Chapter 3

3.1 Methods of moment:

$$E(x^k) = \frac{\sum_{i=1}^{n}(x_i)^k}{n}, k = 1, \dots, m \quad \textit{where } m \textit{ is the number of parameters}$$

- **COMPLETE DATA:**

3.2.1 Maximum likelihood method:

$$L(\theta) = \prod_{i=1}^{n} f(x_i|\theta)$$
$$\ell = \ln\big(L(\theta)\big)$$
$$\frac{\partial \ell}{\partial \theta} = 0, \textit{differentiate with respect to each parameter you need to estimate}$$

3.2.2 Maximum likelihood method for grouped data:

Interval	# of observation
(c_0, c_1)	m_1
:	:
(c_{n-1}, c_n)	m_n

$$L(\theta) = \prod_{i=1}^{n}\big(F(c_i) - F(c_{i-1})\big)^{m_i}$$

3.2.3 Maximum likelihood method for mixed data:

Interval	# of observation
(c_0, c_1)	m_1
:	:
(c_{n-1}, c_n)	m_n
x_1	k_1
:	:
x_p	k_p

$$L(\theta) = \prod_{i=1}^{n}\big(F(c_i) - F(c_{i-1})\big)^{m_i} * \prod_{i=1}^{p}\big(f(x_i)\big)^{k_i}$$

- **INCOMPLETE DATA:**

Censored data:

- Left-censored data: an observation less than value d is recorded as d
- Right-censored data: an observation greater than value u is recorded as u

Truncated data:

- Left-truncated data: an observation less than value d is not recorded
- Right-truncated data: an observation greater than value u is not recorded

3.3 Method of percentiles:

1. Order data
2. Find p

$$\frac{p}{n+1} \leq g < \frac{p+1}{n+1}$$

3. $\hat{\pi}_p = (n+1)\left[\left(\frac{p+1}{n+1} - g\right)x_{(p)} + \left(g - \frac{p}{n+1}\right)x_{(p+1)}\right]$

$$\hat{\pi}_p \in (x_p, x_{p+1})$$

$$\hat{\pi}_p = F_x^{-1}(g)$$

3.4 Fisher information

$$I(\theta) = E\left(\ell'(\theta)\right)^2 = -E\left(\ell''(\theta)\right)$$

$$where\ \ell(\theta) = Ln\left(L(\theta)\right)$$

$$\widehat{Var}\left(\hat{\theta}\right) = \frac{1}{I(\theta)}$$