

## 14: Pulsed lasers: gain-switching and Q- switching

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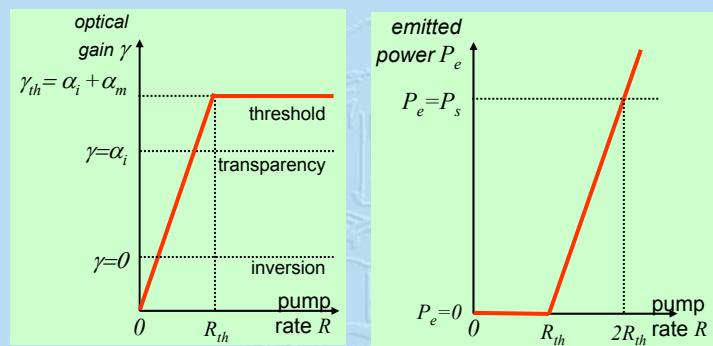
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### Methods for laser switching

So far, we have looked at the steady-state (CW) output from a laser:



But how can we turn a laser on and off ?

1. Change the pump rate  $R$  to modify the gain ("gain switching")
2. Change the cavity loss  $\alpha_i + \alpha_m$  to modify the lasing threshold ("Q-switching")
3. Employ beating between multiple modes of the laser cavity ("mode-locking")

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## Gain switching

**How fast can we switch a laser on and off, by modulating the pumping rate (i.e. what are the dynamics)?**

We've written all these rate equations, so let's use 'em !!

Remember the rate equations for an idealised 4-level system:

Re-writing the energy density in terms of photon number:

Assuming a further idealised system in which  $N_1 \approx 0$ , we end up with coupled rate equations for the **photon density  $\phi$**  and the **population inversion  $N$** .

$$N = \left( N_2 - \frac{g_2}{g_1} N_1 \right)$$

$$\begin{cases} \frac{dN_2}{dt} = R - \frac{N_2}{\tau_{sp}} - \left( N_2 - \frac{g_2}{g_1} N_1 \right) B_{21}\rho(v) \\ \frac{d\phi}{dt} = \left( N_2 - \frac{g_2}{g_1} N_1 \right) B_{21}\rho(v) - \frac{\phi}{\tau_c} \end{cases}$$

$$B_{21}\rho = \frac{c^3 A_{21}}{8\pi h v^3} \rho = c \frac{c^2 A_{21}}{8\pi v^2} \frac{\rho}{h v} = c \sigma_0 \phi$$

$$\begin{cases} \frac{dN}{dt} = R - \frac{N}{\tau_{sp}} - c \sigma_0 N \phi \\ \frac{d\phi}{dt} = N c \sigma_0 \phi - \frac{\phi}{\tau_c} \end{cases}$$

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## Gain switching 2

We can write these more simply in terms of the population inversion at threshold  $N_{th}$ . We should recognise these as a pair of differential equations describing a system of electrons  $N$  coupled to a system of photons  $\phi$  by the stimulated emission term.

We can obtain an analytical solution only in the small-signal limit  $N'(t) \ll N_0$ :

$$\begin{cases} \frac{dN}{dt} = R - \frac{N}{\tau_{sp}} - \frac{N}{N_{th}} \frac{\phi}{\tau_c} \\ \frac{d\phi}{dt} = \left( \frac{N}{N_{th}} - 1 \right) \frac{\phi}{\tau_c} \end{cases}$$

$$\begin{cases} N(t) = N_0 + N'(t) \\ \phi(t) = \phi_0 + \phi'(t) \end{cases}$$

Ignoring the  $N'(t), \phi'(t)$  terms:

$$\begin{cases} \frac{dN'(t)}{dt} = -N'(t) \left\{ \frac{1}{N_{th}} \frac{\phi_0}{\tau_c} + \frac{1}{\tau_{sp}} \right\} - \frac{N_0}{N_{th}} \frac{\phi'}{\tau_c} \\ \frac{d\phi'}{dt} = + \frac{N'(t)}{N_{th}} \frac{\phi_0}{\tau_c} + \left( \frac{N_0}{N_{th}} - 1 \right) \frac{\phi'(t)}{\tau_c} \end{cases}$$

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### Gain switching 3

Differentiating the equation for  $N'(t)$ , substituting for  $d\phi'/dt$  and setting the equilibrium terms to zero, we obtain:

$$\frac{d^2N'(t)}{dt^2} + \frac{dN'(t)}{dt} \left\{ \frac{1}{N_{th} \tau_c} \frac{\phi_0}{\tau_{sp}} + \frac{1}{\tau_c} \right\} + \frac{1}{\tau_c} \left\{ \frac{1}{N_{th} \tau_c} \frac{\phi_0}{\tau_{sp}} \right\} N'(t) = 0$$

With an identical expression for  $\phi'$ . This is the equation for a *damped harmonic oscillator*. It can be solved (an exercise for the student!) by solutions of the form

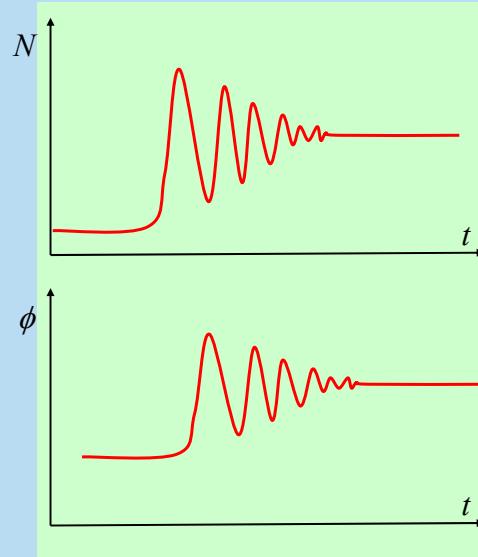
$$\begin{cases} N'(t) = N(\omega) \exp(-t/t_0) \cos(\omega t + \theta) \\ \phi'(t) = \phi(\omega) \exp(-t/t_0) \sin(\omega t + \theta) \end{cases}$$

The strength of the coupling relative to the excitation and decay rates determines whether the system is underdamped (exhibits relaxation oscillations) or overdamped (relaxes exponentially to steady-state values)

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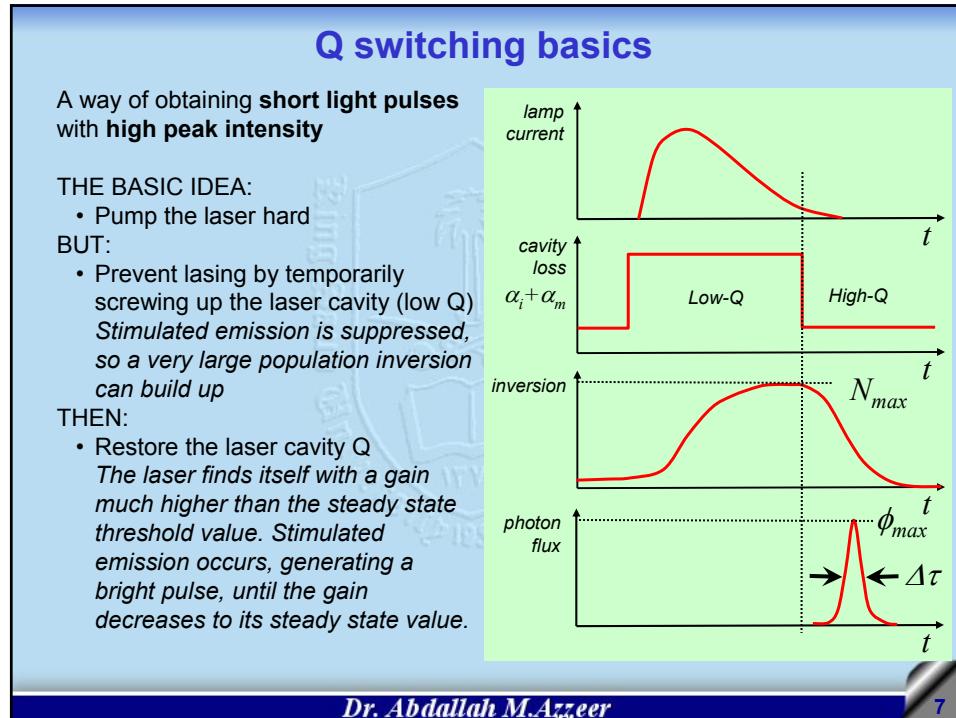
### Gain switching: relaxation oscillations



- Takes a time ( $\sim t_0$ ) to re-establish steady-state after a perturbation
- Limits the modulation rate / switching speed of lasers

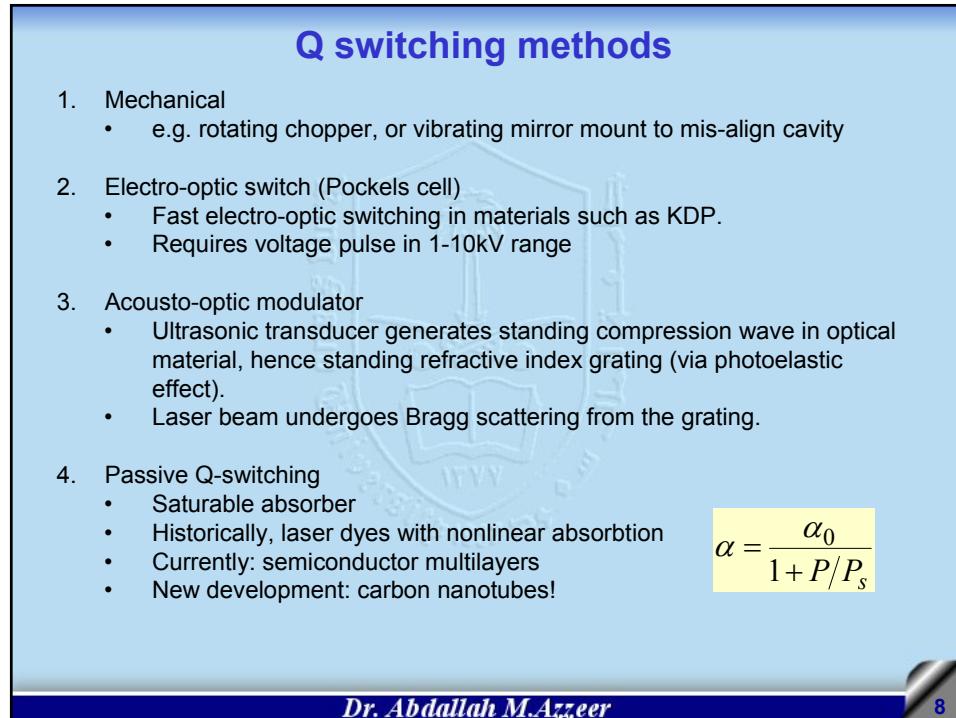
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## Q switching dynamics

The rate equation for photons can be written as:  
where  $N$  is the population inversion  
and  $N_{th}$  is its value at threshold.

$$\frac{d\phi}{dt} = \left( \frac{N}{N_{th}} - 1 \right) \left( \frac{\phi}{\tau_c} \right)$$

The rate equation for  $N$  (if there is no pumping when the Q is restored and spontaneous emission is negligible compared to stimulated emission) is:

$$\frac{dN}{dt} = -2 \frac{N}{N_{th}} \left( \frac{\phi}{\tau_c} \right)$$

A factor of 2 because  $N_2$  decreases by 1 while  $N_1$  increases by 1, for each stimulated emission process.

These equations can be solved, for an initial inversion  $N_0$ : see {AY6.9} or {CD8.16}.

Maximum power is

$$P_{\max} \approx \frac{h\nu}{2\tau_c} N_0$$

Max. number of photons in cavity is:  $\phi_{\max} \approx \frac{N_0}{2}$

Decay rate of Q-switched pulse is:  $\tau_c$

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## Gain switching 4

We can obtain the parameters in "gain switching 3" more simply if we write:

$$\begin{cases} N(t) = N_0 + N' \exp(-\alpha t) \exp(j\omega t) \\ \phi(t) = \phi_0 + \phi' \exp(-\alpha t) \exp(j\omega t) \end{cases}$$

obtaining:

$$\begin{cases} \alpha = \frac{1}{2} \left\{ \frac{1}{N_{th}} \frac{\phi_0}{\tau_c} + \frac{1}{\tau_{sp}} \right\} \\ \omega^2 = \frac{1}{\tau_c} \left\{ \frac{1}{N_{th}} \frac{\phi_0}{\tau_c} \right\} - \alpha^2 \end{cases}$$

If  $\alpha \ll \omega$  (underdamping):

$$\omega \approx \sqrt{\frac{1}{\tau_c} \left\{ \frac{1}{N_{th}} \frac{\phi_0}{\tau_c} \right\}}$$

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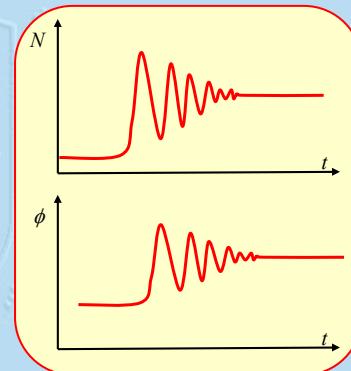
## What U need 2 know: LASER switching 14.1

Be aware of the existence of :  
**coupled equations**  
**for electron and photons**

$$\begin{cases} \frac{dN}{dt} = R - \frac{N}{\tau_{sp}} - \frac{N}{N_{th} \tau_c} \phi \\ \frac{d\phi}{dt} = \frac{N}{N_{th} \tau_c} \phi - \frac{\phi}{\tau_c} \end{cases}$$

pumping      spontaneous emission  
 net stimulated emission      cavity loss

Be aware of the existence of :  
**relaxation oscillations**

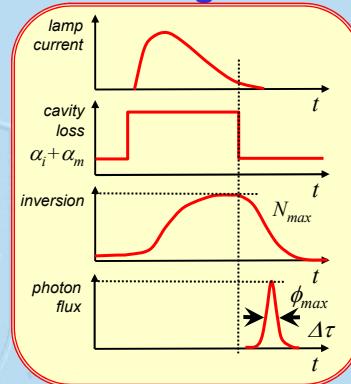


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## What U need 2 know: LASER switching 14.2

Describe briefly with the aid of these diagrams, the basic principles of:  
**Q-switching**



Note that :

**Q-switch pulse duration ~ cavity lifetime**

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