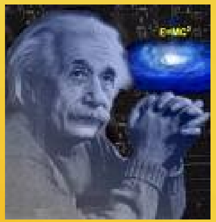
الديناميكا النسبية
RELATIVISTIC DYNAMICS



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الكتلة وكمية الحركة في النسبية
Mass & Momentum Relativity

في الفيزياء التقليدية تعرّف كمية الحركة بالصورة التالية:

$$\vec{p} = m\vec{v}$$

وتكون كمية الحركة الخطية لأي نظام محافظة (conserved) إذا كان:

$$F_{ext} = dP/dt = 0$$

وهذه التعريفات تكون صحيحة كذلك في الميكانيكا النسبية

تتصادم كرتان متماثلتان تصادما مرنا

إذا كان هناك راصدين علي محور y يقفان على مسافة متساوية من نقطة الأصل ويقذفان الكرتين بسرعة متساوية (V_A و V_B') في إتجاه نقطة الأصل

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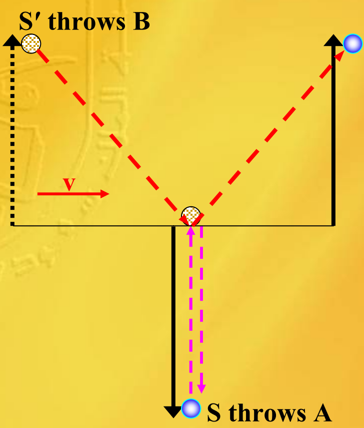
وبإستخدام مبدأ حفظ كمية الحركة ، فإن سرعة الكرتين بعد التصادم تكون متساوية مقدارا ومتعاكسة في الإتجاه .

وعندما يتحرك الراصد في 'S بسرعة v في إتجاه $+x$ فإن الراصد في S سوف يشاهد التصادم حسب الشكل المجاور .



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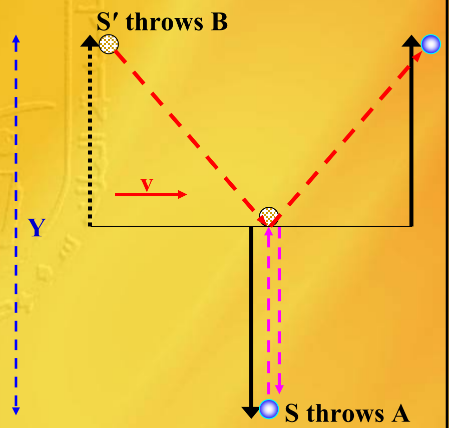
نفترض أن سرعة الكرتين كما تقاس بواسطة الراصد في S هي V_A و V_B وأن المسافة التي قطعتها كل كرة (المركبة في إتجاه y) متساوية Y/2 .

إذا كان الزمن الذي إستغرقته الكرة A (من تحركها وصدمها الكرة B وعودتها مرة أخرى) والمقاس بواسطة الراصد في S هو T_0 .

فإن الزمن الذي يقيسه الراصد في S للكرة B يكون أقصر (ظاهرة تمدد الزمن) ويساوي T .



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حسب الراصد في S :

$$V_A = \frac{Y}{T_0} \qquad T = \frac{Y}{V_B} \qquad T_0 = \frac{Y}{V_A}$$

حسب الراصد في S' :

$$T_0 = \frac{Y}{V'_B}$$

تمدد الزمن:

$$T = \frac{T_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

حسب الراصد في S تكون السرعة V_B :

$$V_B = \frac{Y}{T} = \frac{Y\sqrt{1-\left(\frac{v}{c}\right)^2}}{T_0}$$

$$V_B < V_A$$

حسب الراصد في S فإن كمية الحركة للكرتين يمكن كتابتهما علي الصورة:

$$p_B = m_B V_B = m_B \frac{Y\sqrt{1-\left(\frac{v}{c}\right)^2}}{T_0} \qquad p_A = m_A V_A = m_A \frac{Y}{T_0}$$

فإذا كانت $m_A = m_B$ فإن $p_B < p_A$ (لأن $V_B < V_A$) .

وبالتالي فإن كمية الحركة غير محافظة في هذا التصادم المرن ! !

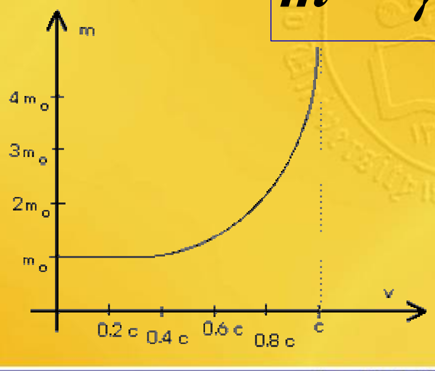
ولكي تكون كمية الحركة محافظة لابد أن تكون $p_A = p_B$ أو:

$$m_B = \frac{m_A}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

وهذه تسمى بتحويلات الكتلة النسبية (‘relativistic mass’)

$$m = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{m_0}{\sqrt{1-\beta^2}}$$

$$m = \gamma m_0$$



(rest mass) تسمى الكتلة السكونية m_0

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متى يمكن إستخدام الكتلة السكونية (rest mass)

$$m(v) = \frac{m}{\sqrt{1-\left(\frac{v}{c}\right)^2}} ?$$

Object	*v*	*v/c*	*m(v)/m*
jogger	10 km/h	.000000009	≈ 1
space shuttle	10^4 m/s	0.000033	1.0000001
electron	10^6 m/s	0.0033	1.001
electron	10^8 m/s	0.333	1.061

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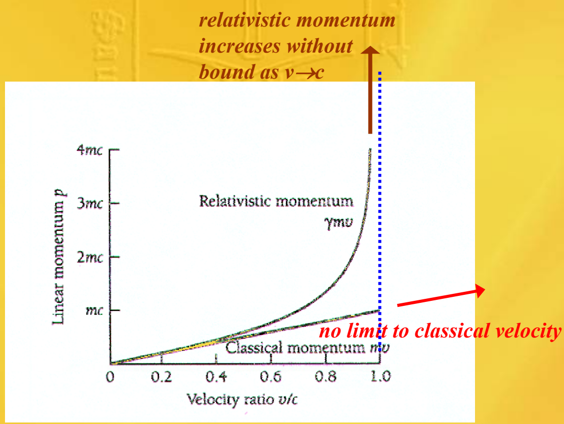
Relativistic momentum

$$\vec{p} = \gamma m \vec{v}$$



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READ Examples from notes

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Relativistic Energy

يمكن إعادة تعريف مفهوم الشغل والطاقة بتعديل قانون نيوتن الثاني ليشمل التعريف الجديد لكمية الحركة

وبالتالي يمكن كتابة قانون نيوتن الثاني على الصورة:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(\gamma m\vec{v}) = \frac{d}{dt}\left(\frac{m\vec{v}}{\sqrt{1-(v/c)^2}}\right)$$

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Mass and Energy

From your first-semester physics course:

$$KE = \int F ds = \int \frac{d(\gamma mv)}{dt} ds.$$

Use the definition of γ and integrate by parts to get

$$KE = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$$

$$\gamma mc^2 = mc^2 + KE.$$

Assuming potential energy is zero (we can always choose coordinates to do this), we interpret γmc^2 as total energy.

$$\boxed{E = mc^2 + KE.}$$

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When an object is at rest KE = 0, and any energy that remains is interpreted as the object's rest energy E_0.

$$E_0 = mc^2.$$

When an object is moving, its total energy is

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}.$$

This is really just a variation of the OSE on the previous slide.

This is the closest you'll come to seeing $E=mc^2$ in this class. In the "old days," $E=\gamma mc^2$ would have been written $E=mc^2$.

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These equations have a number of interesting implications.

Mass and energy are two different aspects of the same "thing."

Conservation of energy is actually conservation of mass-energy.

The c^2 in $E_0=mc^2$ means a little mass is "worth" a lot of energy.

Your lunch: an example of relativity at work in "everyday life."

Total energy is conserved but not relativistically invariant.
Rest (or proper) mass is relativistically invariant.
Mass is not conserved! (But it is for the purposes of chemistry.)

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Example: when 1 kg (how much is that?) of dynamite explodes, it releases 5.4×10^6 joules of energy. How much mass disappears?

If we are to claim relativistic mechanics as a replacement theory for Newtonian mechanics, then relativistic mechanics had better reduce to Newtonian mechanics in the limit of small relative velocities.

$$KE = \gamma mc^2 - mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 .$$

Beiser shows (page 29) that for v<<c,

$$KE \approx \frac{1}{2} mv^2 .$$

When can I get away with using $KE = mv^2/2$, and when do I have to use $KE = \gamma mc^2 - mc^2$?

Use Newtonian KE every time you can get away with it! Use relativistic KE only when you must!

If $v = 1x10^7$ m/s (fast!) then $mv^2/2$ is off by only 0.08%. Probably OK to use $mv^2/2$. If v = 0.5 c, then $mv^2/2$ is off by 19%. Better use relativity.

I won't purposely try to trap you into the "wrong" calculation. Often you will do that without my help. Sometimes I will ask you to make a judgment, but I will always give you the criteria. I might ask, if an error no greater than 5% is tolerable, is a relativistic calculation necessary?

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Energy and Momentum

Total energy and magnitude of momentum are given by

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \qquad p = \frac{mv}{\sqrt{1 - v^2/c^2}} \ .$$

With a bit of algebra, you can show

$$E^2 - p^2c^2 = (mc^2)^2 \ .$$

The quantities on the LHS and RHS of the above equation are relativistically invariant (same for all inertial observers).

Rearranging:

$$\boldsymbol{E^2 = (mc^2)^2 + p^2c^2} \ .$$

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$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \qquad p = \frac{mv}{\sqrt{1 - v^2/c^2}} \ .$$

Is it possible for a particle to have no mass? If $m = 0$, what are E and p?

For a particle with $m = 0$ and $v < c$, then $E=0$ and $p=0$. A "non-particle." No such particle.

But if $m = 0$ and $v = c$, then the two equations above are indeterminate. We can't say one way or the other.

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If $m = 0$ and $v = c$, we must use $E^2 = \left(mc^2\right)^2 + p^2c^2$.

The energy of such a particle is $E = pc$. We could detect this particle! It could exist.

Do you know of any massless particles?

- photon
- neutrino*
- graviton**

graviton is to gravity as photon is to E&M field

**Maybe. Nobel prize for you if you show $m_{neutrino} = 0$.*

***Maybe. Nobel prize for its discoverer. Problem: gravitational fields much, much weaker than E&M fields.*

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Looking ahead…

- Particles having KE >> E_0 (or pc >> mc^2) become more photon-like and behave more like waves.
- The momentum carried by massless particles is nonzero (E = pc).

Could you stop a freight train with a flashlight?

Could you stop a beam of atoms with a laser beam?

A note on units.

We will use the electron volt (eV) as an energy unit throughout this course.

$$1\,\text{eV} = \left(1.6\cdot10^{-19}\ \text{C}\right)\cdot\left(1\ \text{V}\right) = 1.6\cdot10^{-19}\ \text{J}$$

Variations on the eV:

1 meV = 10^{-3} eV (milli)
1 keV = 10^{3} eV (kilo)
1 MeV = 10^{6} eV (mega)
1 GeV = 10^{9} eV (giga)

Because mass and energy are convenient, we sometimes write masses in “energy units.”

An electron has a rest mass of 9.11×10^{-31} kg. If you plug that mass into $E_0 = mc^2$, you get an energy of 511,000 eV, or 511 keV, or 0.511 MeV.

We sometimes write the electron mass as 0.511 MeV/c^2.

It is also possible to express momentum in "energy units." An electron might have a momentum of 0.3 MeV/c.

If you are making a calculation with an equation like

$$E^2 = (mc^2)^2 + p^2c^2$$

and you want to use 0.511 MeV/c^2 for the electron mass, please do. It often simplifies the calculation. But watch out…

What is the total energy of an electron that has a momentum of 1.0 MeV/c?

$$E^2 = (mc^2)^2 + p^2c^2$$

$$E^2 = \left(\frac{0.511\,\text{MeV}}{\cancel{c^2}} \cdot \cancel{c^2}\right)^2 + \left(\frac{1.0\,\text{MeV}}{\cancel{c}}\right)^2 \cancel{c^2}$$

$$E^2 = (0.511\,\text{MeV})^2 + (1.0\,\text{MeV})^2$$

$$E^2 = (1.26\,\text{MeV}^2)$$

$$E = 1.12\ \text{MeV}$$

Notice the convenient cancellation of the c's in the 2nd step.

Avoid the common mistake: don't divide by an extra c^2 or multiply by an extra c^2 in the 2nd step.