Y}(\cdot); \mathcal{Z}(\cdot)$ could be found such that $A(x, y, z)$ satisfies the paraxial Helmholtz equation.

Defining $u = \sqrt{2}x/W(z)$ and $v = \sqrt{2}y/W(z)$

$$\frac{1}{x}\left(\frac{\partial^2 x}{\partial u^2} - 2u\frac{\partial x}{\partial u}\right) + \frac{1}{y}\left(\frac{\partial^2 y}{\partial u^2} - 2u\frac{\partial y}{\partial u}\right) + kW^2(z)\frac{\partial z}{\partial z} = 0$$

∵ Three independent variables => assume the first term = -μ_1, the second term = -μ_2, => the third term = $\mu_1 + \mu_2$,

$$\frac{\partial^2 x}{\partial u^2} - 2u\frac{\partial x}{\partial u} = -\mu_1 x \quad \text{(a)} \qquad \frac{\partial^2 y}{\partial u^2} - 2u\frac{\partial y}{\partial u} = -\mu_2 y \quad \text{(b)}$$

$$kW^2(z)\frac{\partial z}{\partial z} = z_0\left[1 + (z/z_0)^2\right]\frac{\partial z}{\partial z} = \mu_1 + \mu_2 \quad \text{(c)}$$

Equation (a) represents an eigenvalue problem whose eigenvalues are $\mu_1 = l = 0, 1, 2 \ldots$ and whose eigenfunctions are the Hermite polynomials ($H_l(u)$)

$$x(u) = H_l(u)$$

Hermite polynomials is defined by the recurrence relation

$$H_{l+1}(u) = 2uH_l(u) - 2lH_{l-1}(u) \qquad H_0(u) = 1; H_1(u) = 2u$$

$$\Rightarrow H_2(u) = 4u^2 - 2\,; H_3(u) = 8u^3 - 12u\,; \cdots$$

Similarly, solution for equation (b) is (let $\mu_2 = m$)

$$y(v) = H_m(v)$$

Solution for equation (c) is (with $\mu_1 + \mu_2 = l + m$)

$$z(z) = (l + m)\varsigma(z)$$

Substitute all the solution into $A(x, y, z)$ and multiplying by the phase factor exp(-ikz)

=> $U_{l,m}(x,y,z)$

$$U_{l,m}(x,y,z) = A_{l,m}\left[\frac{W_0}{W(z)}\right]G_l\left[\frac{\sqrt{2}x}{W(z)}\right]G_m\left[\frac{\sqrt{2}y}{W(z)}\right]$$

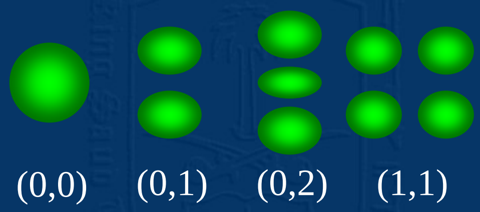
$$\times \exp\left[-ikz - ik\frac{x^2 + y^2}{2R(z)} + i(l + m + 1)\varsigma(z)\right]$$

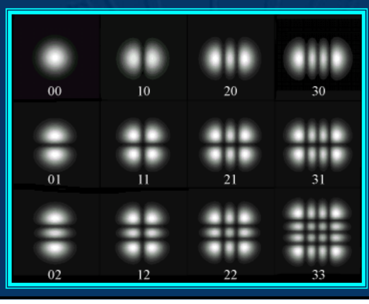
where $G_l(u) = H_l(u)\exp(-u^2/2)$

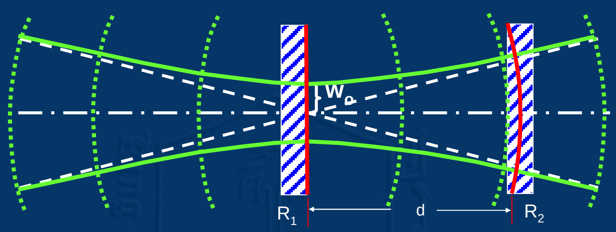
Hermite-Gaussian function

· Intensity distribution

$$I_{l,m}(x,y,z)=\left|A_{l,m}\right|^2\left[\frac{W_0}{W(z)}\right]^2 G_l^2\left[\frac{\sqrt{2}x}{W(z)}\right]G_m^2\left[\frac{\sqrt{2}y}{W(z)}\right]$$

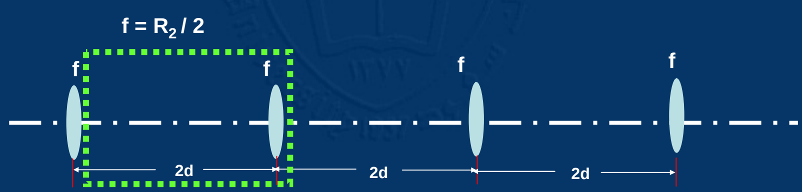


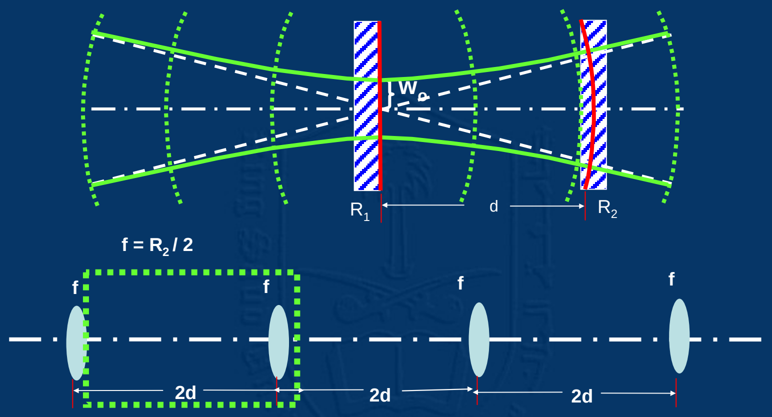




Find the formula for the spot size of a TEM $_{00q}$ mode at the spherical mirror of Fig. 5.1 by following the procedure:

(a) Show an equivalent-lens waveguide for this cavity and identify a unit cell such that the ABCD law will yield the spot size on the spherical mirror directly.





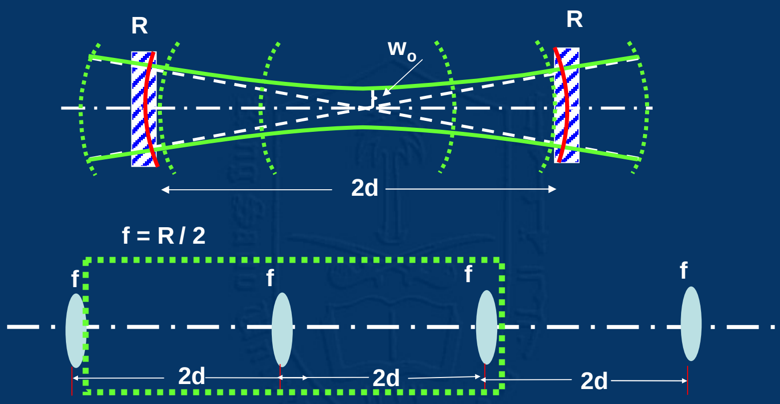
$$T = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2d \\ -\frac{1}{f} & 1 - \frac{2d}{f} \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2d \\ -\frac{1}{f} & 1 - \frac{2d}{f} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\frac{\pi w_1^2(z_1)}{\lambda_o} = \frac{B}{\left[1 - \left(\frac{A+D}{2}\right)^2\right]^{\frac{1}{2}}} = \frac{2d}{\left[1 - \left(\frac{2 - \frac{2d}{f}}{2}\right)^2\right]^{\frac{1}{2}}}$$

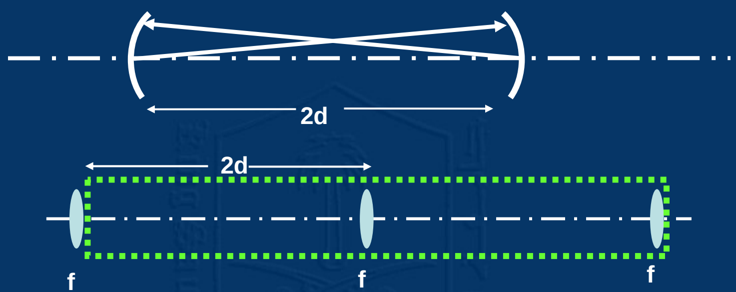
$$= \frac{2d}{\left[\frac{2d}{f} - \left(\frac{d}{f}\right)^2\right]^{\frac{1}{2}}}$$

Confocal Resonator



Stability: $0 \le \dfrac{A+D+2}{4} < 1$

Confocal resonator

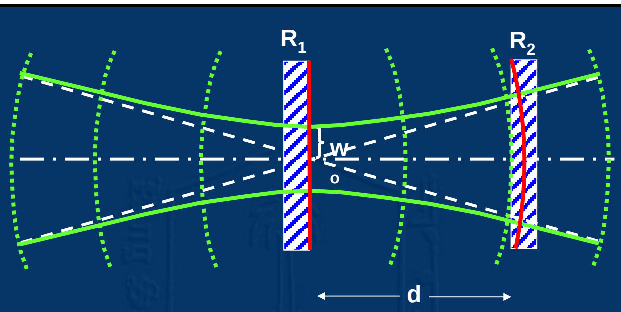


$$\left\{\begin{bmatrix}1 & 0\\ -\frac{1}{f} & 1\end{bmatrix}\begin{bmatrix}1 & 2d\\ 0 & 1\end{bmatrix}\right\}^2 = \begin{bmatrix}1 & 2d\\ -\frac{1}{f} & 1-\frac{2d}{f}\end{bmatrix}^2 =$$

$$0 \le \frac{A+D+2}{4} \le 1 \qquad 0 \le \frac{4-\frac{8d}{f}+\frac{4d}{f^2}}{4} \le 1 \qquad 0 \le \left(1-\frac{d}{f}\right)^2 \le 1$$

$$R = 2f \qquad 0 \le \left(1-\frac{2d}{R}\right)^2 \le 1 \qquad so\ \ 2d = R\ \ gives\ \ \frac{A+D+2}{4} = 0$$

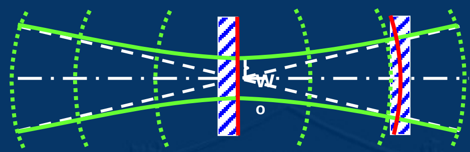
and the resonator is stable, but locateed at the edge of a stable region



$$R(z)=z\left[1+\left(\frac{z_o}{z}\right)^2\right] \qquad R(d)=R_2=d\left[1+\left(\frac{z_o}{d}\right)^2\right]$$

$$w^2(z)=w_o^2\left[1+\left(\frac{z_o}{z}\right)^2\right]$$

$$z_o=\frac{\pi\omega_o^2}{\lambda_o} \qquad z_o=(dR_2)^{\frac{1}{2}}\left(1-\frac{d}{R_2}\right)^{\frac{1}{2}}$$

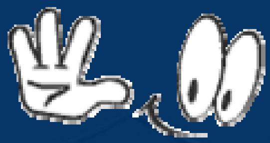


$$\frac{\pi w^2(d)}{\lambda_o}=\frac{\pi w_o^2}{\lambda_o}\left[1+\left(\frac{d}{z_o}\right)^2\right]$$

$$\frac{\pi w^2(d)}{\lambda_o}=(dR_2)^{\frac{1}{2}}\left(1-\frac{d}{R_2}\right)^{\frac{1}{2}}\left[1+\frac{d^2}{dR_2(1-d/R_2)}\right]$$

$$=\frac{(dR_2)^{\frac{1}{2}}}{(1-d/R_2)^{\frac{1}{2}}}$$

A cavity mode is a field distribution that reproduces itself in relative shape and in relative phase after a round trip through the system



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