PHYSICS 501
3rd HOMEWORK
Dr. V. Lempesis

1. In the field of atom cooling we consider the interaction of atoms with a special family of lasers, named *optical vortices*. One family of such beams are the so called Laguerre-Gaussian lasers. These lasers are characterized of a phase and intensity with circular cylindrical symmetry. Indeed their intensity is given by:

$$I = A\left(\frac{\rho}{w_0}\right)^{2\ell} \exp\left(-\frac{2\rho^2}{w_0^2}\right) L_p^{|\ell|}\left(\frac{2\rho^2}{w_0^2}\right)$$

where $L_p^{|\ell|}\left(2\rho^2 / w_0^2\right)$ is the Laguerre polynomial with $x = 2\rho^2 / w_0^2$, w_0 is the laser beam waist (constant) and A a constant. When an atom interacts with such a laser beam under certain conditions it may feel a potential energy known as *optical dipole potential energy* which is given by $U = cI$ where a c is a constant. Calculate the force which corresponds to this potential energy, assuming the p=0, so $L_p^{|\ell|}\left(2\rho^2 / w_0^2\right) = 1$. (Hint: $\mathbf{F} = -\vec{\nabla}U$)

(5 marks)

2. A particle is moving through space. Find the circular cylindrical components of its acceleration. (Hint: this is the second part of Q. 5.10).

(5 marks)

3. From the results of problem 5.21 calculate the partial derivatives of $\hat{\mathbf{r}}$, $\hat{\theta}_0$ and $\hat{\varphi}_0$ with respect to r, θ and φ.

(5 marks)

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4. From the results of problem 5.27 show that

$$-i\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) = -i\frac{\partial}{\partial \varphi}$$

This is the quantum mechanical operator corresponding to the z-component of angular momentum.

(5 marks)

For the girls: Please send your answers in pdf form (typed or in clearly handwritten form) in my email address (vlempesis@ksu.edu.sa). Please use ONE file for your entire homework NOT one file per page. Please do not forget to put your name and your ID number on it AND on your file name.
Your deadline is on Wednesday 7th November 2018 at 23:59.

For the boys: You will hand in your homework in hard copy in my office on Thursday 8th November up to 11:59.