Elementary particle physics (480 phys)
Autumn 2020
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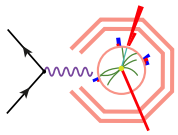

Homework 2

Problem 1.1 – Relativistic electrodynamics (10 Points)

Working in natural units, recall that the electric and magnetic fields are related to the vector and scalar potentials by

$$\vec{E} = -\vec{\nabla}\,\phi - \partial_t \vec{A}$$
$$\vec{B} = \vec{\nabla} \wedge \vec{A}$$

We have defined the 4- potential $A_\mu = (\phi, -\vec{A})$ and the 4-divergence $\partial_\mu = (\partial_t, \vec{\nabla})$, also the 4-current $J^\mu = (\rho, \vec{j})$ *Hint : Repeated induces are summed over and* μ,ν.. $\in$ 0,1,2,3

1. Charge conservation implies the continuity equation $\partial\rho + \vec{\nabla} \cdot \vec{j} = 0$, show that it could be written in the Lorentz index notation

$$\partial_\mu J^\mu = 0$$

2. Show that the Faraday tensor defined as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ takes the matrix form

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix}.$$

Hint : Observe that the vector product in index notation can be expressed as

$$(\vec{a} \wedge \vec{b})_k = \epsilon_{ijk} a^i b^j = (a_i\, b_j - a_j\, b_i)_k \quad i,j,k \in 1,2,3$$

Problem 1.2 – Collider dynamics (5 Points)

1. Usually, in order to detect particle tracks, experimentalists impose a cut on the momentum of the particle to be $|\ \vec{p}\ | > 2$GeV what are the allowed psedorapidity and transverse momentum regions that a particle can have satisfying this cut.

2. At the LHC, we could obtain $\sqrt{s} = 13$TeV by colliding -head on- 2 protons :

   (a) What is the energy of a single proton beam ?

   (b) If we need to achieve the same $\sqrt{s}$ but for fixed target experiment, what is the energy of the incoming proton needed here ?

   (c) If we replaced one of the protons with lead $^{208}_{82}Pb$, compute the head on $\sqrt{s_{NN}}$.

Problem 1.3 – Double Higgs production (5 Points)

One of the anticipated process we are looking forward to seeing is the production of Higgs boson pairs in the High-Luminosity LHC (HL-LHC)
Calculate the expected number of HH events produced at the HL-LHC run 1 and 2 if

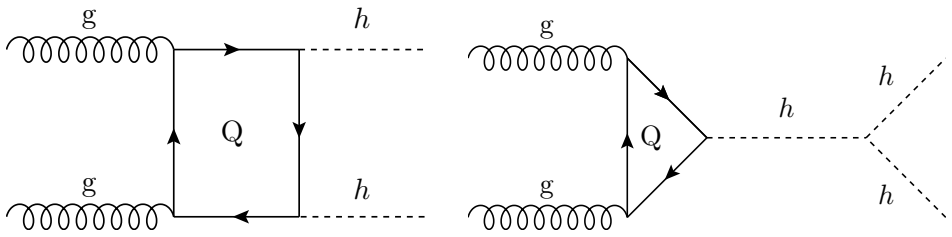

Abbildung 1: Feynman diagrams for the ggF process of Higgs pair production in the SM.

you know that each of the Higgs bosons will decay, and we will consider the final states

$$pp \to HH \to b\bar{b}\gamma\gamma,$$

$$pp \to HH \to b\bar{b}b\bar{b}.$$

Given the information in the table below

Parameter	value	description
$\sigma(pp \to HH)$	$36.4\,\text{fb}$	production cross-section
$\mathcal{L}_1$	$300\,\text{fb}^{-1}$	integrated luminosity for run 1
$\mathcal{L}_2$	$3\,\text{ab}^{-1}$	integrated luminosity for run 2
$\mathcal{B}(H \to b\bar{b})$	5.807×10^{-1}	branching ratio for the decay to b quarks
$\mathcal{B}(H \to \gamma\gamma$	2.27×10^{-3}	branching ratio for the photons

Tabelle 1: The collider and Higgs parameters used for the HL-LHC.