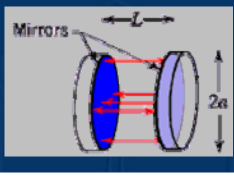
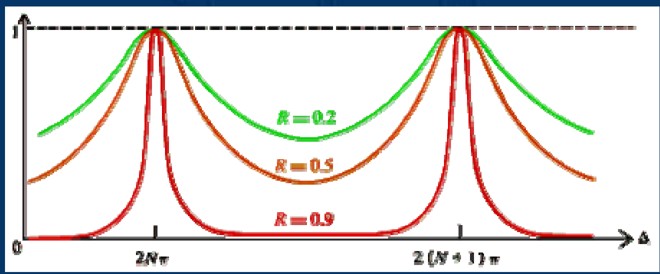
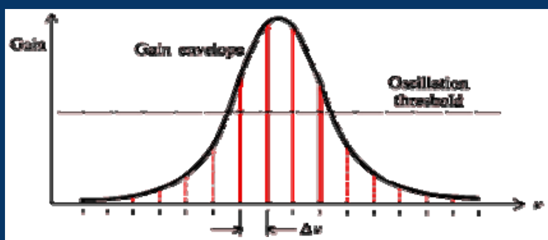
LASER RESONATOR

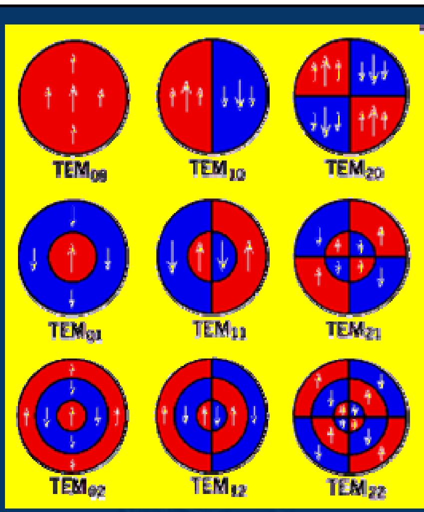




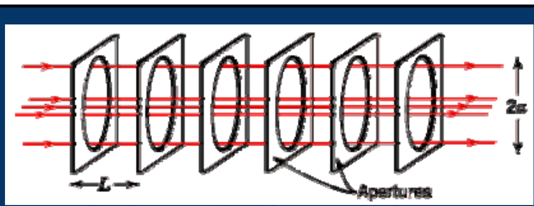
The spectral response of the cavity, where R is cavity reflectivity



Plot of laser gain as a function of frequency.
The natural gain bandwidth (black curve) is further restricted to discrete cavity modes (red), of which only four are above the laser oscillation threshold.

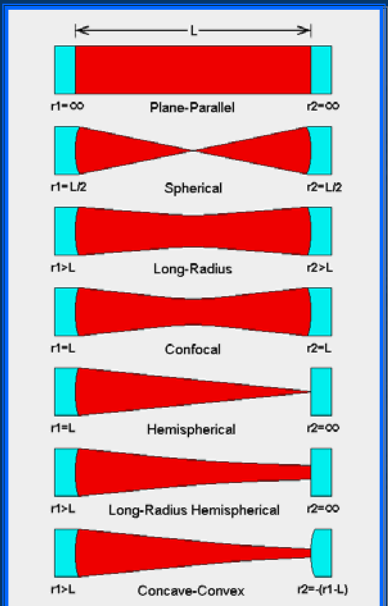


Electric field distribution of a few of the lowest order resonant modes of a cylindrical cavity.
Up or down arrows (red or blue regions) indicate the phase of the electric field and arrow length indicates relative strength.

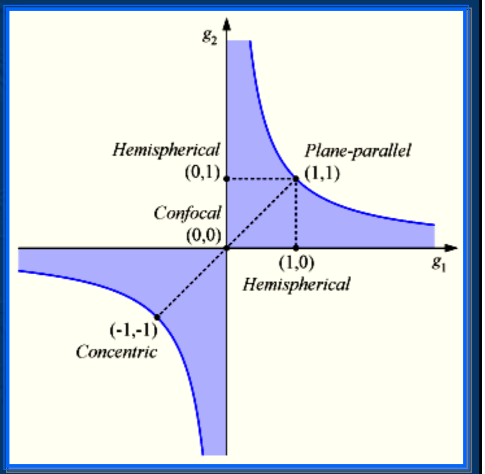


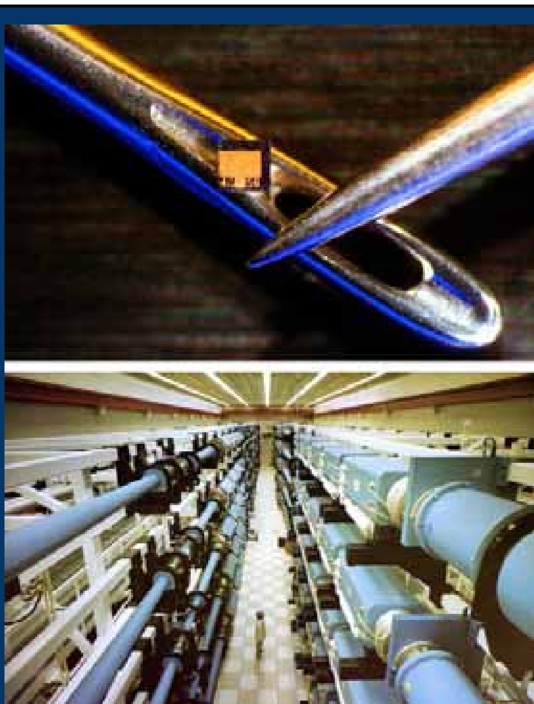
Effect of diffraction in selecting resonant modes.

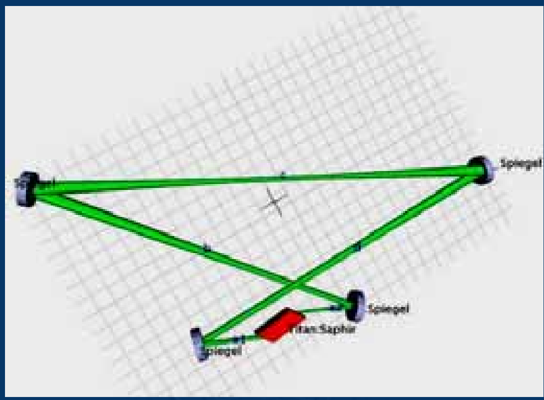
TYPE OF RESONATOR

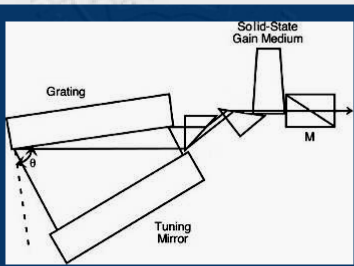


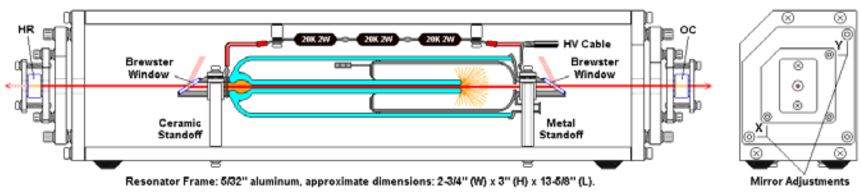
Common Laser Resonator Configurations



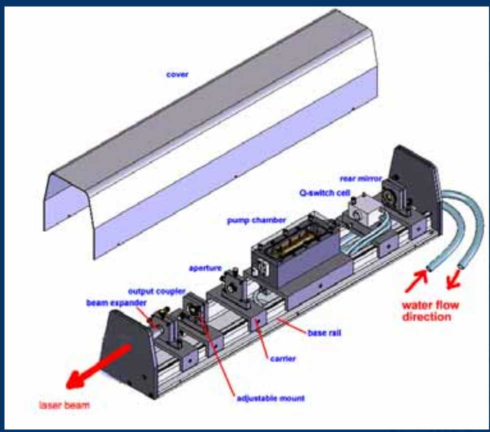




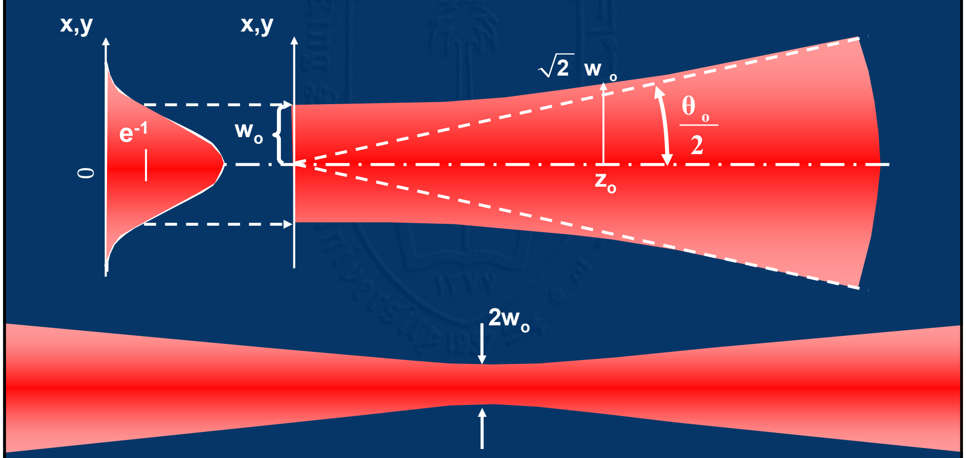




HeNe Laser Tube with Two Brewster Windows Mounted in Home-Built Resonator

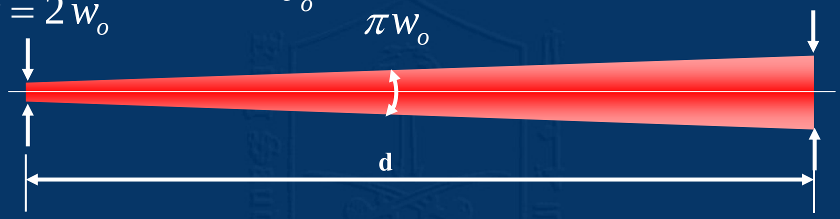


Gaussian Beams in Laser Systems



$$\theta_o = \frac{2\lambda}{\pi w_o}$$

$$\Delta x = 2 w_o$$



$$\Delta x = 2 w_o + \theta_o d = 2 w_o + \frac{2\lambda d}{\pi w_o}$$

Paraxial Wave Equation

$\nabla \cdot \mathbf{E} = 0$ — **Zero charge density**

$\nabla \cdot \mathbf{E} = \nabla_t \cdot \mathbf{E}_t + \frac{\partial}{\partial z}\mathbf{E}_z = 0$ — **Separate transverse and axial components**

$\mathbf{E}_z = \mathbf{E}_o \exp(-jkz)$ $\quad k \sim \omega n/c = 2\pi n/\lambda_o$ — **Major z dependence**

$\frac{\partial \mathbf{E}_z}{\partial z} \sim -j\frac{2\pi n}{\lambda_o}\mathbf{E}_z$ — **Approximate derivative of E wrt z**

$\nabla_t \cdot \mathbf{E}_t \sim \frac{\mathbf{E}_t}{D}$ — **Approximate transverse divergence given a finite beam diameter D**

$|\mathbf{E}_z| \approx \frac{\lambda_o}{2\pi n D}|\mathbf{E}_t|$ — **z component is small compared to transverse component**

Separate the rapid variation in z from the slow spatial variation in x,y and z

$$E(x,y,z) = E_o \psi(x,y,z) e^{-jkz}$$

$$\nabla^2 E + \frac{\omega^2}{c^2} n^2 E = 0$$

Substitute into the wave equation

$$\nabla_t^2 E + \frac{\partial^2 E}{\partial z^2} + \frac{\omega^2}{c^2} n^2 E = 0$$

$$E = E_o \psi(x,y,z) \exp(-jkz)$$

$$k = \frac{\omega}{c} n$$

Gaussian beam equation

Derivatives needed for next step

$$\nabla_t^2 E = E_o \left(\nabla_t^2 \psi\right) exp(-jkz)$$

$$\frac{\partial E}{\partial z} = E_o \left(-jk\psi + \frac{\partial \psi}{\partial z}\right) exp(-jkz)$$

$$\frac{\partial^2 E}{\partial z^2} = E_o \left(-k^2\psi - j2k\frac{\partial \psi}{\partial z} + \frac{\partial^2 \psi}{\partial z^2}\right) exp(-jkz)$$

$$\nabla_t^2 \psi - j2k\frac{\partial \psi}{\partial z} + \frac{\partial^2 \psi}{\partial z^2} = 0$$

Collect terms and cancel out the exponential factor

$$\nabla_t^2 \psi - j2k\frac{\partial \psi}{\partial z} = 0 \quad (paraxial\ \ wave\ \ equation)$$

Neglecting the second partial derivative

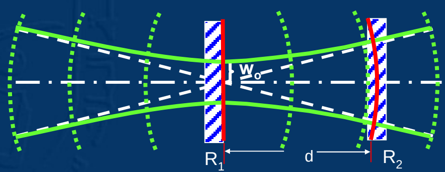
$$\nabla_t^2 \psi - j2k \frac{\partial \psi}{\partial z} = 0$$

$$\psi_o(r) = \exp\left\{ -j\left[P(z) + \frac{kr^2}{2q(z)} \right] \right\}$$

$$E(x,y,z) = \left\{ \frac{w_o}{w(z)} \exp\left[-\frac{r^2}{w^2(z)} \right] \right\}$$
$$\times \exp\left\{ -j\left[kz - \tan^{-1}\left(\frac{z}{z_o} \right) \right] \right\}$$
$$\times \exp\left[-j\frac{kr^2}{2R(z)} \right]$$

$q(z) = z + jz_o$ **Complex beam parameter**

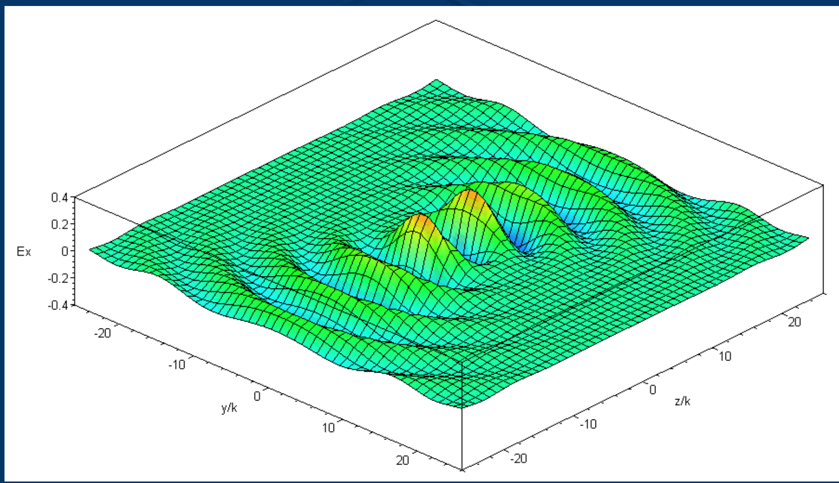
$$\frac{1}{q(z)} = \frac{1}{z + jz_o} = \frac{z}{z^2 + z_o^2} - j\frac{z_o}{z^2 + z_o^2} = \frac{1}{R(z)} - j\frac{\lambda_o}{\pi n w^2}$$



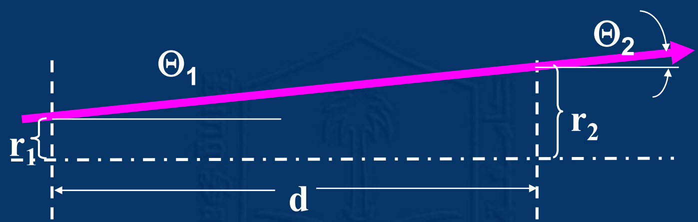
$$w^2(z) = w_o^2\left[1 + \left(\frac{z}{z_o} \right)^2 \right]$$

$$R(z) = \frac{z^2 + z_o^2}{z} = z\left[1 + \left(\frac{z_o}{z} \right)^2 \right]$$

$$z_o = \frac{\pi w_o^2}{\lambda_o}$$



Ray Tracing in an Optical System



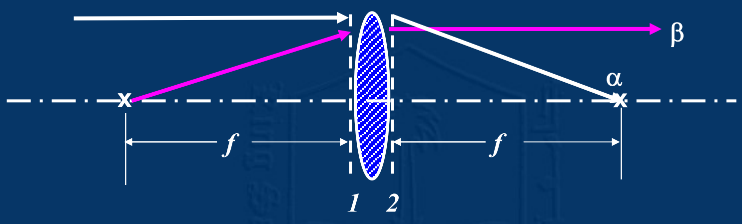
$$r_2 = 1 \times r_1 + d \times r_1'$$

$$r_2' = 0 \times r_1 + 1 \times r_1'$$

$$\begin{bmatrix} r_2 \\ r_2' \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_1' \end{bmatrix}$$

$$\begin{bmatrix} r_{out} \\ r_{out}' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_{in} \\ r_{in}' \end{bmatrix}$$

Thin Lens Ray Matrix



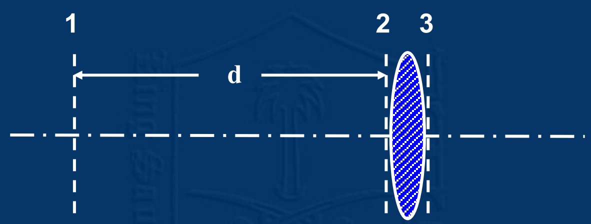
$r_1 = r_2$ **A=1 B=0**

Consider α C=-1/*f*

Consider β D=1

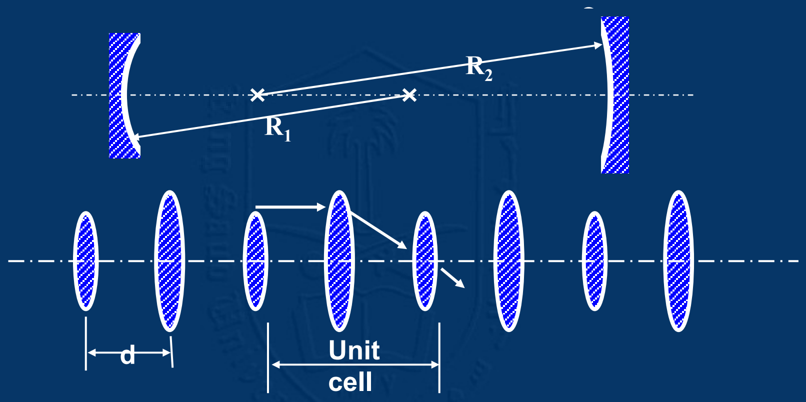
$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix}$$

Combination of lens and free space

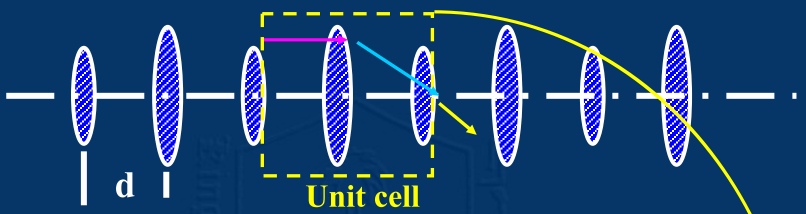


$$T = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & d \\ -\dfrac{1}{f} & 1-\dfrac{d}{f} \end{bmatrix}$$

Optical Resonator



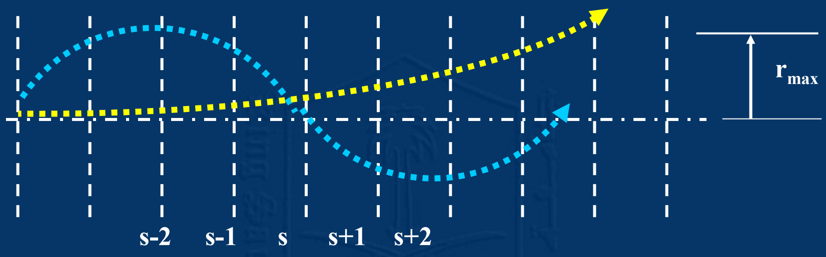
$$T = \begin{bmatrix} 1 & d \\ -\frac{1}{f_1} & 1-\frac{d}{f_1} \end{bmatrix} \begin{bmatrix} 1 & d \\ -\frac{1}{f_2} & 1-\frac{d}{f_2} \end{bmatrix}$$



$$T = \begin{bmatrix} 1 & d \\ -\frac{1}{f_1} & 1-\frac{d}{f_1} \end{bmatrix} \begin{bmatrix} 1 & d \\ -\frac{1}{f_2} & 1-\frac{d}{f_2} \end{bmatrix}$$

$$T = \begin{bmatrix} 1-\frac{d}{f_2} & d + d\left(1-\frac{d}{f_2}\right) \\ -\frac{1}{f_1}-\frac{1}{f_2}\left(1-\frac{d}{f_1}\right) & \left(1-\frac{d}{f_1}\right)\left(1-\frac{d}{f_2}\right)-\frac{d}{f_1} \end{bmatrix}$$

Stability Conditions



$$r_{s+1} = Ar_s + Br_s' \quad or \quad r_s' = \frac{1}{B}\left(r_{s+1} - Ar_s\right)$$

$$r_{s+1}' = \frac{1}{B}\left(r_{s+2} - Ar_{s+1}\right) = Cr_s + Dr_s'$$

$$\frac{1}{B}\left(r_{s+2} - Ar_{s+1}\right) = Cr_s + \frac{D}{B}\left(r_{s+1} - Ar_s\right)$$

$$r_{s+2} - 2\left(\frac{A+D}{2}\right)r_{s+1} + r_s = 0 \quad (use \;\; AD - BC = 1)$$

$$assume \;\; r_s = r_o\left(e^{j\theta}\right)^s = r_o e^{js\theta}$$

$$r_o\left[e^{j2\theta} - 2\left(\frac{A+D}{2}\right)e^{j\theta} + 1\right] = 0$$

$$e^{j\theta} = \frac{A+D}{2} \pm j\left[1 + \left(\frac{A+D}{2}\right)^2\right]^{\frac{1}{2}}$$

Form of Solution

$$r_s = r_o e^{js\theta} + r_o^* e^{-js\theta}$$

$$r_s = r_{\max} \sin(s\theta + \alpha)$$

Stability Diagram

$$-1 \le \left(\cos\theta = \frac{A+D}{2} \right) \le 1$$

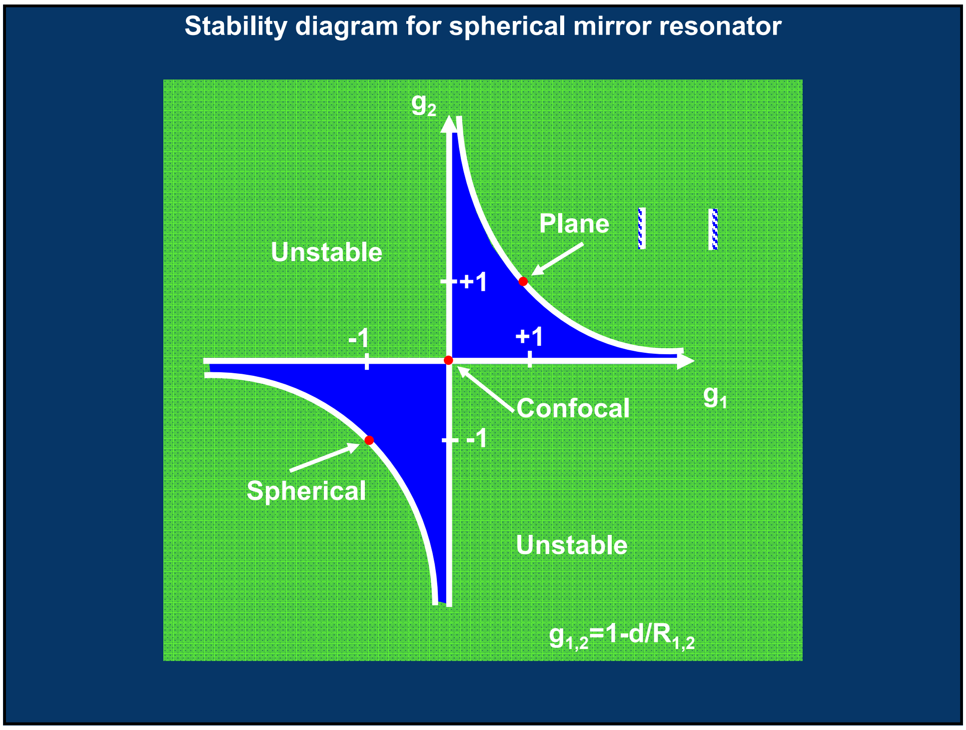
$$0 \le \frac{A+D+2}{4} \le 1 \quad stable \quad regime$$

$$\frac{A+D+2}{4} = \frac{1}{4}\left[1 - \frac{d}{f_2} - \frac{d}{f_1} + \left(1 - \frac{d}{f_2}\right)\left(1 - \frac{d}{f_1}\right) + 2\right]$$

$$= 1 - \frac{d}{2f_1} - \frac{d}{2f_2} + \frac{d}{4f_1 f_2} = \left(1 - \frac{d}{2f_1}\right)\left(1 - \frac{d}{2f_2}\right)$$

since $2f_1 = R_1$ *and* $2f_2 = R_2$

$$0 \le g_1 g_2 \le 1 \quad where \quad g_{1,2} = 1 - \frac{d}{R_{1,2}}$$


Stability diagram for spherical mirror resonator
g2
Plane
Unstable
+1
-1
+1
Confocal
g1
-1
Spherical
Unstable
g1,2=1-d/R1,2