16: Cavities, modes and mode-locking

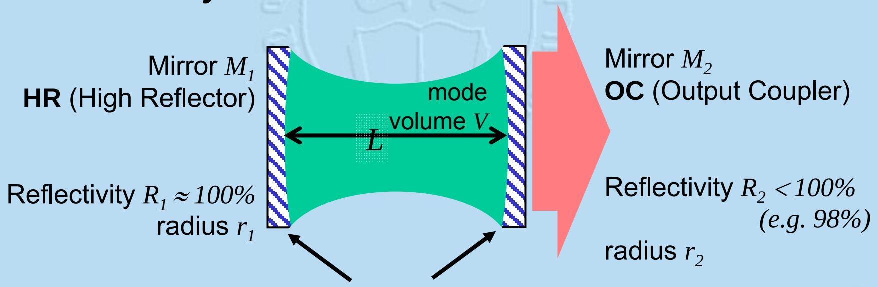
- Laser cavities and modes
  - Simple laser cavities
  - Types of resonator
  - Transverse EM modes
  - Longitudinal modes
  - Real laser cavities
- Gain depletion
  - Unbroadened
  - Homogeneously-broadened
  - In homogeneously-broadened
- Single-mode and multi-mode emission
- Mode-locking

Simple laser cavities

Why a resonator / cavity?

- γ is typically small (~0.1 m^{-1}) so **multiple passes** needed for sufficient amplification(exception is Nd-glass lasers)
- **optical feedback** gives self-sustained laser **oscillation** rather than laser **amplification** - and hence **longitudinal coherence** of beam
- optical resonator determines frequency of oscillation → **tunability**

Generic cavity:



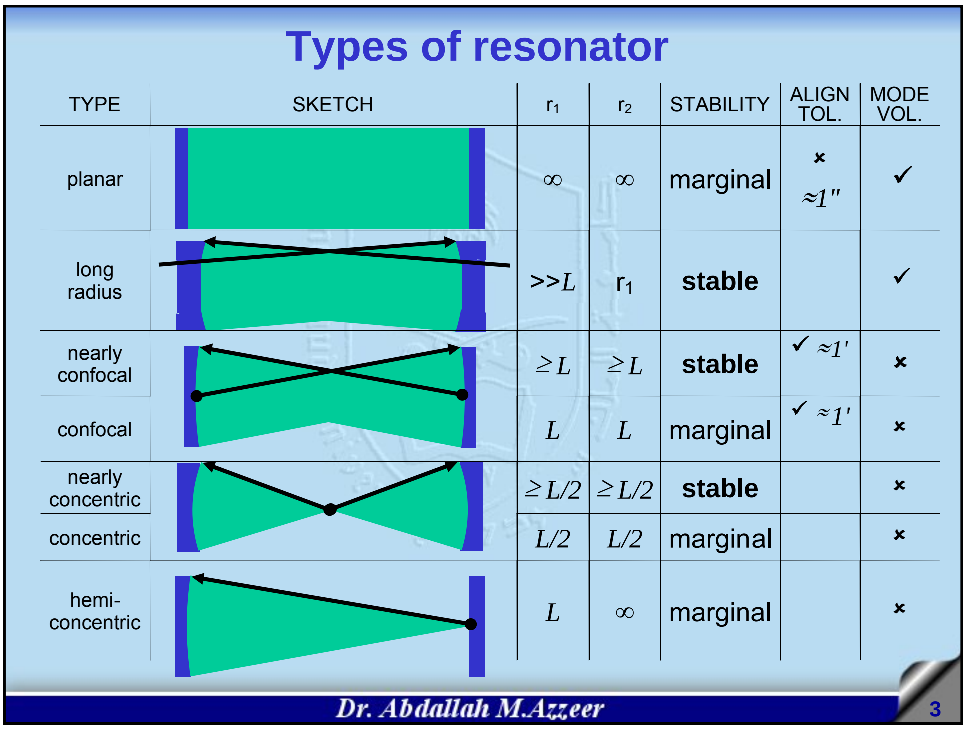
Mirror M_1
HR (High Reflector)

Reflectivity $R_1 \approx 100\%$
radius r_1

Mirror M_2
OC (Output Coupler)

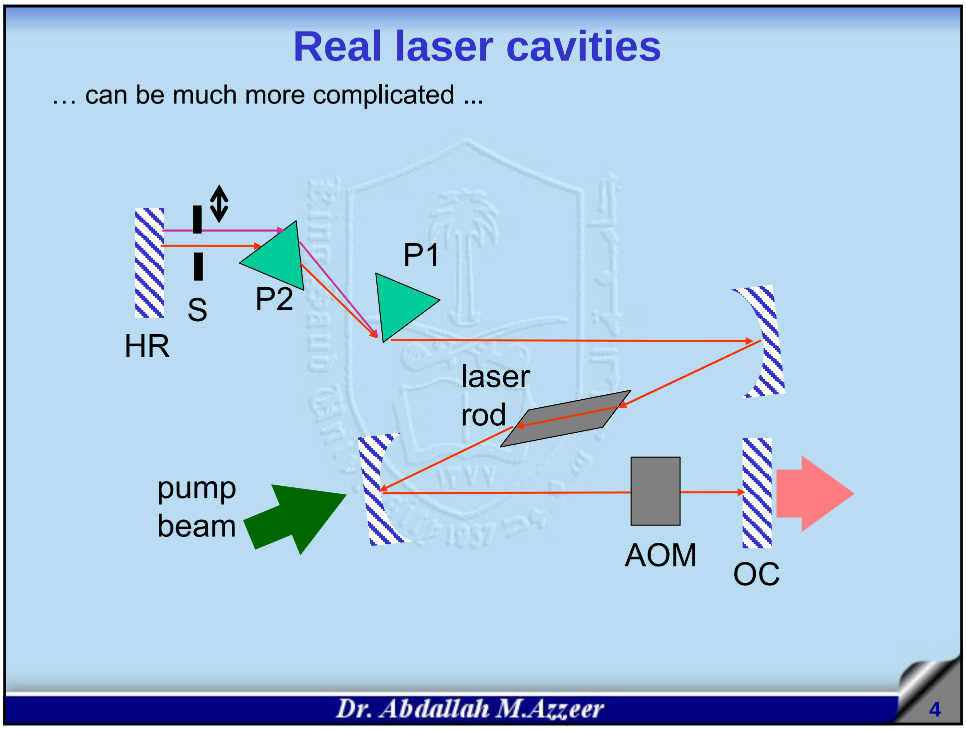
Reflectivity $R_2 < 100\%$
(e.g. 98%)
radius r_2

High reflectance, low absorption, flat $(\pm\lambda/20)$mirrors



Types of resonator

TYPE	SKETCH	r_1	r_2	STABILITY	ALIGN TOL.	MODE VOL.
planar		∞	∞	marginal	✗ $\approx 1''$	✓
long radius		$>>L$	r_1	**stable**		✓
nearly confocal		$\geq L$	$\geq L$	**stable**	✓ $\approx 1'$	✗
confocal		L	L	marginal	✓ $\approx 1'$	✗
nearly concentric		$\geq L/2$	$\geq L/2$	**stable**		✗
concentric		$L/2$	$L/2$	marginal		✗
hemi-concentric		L	∞	marginal		✗



Real laser cavities

… can be much more complicated ...

Transverse Electro-Magnetic Modes 1

The solution of Maxwell’s equations for a beam emitted from a simple laser cavity has transverse solutions (“TEM modes”) of the form:

$$E_{mn}(x,y) = E_0 H_m\left(\frac{\sqrt{2}x}{w}\right) H_n\left(\frac{\sqrt{2}y}{w}\right) \exp\left(-\frac{x^2+y^2}{w^2}\right)$$

H_m and H_m are Hermite polynomials, with the first few terms:

$H_0(x) = 1$; $H_1(x) = 2x$; $H_2(x) = 2(2x^2-1)$.

The most important mode is the TEM_{00} mode, which has a Gaussian radial distribution:

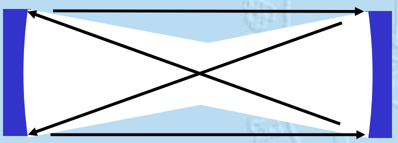
$$E_{00}(x,y) = E_0 \exp\left(-\frac{x^2+y^2}{w^2}\right) = E_0 \exp\left(-\frac{r^2}{w^2}\right)$$

w is a measure of the beam size.

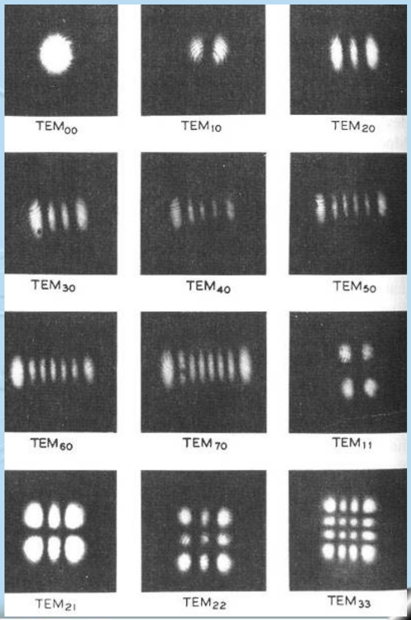
The TEM_{00} mode has the smallest divergence and can be focussed to the smallest spot size.

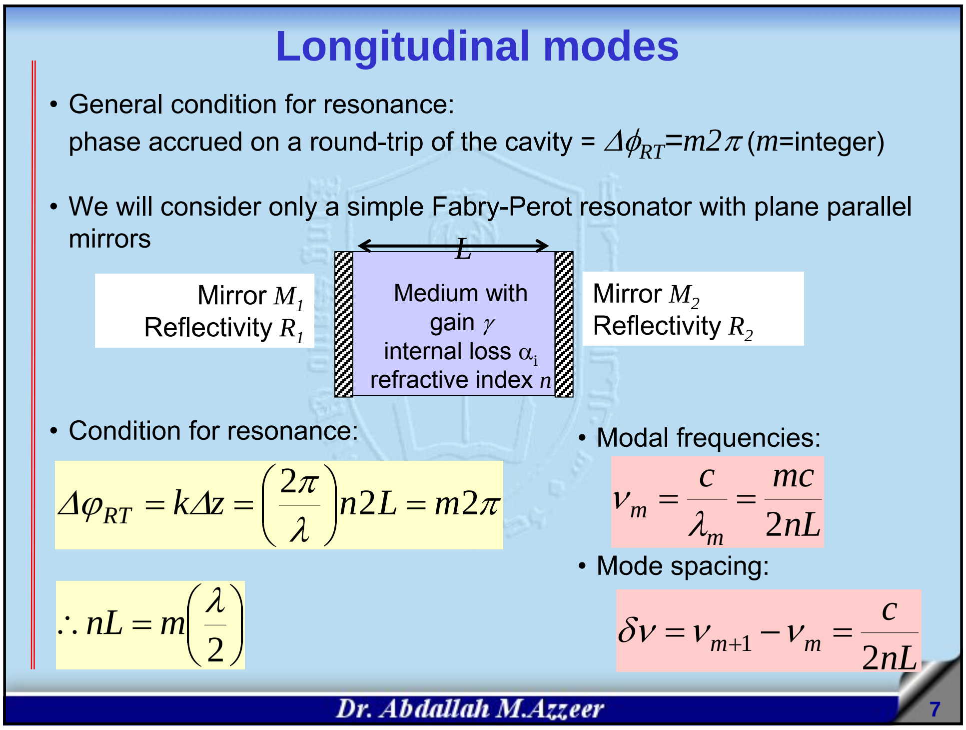
Transverse Electro-Magnetic Modes 2

- The 00 mode is the axial Gaussian beam.
- Non-axial self-replicating rays give rise to non-axial modes:



- Higher-order modes can be suppressed by aperturing





Longitudinal modes

- General condition for resonance:
  phase accrued on a round-trip of the cavity = $\Delta\phi_{RT}=m2\pi$ (m=integer)
- We will consider only a simple Fabry-Perot resonator with plane parallel mirrors

- Condition for resonance:

$$\Delta\varphi_{RT}=k\Delta z=\left(\frac{2\pi}{\lambda}\right)n2L=m2\pi$$

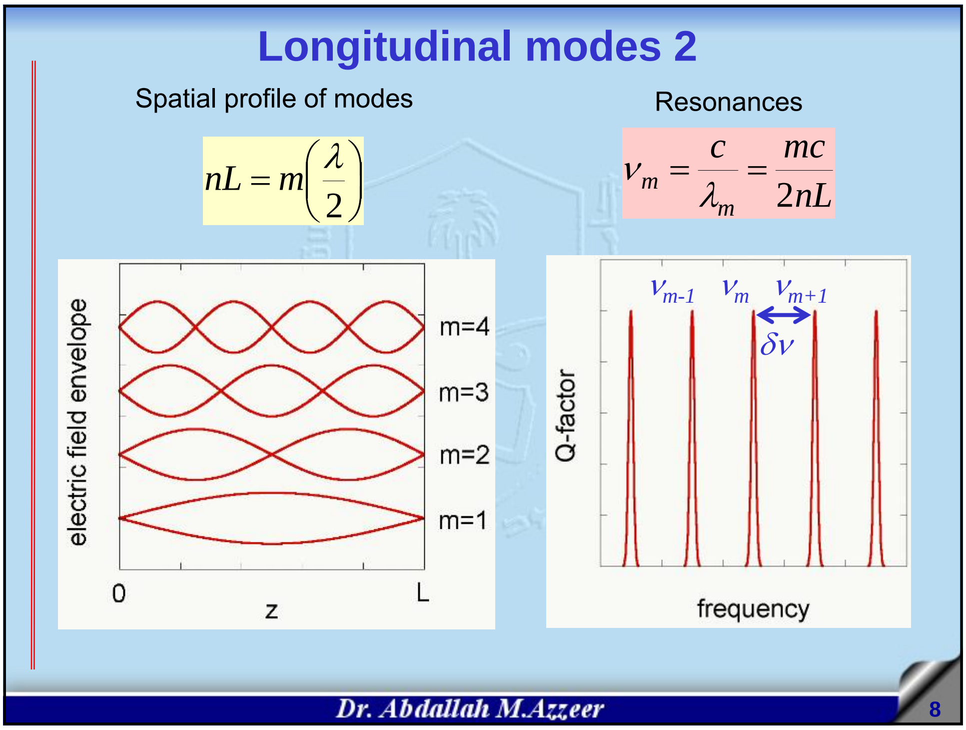
$$\therefore nL=m\left(\frac{\lambda}{2}\right)$$

- Modal frequencies:

$$\nu_m=\frac{c}{\lambda_m}=\frac{mc}{2nL}$$

- Mode spacing:

$$\delta\nu=\nu_{m+1}-\nu_m=\frac{c}{2nL}$$



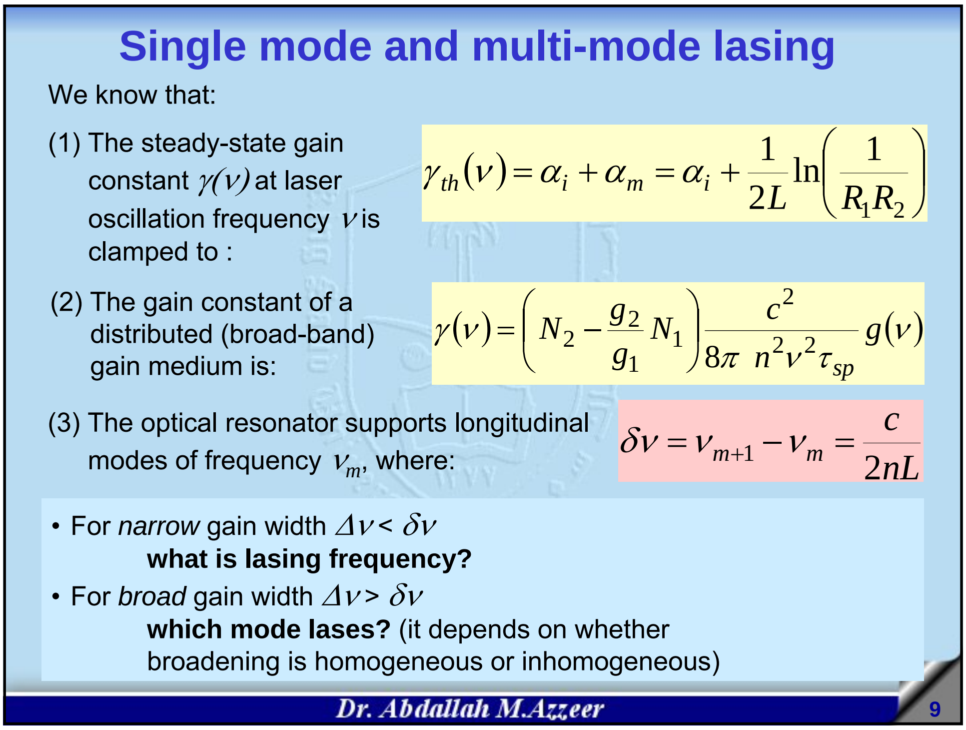
Longitudinal modes 2

Spatial profile of modes

$$nL=m\left(\frac{\lambda}{2}\right)$$

Resonances

$$\nu_m=\frac{c}{\lambda_m}=\frac{mc}{2nL}$$



Single mode and multi-mode lasing

We know that:

(1) The steady-state gain constant $\gamma(\nu)$ at laser oscillation frequency ν is clamped to :

$$\gamma_{th}(\nu) = \alpha_i + \alpha_m = \alpha_i + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)$$

(2) The gain constant of a distributed (broad-band) gain medium is:

$$\gamma(\nu) = \left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{c^2}{8\pi\ n^2\nu^2\tau_{sp}}\,g(\nu)$$

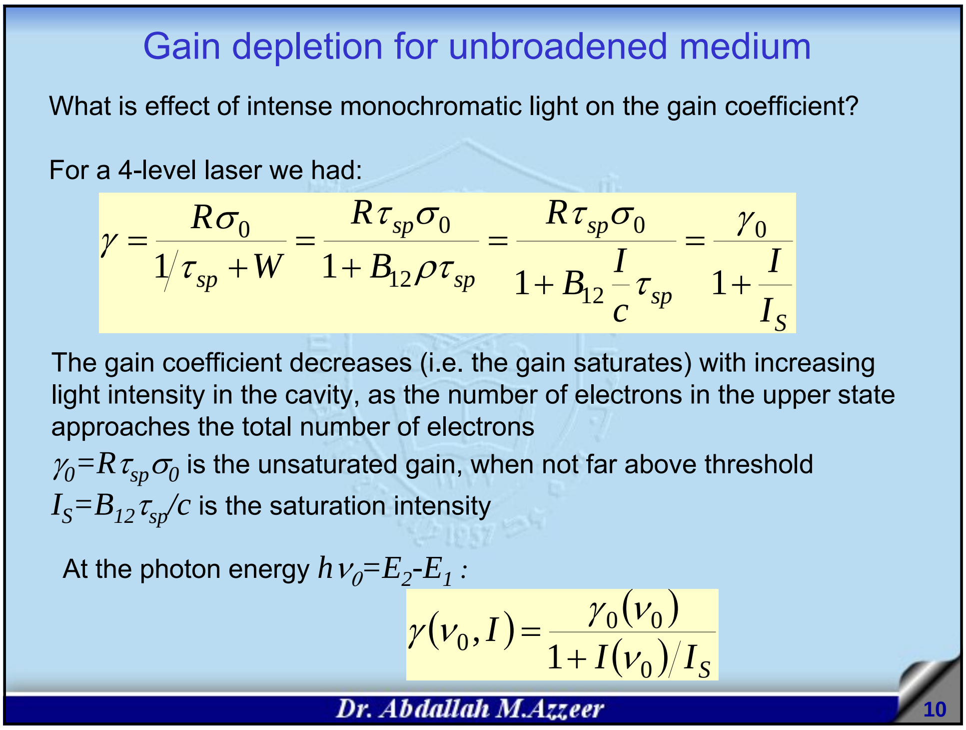
(3) The optical resonator supports longitudinal modes of frequency ν_m, where:

$$\delta\nu = \nu_{m+1} - \nu_m = \frac{c}{2nL}$$

- For *narrow* gain width $\Delta\nu < \delta\nu$

  what is lasing frequency?
- For *broad* gain width $\Delta\nu > \delta\nu$

  which mode lases? (it depends on whether broadening is homogeneous or inhomogeneous)



Gain depletion for unbroadened medium

What is effect of intense monochromatic light on the gain coefficient?

For a 4-level laser we had:

$$\gamma = \frac{R\sigma_0}{1/\tau_{sp} + W} = \frac{R\tau_{sp}\sigma_0}{1 + B_{12}\rho\tau_{sp}} = \frac{R\tau_{sp}\sigma_0}{1 + B_{12}\frac{I}{c}\tau_{sp}} = \frac{\gamma_0}{1 + \frac{I}{I_S}}$$

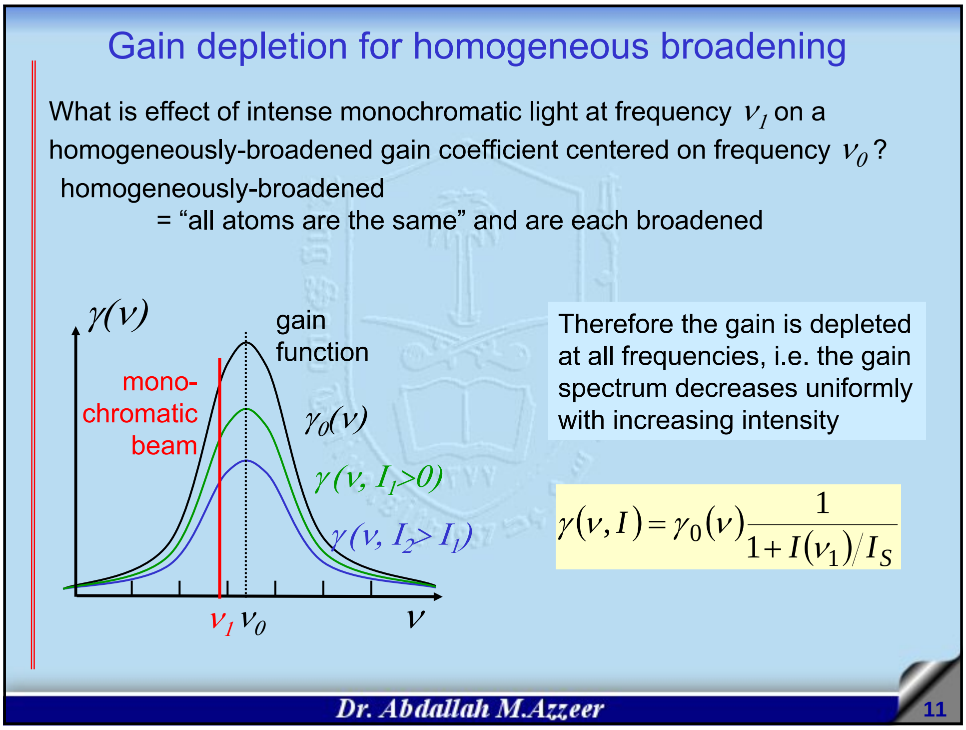
The gain coefficient decreases (i.e. the gain saturates) with increasing light intensity in the cavity, as the number of electrons in the upper state approaches the total number of electrons

$\gamma_0 = R\tau_{sp}\sigma_0$ is the unsaturated gain, when not far above threshold

$I_S = B_{12}\tau_{sp}/c$ is the saturation intensity

At the photon energy $h\nu_0 = E_2 - E_1$:

$$\gamma(\nu_0, I) = \frac{\gamma_0(\nu_0)}{1 + I(\nu_0)/I_S}$$



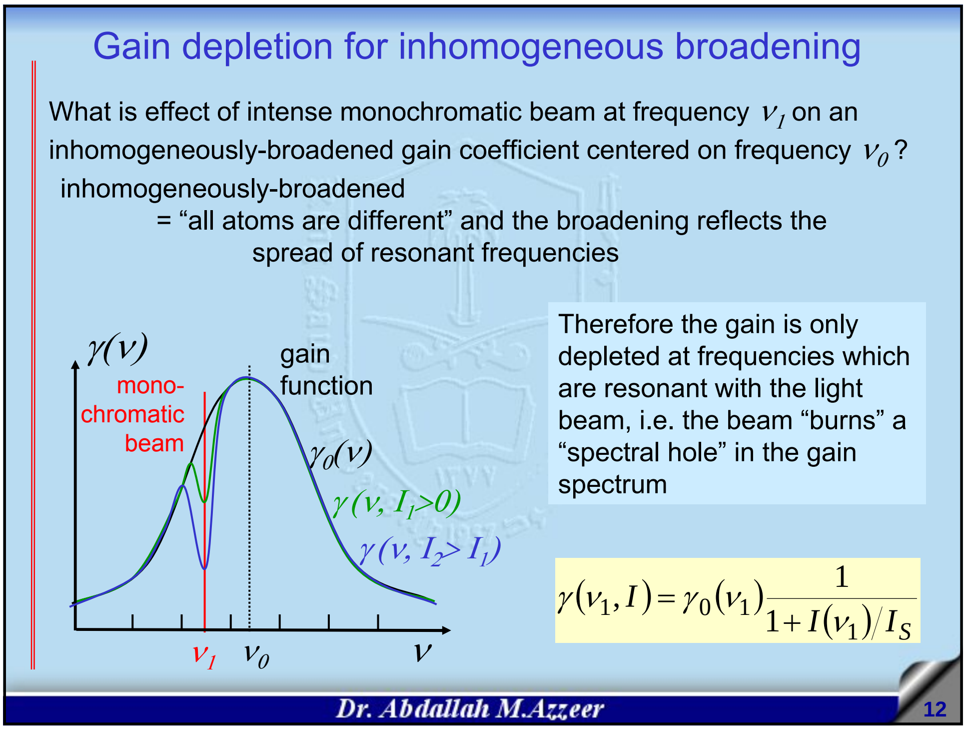
Gain depletion for homogeneous broadening

What is effect of intense monochromatic light at frequency ν_1 on a homogeneously-broadened gain coefficient centered on frequency ν_0?

homogeneously-broadened
= "all atoms are the same" and are each broadened

Therefore the gain is depleted at all frequencies, i.e. the gain spectrum decreases uniformly with increasing intensity

$$\gamma(\nu, I) = \gamma_0(\nu)\frac{1}{1+I(\nu_1)/I_S}$$



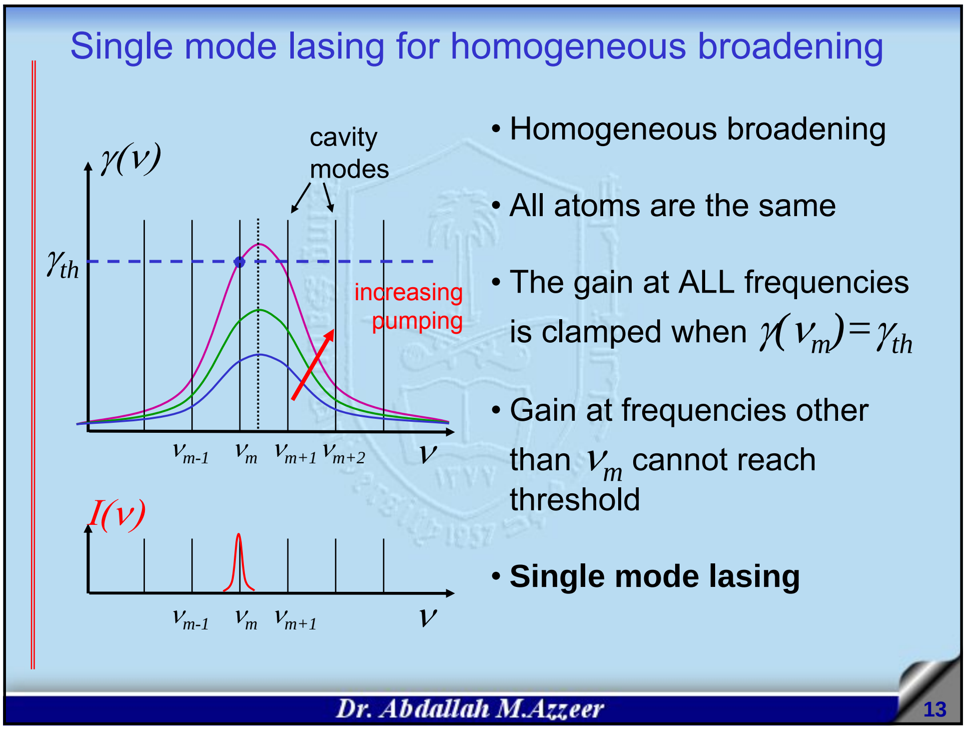
Gain depletion for inhomogeneous broadening

What is effect of intense monochromatic beam at frequency ν_1 on an inhomogeneously-broadened gain coefficient centered on frequency ν_0?

inhomogeneously-broadened
= "all atoms are different" and the broadening reflects the spread of resonant frequencies

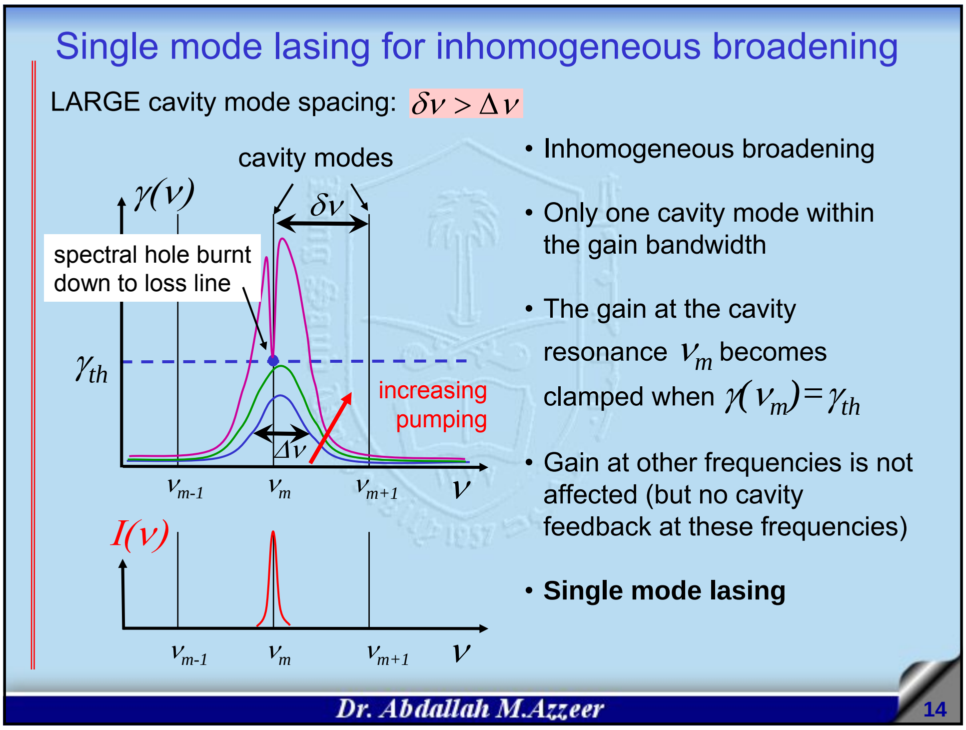
Therefore the gain is only depleted at frequencies which are resonant with the light beam, i.e. the beam "burns" a "spectral hole" in the gain spectrum

$$\gamma(\nu_1, I) = \gamma_0(\nu_1)\frac{1}{1+I(\nu_1)/I_S}$$



Single mode lasing for homogeneous broadening

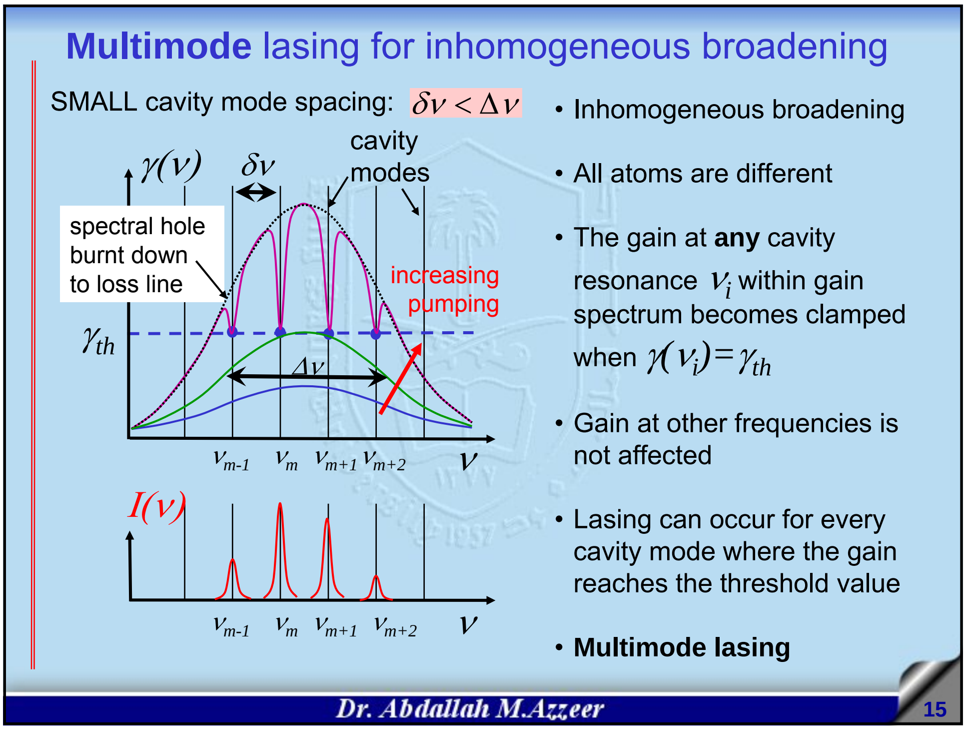
- Homogeneous broadening
- All atoms are the same
- The gain at ALL frequencies is clamped when $\gamma(\nu_m)=\gamma_{th}$
- Gain at frequencies other than ν_m cannot reach threshold
- **Single mode lasing**



Single mode lasing for inhomogeneous broadening

LARGE cavity mode spacing: $\delta\nu > \Delta\nu$

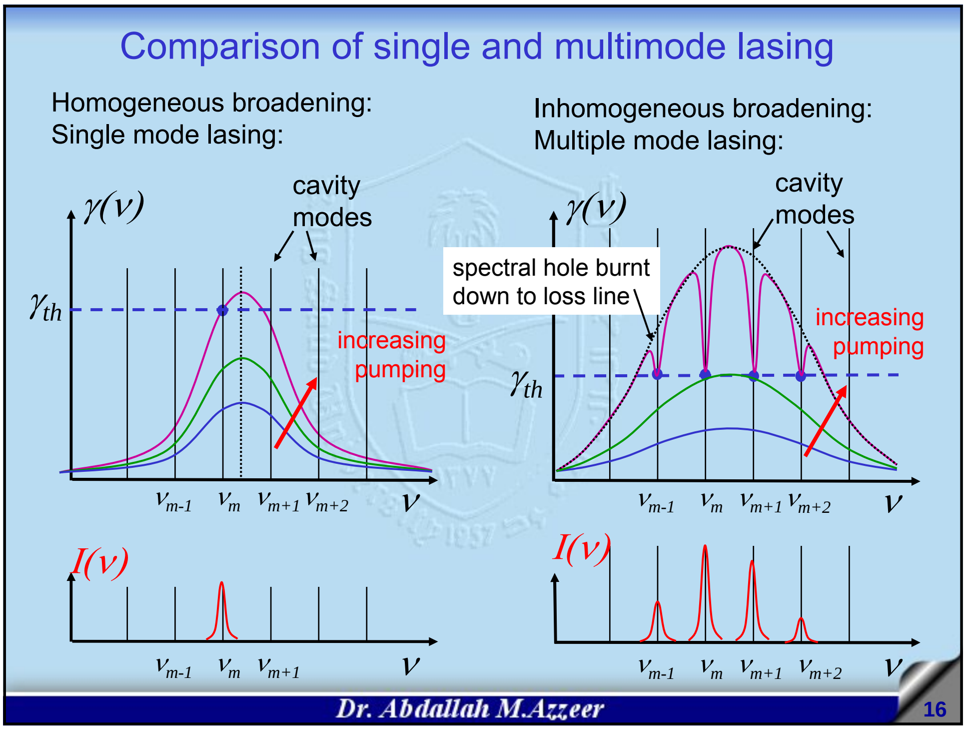
- Inhomogeneous broadening
- Only one cavity mode within the gain bandwidth
- The gain at the cavity resonance ν_m becomes clamped when $\gamma(\nu_m)=\gamma_{th}$
- Gain at other frequencies is not affected (but no cavity feedback at these frequencies)
- **Single mode lasing**



Multimode lasing for inhomogeneous broadening

SMALL cavity mode spacing: $\delta\nu < \Delta\nu$

- Inhomogeneous broadening
- All atoms are different
- The gain at **any** cavity resonance ν_i within gain spectrum becomes clamped when $\gamma(\nu_i)=\gamma_{th}$
- Gain at other frequencies is not affected
- Lasing can occur for every cavity mode where the gain reaches the threshold value
- **Multimode lasing**



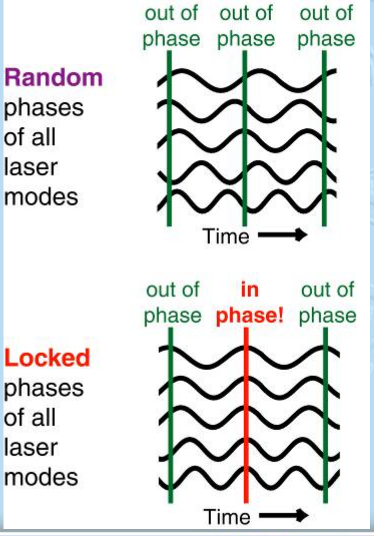
Comparison of single and multimode lasing

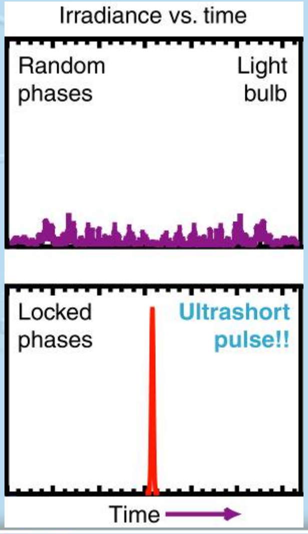
Homogeneous broadening:
Single mode lasing:

Inhomogeneous broadening:
Multiple mode lasing:

Generating short pulses = “mode-locking”

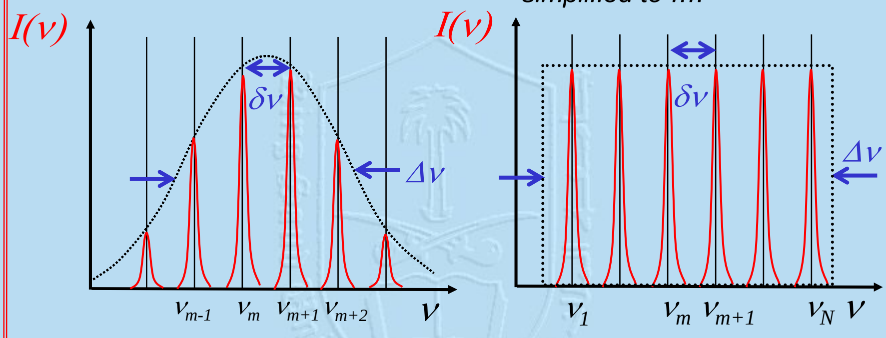
- Locking the phases of the laser modes yields an ultrashort pulse.





Simple approach to mode-locking

simplified to ….

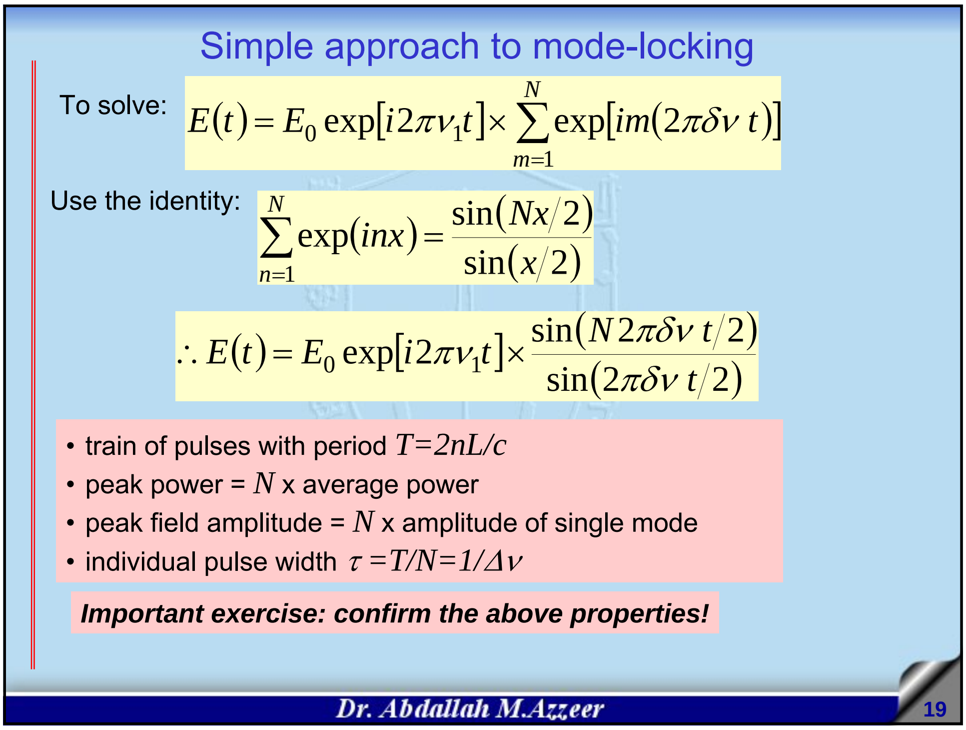


Each mode has an amplitude E_m, frequency $\nu_0+m\delta\nu$, phase ϕ_m

$$E(t) = \sum_m E_m \exp[i2(\nu_0 + m\delta\nu)t + i\phi_m]$$

Constant amplitude E_0, constant phase $\phi_0=0$

$$E(t) = E_0 \exp[i2\pi\nu_1 t] \times \sum_{m=1}^{N} \exp[im(2\pi\delta\nu\, t)]$$



Simple approach to mode-locking

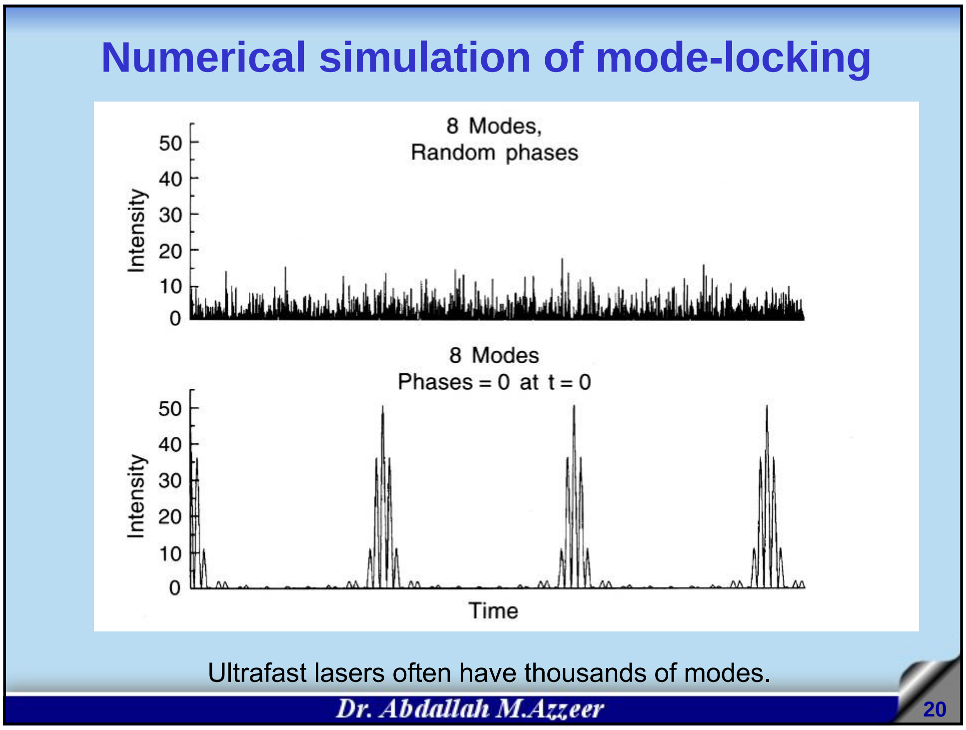
To solve: $E(t) = E_0 \exp[i2\pi\nu_1 t] \times \sum_{m=1}^{N} \exp[im(2\pi\delta\nu\, t)]$

Use the identity: $\sum_{n=1}^{N} \exp(inx) = \frac{\sin(Nx/2)}{\sin(x/2)}$

$$\therefore E(t) = E_0 \exp[i2\pi\nu_1 t] \times \frac{\sin(N 2\pi\delta\nu\, t/2)}{\sin(2\pi\delta\nu\, t/2)}$$

- train of pulses with period $T=2nL/c$
- peak power = N x average power
- peak field amplitude = N x amplitude of single mode
- individual pulse width $\tau = T/N = 1/\Delta\nu$

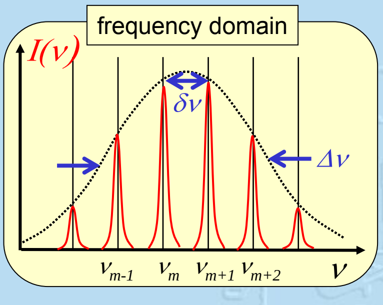
Important exercise: confirm the above properties!

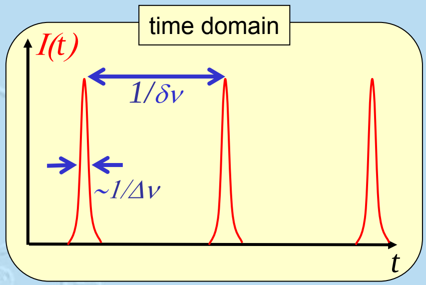


Numerical simulation of mode-locking

Ultrafast lasers often have thousands of modes.

How to achieve mode-locking?

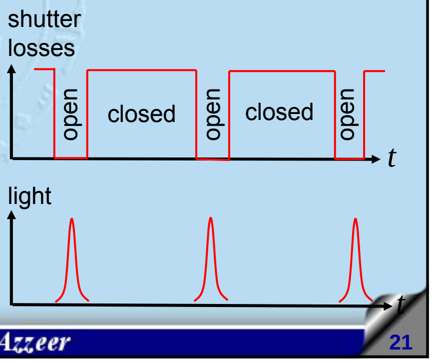




How to lock the phases ϕ_m?

Do it in the time domain, using a fast shutter, synchronised to cavity round-trip time, to modulate the cavity loss

CW modes cannot oscillate due to cavity losses. Only the mode-locked train is unperturbed by the shutter.



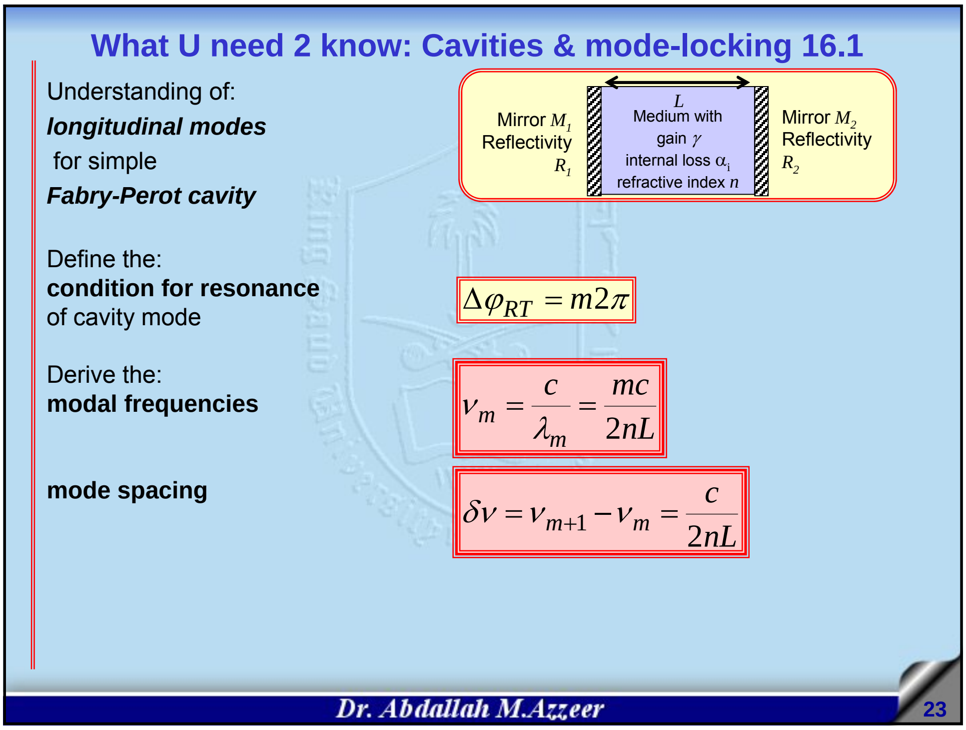
Methods of mode-locking

Active mode-locking:

- Modulate cavity losses using active device such as electro-optic or acousto-optic modulator
- Modulator induces sidebands in longitudinal mode ν_m which each corresponds to another longitudinal mode ν_{m+k} (k=0, ±1, ± 2, …)
- Can actually mode-lock inhomogeneously- and homogeneously- broadened lasers this way
- How to synchronise modulator to the cavity round-trip time? “Regenerative mode-locking” is one way (light detector→phase-locked loop→modulator)

Passive mode-locking:

- Use the short, intense light pulse of the mode-locked pulse train as its own shutter, inducing a nonlinear effect in either the laser material or a separate nonlinear element.
- Nonlinear interactions:
  - Saturable absorption (dye jet or semiconductor layers “SESAM”)
  - Kerr effect (intensity-dependent refractive index – see Ti-sapphire laser)



What U need 2 know: Cavities & mode-locking 16.1

Understanding of:
longitudinal modes
for simple
Fabry-Perot cavity

Define the:
condition for resonance
of cavity mode

$$\Delta\varphi_{RT} = m2\pi$$

Derive the:
modal frequencies

$$\nu_m = \frac{c}{\lambda_m} = \frac{mc}{2nL}$$

mode spacing

$$\delta\nu = \nu_{m+1} - \nu_m = \frac{c}{2nL}$$

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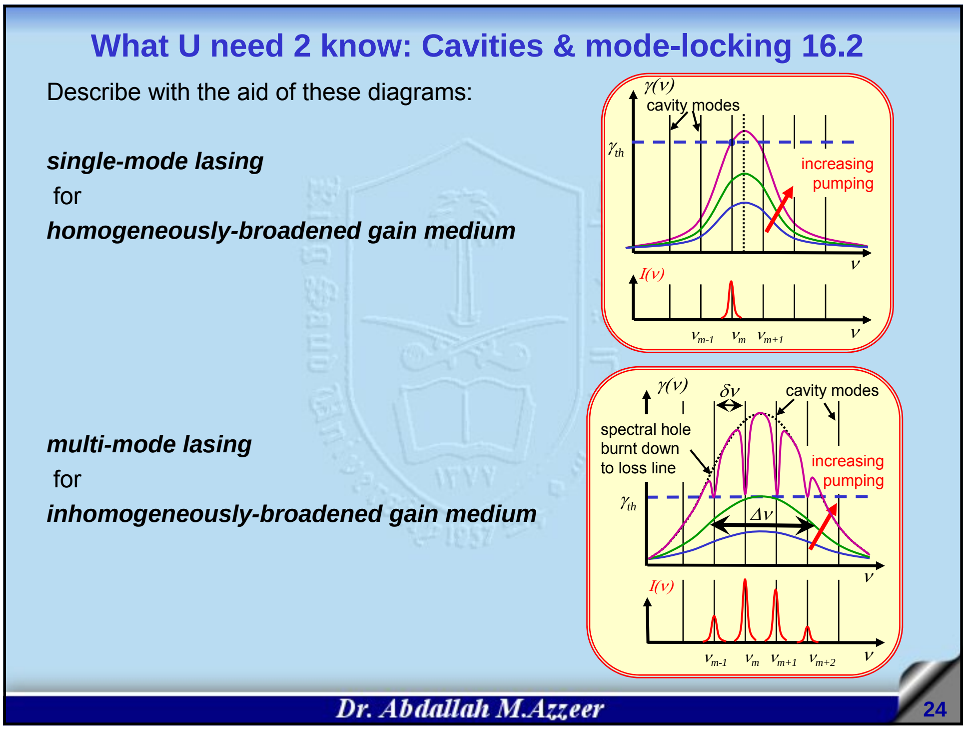

What U need 2 know: Cavities & mode-locking 16.2

Describe with the aid of these diagrams:

single-mode lasing
for
homogeneously-broadened gain medium

multi-mode lasing
for
inhomogeneously-broadened gain medium

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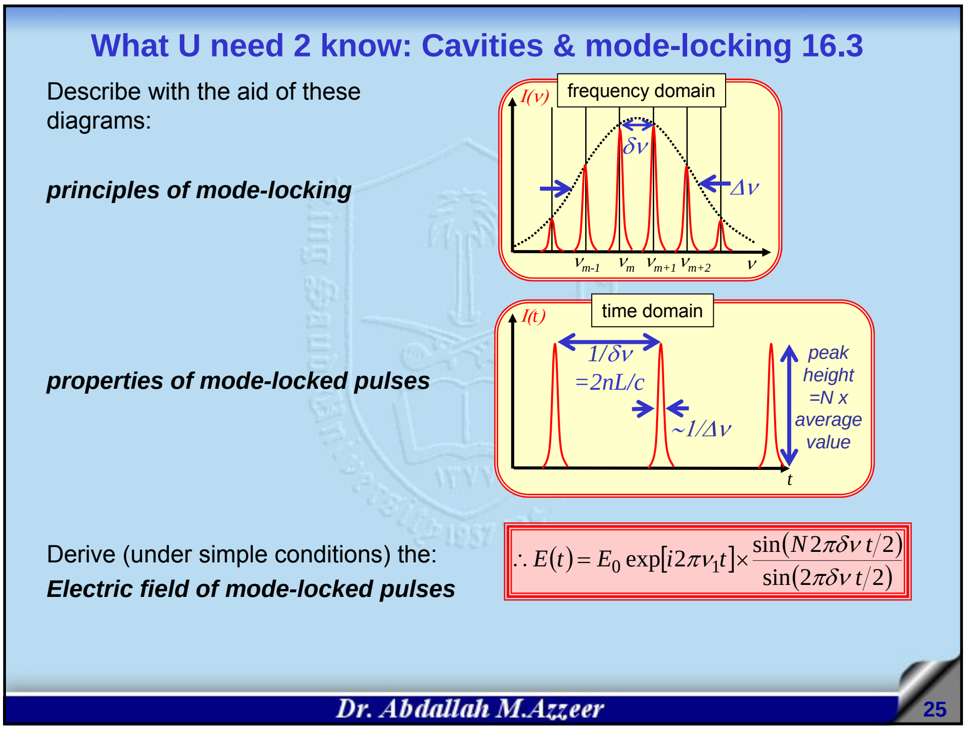

What U need 2 know: Cavities & mode-locking 16.3

Describe with the aid of these diagrams:

principles of mode-locking

properties of mode-locked pulses

Derive (under simple conditions) the:

Electric field of mode-locked pulses

$$\therefore E(t) = E_0 \exp[i2\pi\nu_1 t] \times \frac{\sin(N2\pi\delta\nu\, t/2)}{\sin(2\pi\delta\nu\, t/2)}$$