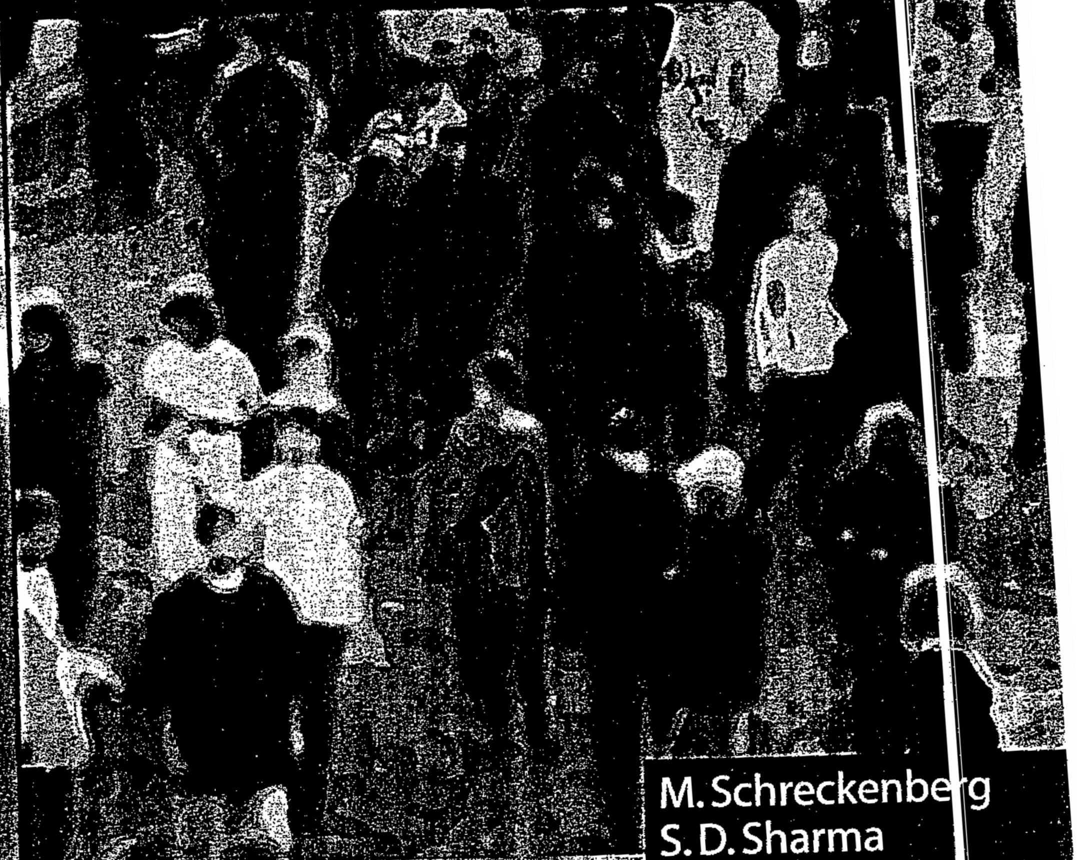

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A Speed-Concentration Relation for Bi-Directional Crowd Movements with Strong Interaction

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A bi-directional speed concentration relation is developed for crowd movements in opposite directions with strong interactions. Two types of formulations of this relation are investigated: static and dynamic. The investigation is based on observations of crowd movement in the "Jamarat" system, in connection with the pilgrimage rites in Makkah. The estimated models revealed that the impedance to one's movement due to pedestrians going in the same direction is more than twice that due to those going in the opposite direction. Furthermore, the analysis shows that there is no significant interaction term, between the concentrations of the two streams, in the estimated speed-concentration models.

1 Introduction

1.1 Background

Uni-directional Speed-Concentration Relation Studies.

These studies deal primarily with uni-directional pedestrian flows encountered on a daily basis in high traffic areas such as passageways, stairways, plazas, sidewalks, etc.

The Institute of Architecture of the Russian Academy of Arts (VAKH) appears to have been the first to study pedestrian traffic flow, in 1937 [24]. The principal scientific achievement of that study was establishing that the speed of pedestrian flow, u, is inversely proportional to the prevailing concentration of pedestrians, k, though the specific quantitative results obtained were unreliable because of the relatively small number of actual observations, and other inadequacies discussed in Predtechenskii and Milinski [24].

The Central Scientific Research Institute of the Russian Fire Protection Service (VNIIPO) conducted a study, between 1946 and 1948, to obtain reliable quantita-

† This paper was completed after the regrettable passing away of Professor Robert Herman, whose contribution to the research was immense.

tive expressions characterizing pedestrian traffic flows. Average sizes of the horizontal projection of the human body (approximated by an ellipse) for different age groups, in both winter and summer clothing, were established, based on a large number of observations made in public buildings such as theaters, and industrial, educational and transportation facilities. The study introduced the measurement of the concentration of pedestrian traffic as the sum of the horizontal projections of the individuals occupying a unit area, or the occupancy ratio. This work observationally verified that the pedestrian flow rate, q, equals the product of the stream's speed times its concentration, that is $q = u\ k$, which is a well known identity in traffic flow theory [24].

Hankin and Wright [8] investigated the flow of passengers in London subways (passageways), and observationally established relations between speed, concentration and flow for uni-directional pedestrian flow. More recent studies of the relationships among these variables include those by Oeding [17] on mixed traffic (e.g., shoppers, commuters, sports spectators), Predtechenskii [23] on mixed mass[1], Older [18] on shoppers, Navin and Wheeler [16] on students, and Fruin [7] on commuters. A synthesis of the above speed-concentration relations is shown in Fig. 1, which depicts a family of curves constructed from the measurements reported by the investigators cited previously and converted to common units. It should be noted that all of these studies applied only to streamlined and orderly pedestrian movements under approximately steady-state conditions.

While most of these studies have dealt with uni-directional pedestrian flows, Older [18], Navin and Wheeler [16] and Fruin [7] studied pedestrian flows in two opposing directions, in a sidewalk environment. These studies found that the two groups of pedestrians split into two distinct streams. Pedestrians travelling in the same direction tend to follow one another in files which interweave with those from the opposite direction. Thus, the interaction between the two flows is reduced and limited to the interface line of the two streams [18].

A linear relation was specified between speed and concentration in all the reported studies, with the exception of those by Hankin and Wright [8] and Predtechenskii [23], (Fig. 1), according to the following form:

$$u = u_f(1 - k/k_{jam}) \quad (1)$$

where u_f represents the theoretical speed attained by a traffic stream under conditions of completely free flow, with a practically unlimited amount of space per pedestrian, and k_{jam} is the jam concentration, at which all movement in a traffic stream grinds to a halt and speed becomes zero.

[1] Predtechenskii and Milinski [24] defined mass movement as that characterized by simultaneous displacement of a large number of people within a relatively limited area.

Of course, the linearity of the speed-concentration relation has long been questioned for both vehicular [9] and pedestrian flows [25], because observations generally depart from a constant slope both in the high- and low-concentration ranges, for both vehicles and pedestrians. At very high concentrations, flow resists grinding to a complete halt and attempts to maintain some speed (i.e. shuffling) even with minimal allocation of space [8], while at very low concentrations, speeds may not increase proportionately to rising space allocations because of imposed or inherent speed limits [25]. However, a thorough statistical investigation of seven different hypotheses concerning the shape of the speed-concentration relationship for vehicular flow indicated that the differences among them are rather small [5]. No such testing appears to have been performed for pedestrian flows.

1.2 Jamarat Model Framework

The Jamarat system of the Hajj is composed of three stone monuments symbolizing the devil, which are supposed to be stoned (i.e. pebbles thrown at each of them) by pilgrims in a given sequence, starting with the first, "Small", second, "Middle", then the third, "Big" Jamarah. Each monument (called Jamarah in the Arabic language) is enclosed by a ring of about 16 meters in diameter, referred to hereafter as the Jamarah ring, to collect pebbles. Pilgrims squeeze through the crowd to get their pebbles into the ring, and preferably, to hit the monument. At the same time, others who have finished stoning squeeze and push their way out. On each of the three days of stoning[2], pilgrims arrive at about the same time, creating extremely crowded conditions.

In a previous paper, the first two authors developed a general crowd behavior and movement model, which consists of a set of simultaneous partial differential equations describing the principal processes that govern the dynamics of the system [1]. The system of governing differential equations can be solved numerically by discretizing in time and space. Such solution yields a profile of the system's evolution and allows the computation of various performance measures and figures of merit. The details of the application of this general model to the Jamarat system of the Hajj, which focuses on the processes taking place around an individual Jamarah ring, are given elsewhere [3].

A fundamental concept in the Jamarat model is the classification of the pilgrims in the Jamarat system into two major classes: class 1 users, who have the intention of performing stoning, and class 2 users, who intend to leave the Jamarah after having completed the rite. For each class of users, two types of movements around the Jamarah can be identified: in the radial direction (forward or backward), and in the circumferential direction (clockwise or counter-clockwise). In the model, mathematical relations are defined for three fundamental processes: 1) radial

[2] On the first day of stoning, only the Big Jamarah is stoned by pilgrims over a period extending from Sunrise to Sunset. However, on each of the other days of stoning all three Jamarat are stoned in sequence, over a period extending from Noon till Sunset.

movement, 2) lateral movement, and 3) stoning process. These relations were developed and calibrated using actual measurements taken at the site. The overall model was applied to the evaluation of possible design and control strategies aimed at improving the efficiency and throughput of the system.

The radial movements of pilgrims at the Jamarah were modeled by the mathematical product of the prevailing radial speed and the corresponding concentration. The average radial speed of a given class of users at a given location around the Jamarah ring is a function not only of the concentration of users of the same class in that location, but the concentration of both classes of users.

The speed-concentration model, for pedestrian traffic in a bi-directional movement setting, is one of the fundamental relations used in the Jamarat model. A discussion of the theoretical aspects of the bi-directional speed-concentration relation specification and calibration is given next, followed by a description of the data collection and reduction procedures employed in the observational study. Data analysis and results are then presented.

2 Theoretical Setting

As stated earlier, previous research on pedestrian movement has dealt mainly with orderly and streamlined flows; hence, the speed-concentration relations developed by such studies were for uni-directional pedestrian streams. A linear speed-concentration formulation was used by most of these studies, but with different parameter values for different types of pedestrians (Eq. 1; Fig. 1).

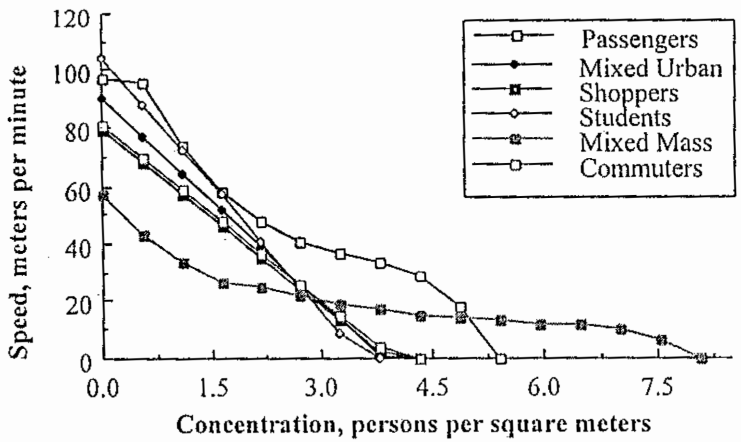

Figure 1: Speed-concentration relations for normal steady-state uni-directional pedestrian flows.

Although previous research on pedestrian movement has considered such areas as stairways, escalators, and merging-crossing situations, no formal models of pedestrian traffic flow in multi-directional movement settings have been developed [2]. It is the objective of this paper to develop such models for pedestrian movement in

two opposite directions with strong interactions. This section starts by a presentation of the relevant traffic stream models of vehicular traffic flow, then analogies are drawn to develop pedestrian speed-concentration models.

2.1 Model Specification

In modeling vehicular traffic flow, the most widely known dynamic formulations can be classified as either simple continuum or high order continuum, e.g., [10-12, 14, 19-21]. The simple continuum formulation is based on the conservation equation supplemented by a quasi static equation of state. In the higher order continuum models, a so-called "momentum equation" (actually an acceleration equation) is added to the conservation in order to account for purely dynamic effects [14]. By analogy, pedestrian traffic flow can be modeled using both types of formulations. The simple continuum formulation is considered first, where a "static" speed-concentration relation is specified. The static model is most naturally applicable under equilibrium conditions, where speeds and concentrations are only slowly-varying over time. However, in the simulation of both vehicular and pedestrian traffic, this type of model has also been considered as an instantaneous relation between prevailing speeds and concentrations. Such interpretation may be questionable. Thus, the discussion is expanded to the higher order continuum formulation and a "dynamic" speed-concentration model specification.

2.1.1 Static Model

It was postulated, in the previous section, that the radial speed of either class of users (i.e. pilgrims seeking to perform and those who have completed the stoning process at the Jamarah) is a function of the total concentration of both classes:

$$u_{1F} = \text{Max}(u_{min}, f_1(k_1, k_2)) \quad (2a)$$

$$u_{2B} = \text{Max}(u_{min}, f_2(k_1, k_2)) \quad (2b)$$

where u_{1F} is the speed of class 1 users moving towards the Jamarah ring (i.e. forward radial direction), u_{2B} is the speed of class 2 users moving away from the Jamarah ring (i.e. backward radial direction), u_{min} is the shuffling speed, and k_m is the class m users concentration (m=1, 2). The presence of the laterally moving pedestrians will also impede forward and backward radial movement. Thus, the radial speed of a class m user (m=1, 2) is a function not just of the radial concentrations, k_{mr} but of the total concentrations (i.e. radial plus lateral), k_m. The functional forms, $f_1(.)$ and $f_2(.)$, relating pedestrians' radial speed to the two types of concentrations, are specified below.

First, we consider a linear speed-concentration model with speed as a function only of the concentrations of both user types:

$$u_{1F} = u_{f1} [1 - (\alpha_{11} k_1 + \alpha_{21} k_2)/k_{jam}], \text{ for } u_{1F} > u_{min} \quad (3a)$$

$$u_{2B} = u_{f2}\,[1 - (\alpha_{12}\,k_2 + \alpha_{22}\,k_1)/k_{jam}],\ \text{for}\ u_{2B} > u_{min} \tag{3b}$$

where u_{fm} is the class m users' free-flow speed (m=1, 2), and α_{1m} and α_{2m} are parameters (expected to be positive) that reflect the relative impedance, on one's speed, of the concentration of the pedestrian's own stream vs. that of the opposing stream, respectively. A key question that arises is: which concentration type exerts greater impedance on an individual's speed, his/her own stream concentration or that of the opposite stream? It seems reasonable to argue that the impedance, on an individual's speed, of pedestrians moving in same direction is greater than that of those moving in the opposite one, i.e., $\alpha_{1m} > \alpha_{2m}$, (m=1, 2). The speed of an individual moving within a group of people in one direction is controlled by the aggregate speed of the group, leaving the pilgrim little control over his/her own speed. On the other hand, an individual going against the main flow can often exert some degree of will in maneuvering his/her way in the crowd. It seems that a counter-argument (i.e., $\alpha_{2m} > \alpha_{1m}$) might be more plausible for pedestrian flows under low levels of crowding, and hence higher speed of movements, than under crowded conditions. This question is investigated observationally in Section 4.

Another important question is whether the speed-concentration relation for class 1 users is different from that for class 2 users, or both exhibit the same behavior. In other words, do these two equations (Eqs. 3a,b) share the same underlying parameters? If so, then the calibrated speed-concentration model would not be limited to conditions prevailing for a specific direction of movement (i.e. forward or backward), but would have more general applicability.

A more general linear speed-concentration formulation includes the interaction between the two opposing streams, captured by a multiplicative interaction term in the specification, namely:

$$u_{1F} = u_{f1}\,[1-(\alpha_{11}\,k_1 + \alpha_{21}k_2 + \alpha_{31}k_1k_2/k_{jam})],\ \text{for}\ u_{1F} > u_{min} \tag{4a}$$

$$u_{2B} = u_{f2}\,[1-(\alpha_{12}\,k_2 + \alpha_{22}\,k_1 + \alpha_{32}k_1k_2/k_{jam})],\ \text{for}\ u_{2B} > u_{min} \tag{4b}$$

The inclusion of the interaction term is useful for investigating if the effect of the concentration of class m users (m=1, 2) on the speed of movement is independent of the concentration of the other class of users. For example, consider the impact of a change in k_1 on its forward radial speed, u_{1F}. Without the interaction term, the effect $(\partial u_{1F}/\partial k_1)$ would be $(-\alpha_{11})(u_{f1}/k_{jam})$. However, with the interaction, the effect would be $(-\alpha_{11} - \alpha_{31}\,k_2)(u_{f1}/k_{jam})$. Thus, the effect of k_1 on u_{1F} would depend on the level of k_2. If α_{31} is positive, then the negative effect of k_1 on u_{1F} will increase as the value of k_2 increases. The two static models, with and without the interaction term, are calibrated in Section 4.

2.1.2 Dynamic Model

The simple continuum model does not include acceleration and inertia effects; these can be taken into account by the momentum equation in the higher order models. For example, Payne's model for vehicular traffic [19] employs the usual continuity equation and the following "momentum equation" :

$$\partial u/\partial t = -u\,(\partial u/\partial x) - 1/T\,(u - ue(k)) - (\upsilon/T)\,(1/k)\,(\partial k/\partial x) \tag{5a}$$

where $u=u(x,t)$ is the vehicle speed at time t at road location x, T represents a constant reaction time, $u_e(k)$ is the equilibrium speed (which would be compatible with the concentration level, k, at steady state) and $\upsilon = -1/2[du_e(k)/dk]$. Equation 5a can be written in the following difference form [19]:

$$u_j^{t+1} = u_j^{\,t} - \Delta t\,\{u^t_j\,[(u^t_j - u^t_{j-1})/\Delta x_j]$$
(convection term)
$$+\ 1/T_j\,[(u^t_j - u_e(k^t_j)]$$
(relaxation to equilibrium term)
$$+\ (\upsilon_j/T_j)\,(1/k^t_j)\,(k^t_{j+1} - k^t_j/\Delta x_j)\},\ j=1, 2, \ldots, N \tag{5b}$$
(anticipation term)

where j is the roadway section index, t is the time, Δx_j is the length of section j, Δt is the numerical solution time step, N is the total number of sections in the roadway and other terms are as previously defined. The three groups of terms express three physical processes. The first of these is convection, i.e., the fact that vehicles traveling at speed u_{j-1} in the upstream section (section j-1) will tend to continue to travel at that speed as they enter section j. The second represents the tendency of drivers to adjust their speeds to the equilibrium one. The third term is a model of anticipation of travel conditions ahead; i.e. drivers tend to slow down if the concentration is seen to be increasing.

However, in their review of continuum models of freeway vehicular flow, Michalopoulos and Beskos [14] concluded that Payne's model is valid only at low concentrations, and they proposed that an improved practical treatment would be to drop the "momentum equation" at high concentrations; they suggested a concentration threshold of $k_{jam}/3$, beyond which the "momentum equation" can be dropped.

By analogy to vehicular traffic flow, we propose here to explore a dynamic pedestrian speed-concentration relationship. However, data collected in connection with this study, as described in the next sub-section, does not allow the investigation of all the three processes proposed in connection with Eq. 5. Only the relaxation to equilibrium term is applicable since the data are restricted to a single section of the physical space. By dropping the convection and anticipation terms, and changing the notation to fit ours, Eq. 5b can be written as:

$$u_{1F,t+1} = u_{1F,t} - \Delta t/T\,[(u_{1F,t} - u_e(k_{1,t}, k_{2,t})] \quad (6a)$$

$$u_{2B,t+1} = u_{2B,t} - \Delta t/T\,[(u_{2B,t} - u_e(k_{2,t}, k_{1,t})] \quad (6b)$$

where $u_e(k_{1,t}, k_{2,t})$ is the equilibrium speed-concentration function, which could take either form of Eqs. 3 or 4. One useful aspect of this formulation is that it enables one to calibrate the parameters of the equilibrium speed-concentration model, $u_e(k_1, k_2)$, and test whether they are equal to those obtained from a "direct" calibration of the static model.

3 Observational Work

The objective of the observational component of this study is to obtain the data needed to calibrate the speed-concentration models described in the previous section and to quantify the interactions between pilgrims approaching the Jamarah ring and those leaving it. This can be accomplished by measuring the speeds and the corresponding concentrations of both classes of pilgrims: class 1 users, who have the intention of performing stoning, and class 2 users, who are leaving the Jamarah ring after having completed stoning. Field procedures employed in this observational study are described next, followed by an outline of the data reduction method used.

3.1 Field Procedures

The study was conducted at the "Big" Jamarah during the 1989 Hajj season. Finding a vantage point from which to view the entire scene to be studied is of critical importance to providing sufficient field length and width for analysis. The observation station of the Hajj Research Center (HRC) of Umm-Al-Qura University, Makkah, Saudi Arabia, was used as the study's observation post. It is located on top of a (small) mountain overseeing the Big Jamarah, about 120 meters across from it, and at an elevation of 20 meters.

Time-lapse photography was the principal method used in this study. This method has been effectively used for pedestrian movement studies, especially for complex heavy flows that preclude manual recording [15]. It is useful in measurements of pedestrian stream speed, concentration, and flow, identification of conflicts, interactions, and interferences, as well as tracking specific movements within pedestrian streams.

The camera system, mounted on a tripod for stability and using a telephoto lens (800 mm focal length), was focused on a relatively small area (5 x 18 = 90 square meters) upstream of the Big Jamarah. This area constitutes the observation frame of this study. The selection of its location with respect to the Jamarah ring is critical in the sense that if it is adjacent to the ring, the movement of class 1 pilgrims towards the ring will be difficult to detect because the majority will be in a stationary condition, busy casting their pebbles. On the other hand, an observation frame far from the Jamarah ring would not serve the purpose of this study, because class 2 pilgrims who have finished stoning would have already chosen to go to the sides of the structure to avoid interaction with oncoming pilgrims from the middle Jamarah. The center of the selected observation frame is about 6 meters from the rim of the ring, and along its center-line.

After the initial setup, the camera was focused on that observation frame and ground measurements were conducted by two teams, one at the observation post and the other on the Jamarat bridge. Coordination was achieved through a two-way wireless communication system (walkie-talkie), with the ground team physically measuring, using a measurement tape, the dimensions of the observation frame as it appeared in the camera field of view, in addition to measuring its position relative to the Jamarah ring.

Using the time-lapse photography, with a sequence of one exposure every 2 seconds, filming was conducted during the two peak periods of the first and second days of stoning (July 13 and 14,1989). On the first day, only the Big Jamarah was stoned by pilgrims, with the peak taking place in the period of 6:00-9:00 A.M. On that morning, photography started at 6:20 A.M. for one hour, during which 15 rolls of 36-exposure colored films were used, and the average time taken for unloading the exposed film and loading the new one was about 3 minutes. The same procedure was repeated on the next day, during which all three Jamarat were stoned, with photography taking place during the different peak period of 1:00 to 4:00 P.M. (photography started at 1:20 P.M. for one hour).

3.2 Data Reduction

Using a standard slide projector, each of the above 1100 exposures were projected on a grid sheet mounted on a wall at a fixed distance from the projector. As mentioned earlier, the observation frame covers an actual area of 90 square meters, which can accommodate up to 630 pilgrims (at the observed maximum concentration of 7 p/m^2). To manually count the total number of pilgrims within this area for each slide would have been cumbersome and time consuming. A more efficient approach was employed to facilitate the data reduction process, by restricting it to a smaller frame of (2.5 m x 4 m) 10 m^2 in size at the center of the larger one. Its size (length) is such that no pilgrim can cross it in one two second interval.

For each slide, the number of class 1, class 2 and the sum of both classes of pilgrims were counted (heads only). A person was considered of class 1 if his/her face was pointed towards the Jamarah ring region, that is if his/her speed vector was directed towards the center of the Jamarah ring or within 90 degrees from it, and vice versa. In case of ambiguity, the class was identified simply by advancing or backing the slide projector by a few slides to ascertain the pilgrim's intention. The corresponding concentrations, k_1, k_2, and k, were obtained by dividing the above numbers of pilgrims by the ground area of the observation frame.

Due to the limitations of the manual data reduction method employed, it was not possible to measure the speed of movement for all pilgrims; a representative sample of each class of users was obtained instead. Each 36-exposure film was broken into four groups of 9 slides each, such that the frame length (2.5 meters) could be traversed during that period by a pilgrim moving at a shuffling speed of 500 meters per hour. Within each group of slides, and starting with the first one, five representative pilgrims of each class were selected as they entered the frame. Each of them was traced as he/she traversed the frame until one of the following three conditions occurred: (1) the pilgrim left the frame, (2) the end of the slides group was reached, or (3) he/she stopped to perform stoning (class 1 only). The pilgrim's time of travel and distance traversed were then recorded.

The selection of the representative pilgrims for speed measurement was based mainly on their identifiability and traceability, by tagging each of them with a special reference mark, such as one's clothes type and color, height, hair shape and color, and the presence of eye glasses. On the first day of stoning, it was more difficult to identify and tag individual pilgrims, because all male pilgrims were required to wear the same clothes, consisting of two pieces of seamless, stitchless white cloth, one covering the body from waist to ankle and the other worn over the pilgrim's shoulder. The difficulties associated with identifying and tracing the same pilgrim, the size of the observation frame, and the period of tracing, 18 seconds, limited the total number traced to five of each class of pilgrims. Using this approach it was possible to measure the speeds of 1200 pilgrims, of both classes.

For each class of pilgrims, within each group of slides, the space-mean speeds were then calculated:

$$u_m = \sum_{i=1}^{5} D_{im} \Big/ \sum_{i=1}^{5} t_{im} \text{ , } m = 1,2$$

where u_m is the space-mean speed of class m users (m=1, 2), in meters per hour, and D_{im} is the distance, in meters, traversed by pilgrim i of class m in time t_{im}, hours. The above speed-concentration measurements are then used to calibrate the models specified earlier, as discussed next.

4 Data Analysis and Results

Measurements of each class of pilgrims' radial speeds and the corresponding concentrations, which were collected during the first two days of stoning around the Big Jamarah, are now used to estimate the parameters of the static and dynamic speed-concentration models specified in Section 2.

The speeds of movement and concentration levels, for each class of pilgrims, were measured over a time interval of 18 seconds (i.e. 9 slides) and 2 seconds, respectively. Thus, for each speed measurement, the corresponding concentrations were obtained by averaging the respective individual measurements over the corresponding period of 18 seconds. Table 1 present summary statistics for the principal variables in the sample of observations used for estimation.

Variable	Day	Average	St. Dev.	Maximum	Minimum
Speed of Class 1 Users	1	727	197	1306	300
u_{1F}(m/hr)	2	1200	410	2284	214
	both	965	400	2284	214
Speed of Class 2 Users	1	1021	267	1667	548
u_{2B}(m/hr)	2	1519	477	2606	587
	both	1272	460	2606	548
Concentration of Class 1 Users	1	2.91	0.72	4.72	1.40
	2	1.88	0.35	2.58	1.12
k_1(p/m^2)	both	2.39	0.76	4.72	1.12
Correlation of Class 2 Users	1	1.04	0.38	1.98	0.38
	2	0.74	0.21	1.27	0.37
k_2(p/m^2)	both	0.89	0.34	1.98	0.37
Total Concentration	1	3.95	0.67	5.86	2.70
k (p/m^2)	2	2.62	0.32	3.29	1.92
	both	3.28	0.85	5.86	1.92

Table 1: Statistical Summary of Observed Speed and Concentration Variables (n = 119 groups of slides).

4.1 Static Model

For estimation purposes, Eq. 3 can be rewritten as:

$$u_{1Fi} = \beta_{01} + \beta_{11} k_{1i} + \beta_{21} k_{2i} + \varepsilon_{1i} \quad (7a)$$

$$u_{2Bi} = \beta_{02} + \beta_{12} k_{2i} + \beta_{22} k_{1i} + \varepsilon_{2i} \quad (7b)$$

where the i subscript denotes the i-th observation, i =1, ..., n (number of observations). The terms ε_{1i} and ε_{2i} are random error terms reflecting the discrepancies between observed and predicted values; β_{0m}, β_{1m} and β_{2m} (m=1,2) are the parameters to be estimated.

More generally, let u_i denote the average speed of users of a given class going in a particular direction, k_i the corresponding concentration of users of the same class, and $k_{opp,i}$ the concentration of users in the opposite direction. The corresponding speed - concentration relation can be expressed as:

$$u_i = \beta_0 + \beta_1 k_i + \beta_2 k_{opp,i} + \varepsilon_i \quad (7c)$$

The above specification can be calibrated using a pooled sample of class 1 and class 2 users, and then used to test if the two classes of users share the same underlying coefficients (i.e. that $\beta_{j1} = \beta_{j2} = \beta_j$, j = 0, 1, 2).

Calibration is performed using OLS estimation. One of the assumptions of OLS is that the random error terms are independently and identically normally distributed, i.e. that $\varepsilon_i \sim N(0, \sigma^2)$. However, because the field measurements were obtained from a time-series study, the above assumption may not hold as the errors associated with consecutive observations may be serially correlated. It is therefore necessary to test for the presence of serial correlation and correct for it if present. The standard Durbin-Watson (DW) test is used this purpose [6, 22].

By employing OLS estimation method and using the functional form specified by Eq. 7, the following static speed-concentration relations for class 1, class 2, and both classes of pilgrims were obtained, respectively:

$$u_{1F} = 1775 - 289\, k_1 - 135\, k_2 \quad (14.1)\ (-7.2)\ (-1.5) \quad R^2 = 0.32,\ DW = 1.3,\ n = 119 \tag{8a}$$

$$u_{2B} = 2194 - 341\, k_2 - 259\, k_1 \quad (14.4)\ (-3.1)\ (-5.4) \quad R^2 = 0.26,\ DW = 1.57,\ n = 119 \tag{8b}$$

$$u = 1996 - 353\, k - 182\, k_{opp} \quad (20.7)\ (-11.1)\ (-5.7) \quad R^2 = 0.35,\ DW = 1.43,\ n = 238 \tag{8c}$$

where speeds are in meter/hr, concentrations are in p/m^2, R^2 is the coefficient of determination, DW is the Durbin-Watson serial correlation test statistic, n is the sample size and the numbers in parentheses are the t-statistics for the null hypothesis that the corresponding parameter is equal to zero.

The DW test statistic calculated for each of the above models reveals that positive serial correlation is present in all three models (d_l=1.63 and d_u=1.72, at the 5 percent level of significance). These models were then corrected for serial correlation by utilizing the Durbin procedure [6, 22][1] resulting in the following model parameter estimates[2]:

$$u_{1F} = 1708 - 275\, k_1 - 80\, k_2^3 \quad (8.6)\ (-4.7)\ (-0.7) \quad R^2 = 0.2,\ DW = 2.26,\ n = 89 \tag{9a}$$

$$u_{2B} = 2278 - 450\, k_2 - 239\, k_1 \quad (8.5)\ (-2.9)\ (-2.7) \quad R^2 = 0.14,\ DW = 1.91,\ n = 89 \tag{9b}$$

$$u = 1907 - 336\, k - 128\, k_{opp} \quad (12.0)\ (-6.4)\ (-2.5) \quad R^2 = 0.2,\ DW = 2.0,\ n = 178 \tag{9c}$$

We then test if the two classes of pilgrims share the same underlying coefficient values. The Chow test is used for this purpose [13]. Under the null hypothesis that the restrictions are valid, the test statistic is F-distributed with (r,N-GP) degrees of freedom, where r denotes the number of restricted parameters values, N the number of observations, G is the number of groups and P is the numbers of parameters to be estimated in the model.[3] For our case, the calculated F^* statistic equals 1.51, while the theoretical value of $F_{3,172}$ at the 5 percent level of significance is 2.68; therefore, we can conclude that the restrictions are valid, i.e., coefficients of the speed concentration relation are the same for the two classes of pilgrims and are best estimated using the pooled model (Eq. 9c).

Under uni-directional pedestrian flow (i.e. k_{opp}=0), and assuming that the intercept of Eq. 9c represents the free-flow speed (u_f=1907 m/hr), the jam concentration k_{jam}, can be found to equal 5.68 p/m^2 (=1907/336). Using this jam concentration, the general static bi-directional speed concentration model can be rewritten as:

$$u = 1907\ [1 - (k + 0.38\, k_{opp})/k_{jam}] \tag{10}$$

An interesting aspect of this relation is that the impedance exerted on the average speed of a stream of pilgrims by pilgrims moving in the same direction is more than twice that of those moving in the opposing one.

The other linear speed-concentration formulation, with multiplicative interaction terms (Eq. 4,) was calibrated, using the OLS estimation method, and the following models were obtained for class 1, class 2, and both classes of pilgrims, respectively, after correcting for (positive) serial correlation that was present in all models:

[1] The Durbin procedure assumes a first-order auto-correlation model of the form $\varepsilon_i = \rho\, \varepsilon_{i-1} + \upsilon_i$, where $1<\rho\leq 0$ and $\upsilon_i \sim N(0, \sigma^2)$ is independent of other errors over time as well as of the ε's. Any standard econometrics textbook can be consulted for a review of serial correlation correction methods [13,22].

[2] Note the lower values of R^2 after correcting for positive serial correlation, where its presence resulted in underestimation of the true residual variance, hence over-estimating R^2.

[3] The test statistic is calculated as $F^* = [(Q^R - Q^U)/r]/[Q^U/(N-GP)]$, where Q^R is the sum of squared errors in the pooled (restricted model), Q^U is the total sum of squared errors taken over all the separate models, and the other terms are defined in the text.

$$u_{1F} = 1960 - 380\ k_1 - 345\ k_2 + 111\ k_1 k_2$$
(4.8) (-2.3) (-0.9) (0.66)
$R^2 = 0.22$, DW = 2.27, n = 89 (11a)

$$u_{2B} = 2834 - 1174\ k_2 - 469\ k_1 + 302\ k_1 k_2$$
(5.2) (-2.1) (-2.1) (1.3)
$R^2 = 0.16$, DW = 1.89, n = 89 (11b)

$$u = 1927 - 348\ k - 141\ k_{opp} + 9.2\ k\, k_{opp}$$
(10.2) (-3.9) (-1.6) (0.14)
$R^2 = 0.21$, DW = 2.0, n = 178 (11c)

The null hypothesis that the coefficient of the interaction term is zero cannot be rejected at the 5 percent level of significance, for all the three models. Hence, the original formulation without the interaction term (Eq. 10) is retained as the static speed-concentration relationship for pedestrian movement in bi-directional movement setting.

4.2 Dynamic Model

Consider the dynamic speed-concentration formulation of Eq. 6. Assuming that the equilibrium speed-concentration function, $u_e(k_1, k_2)$, takes the form of Eq. 3, the following models were estimated for class 1, class 2, and both classes of users, respectively:

$$u_{1F,t+1} = 849 + 0.47\ u_{1F,t} - 96\ k_{1,t} - 108\ k_{2,t}$$
(4.1) (4.8) (-2.0) (-1.2)
$R^2 = 0.39$, DW = 2.17, n = 89 (12a)

$$u_{2B,t+1} = 1327 + 0.36\ u_{2B,t} - 165\ k_{2,t} - 141\ k_{1,t}$$
(4.6) (3.3) (-1.3) (-2.3)
$R^2 = 0.3$, DW = 1.85, n = 89 (12b)

$$u_{t+1} = 1042 + 0.43\ u_t - 166\ k_t - 67\ k_{opp,t}$$
(6.0) (6.0) (-4.0) (-1.8)
$R^2 = 0.4$, DW = 1.93, n = 178 (12c)

Note that the DW test statistic calculated for each of the above models indicates that, as expected, serial correlation is not significant. This is due to the inclusion of the lagged speed variable in the specification.

The Chow test is then performed to determine if the two classes share the same parameter values. The calculated F^* statistic is 1.13, while the theoretical value of $F_{4,170}$ at the 5 percent level of significance is 2.45, thus, it can be concluded that the coefficients of the dynamic model are the same for the two classes, and are best

estimated by the values shown in Eq. 12c. By comparing Eqs. 5b and 12c, the implied value of the relaxation reaction time, T, can be found to be ≃ 32 seconds, by substituting the value of Δt=18 seconds in the expression of the coefficient of u_t [(1-Δt/T)=0.43]. Hence the implied equilibrium speed-concentration model takes the form:

$$u_e(k, k_{opp}) = 1828 - 291\ k - 118\ k_{opp} \quad (13)$$

When the equilibrium speed-concentration model was assumed to take the form of Eq. 4, which involves an interaction term, the following dynamic models were estimated, for class 1, class 2, and both classes of users:

$$u_{1F,t+1} = 2165 + 0.44\ u_{1F,t} - 234\ k_{1,t} - 455\ k_{2,t} + 141\ k_{1,t}\ k_{2,t}$$
(3.2) (4.5) (-1.8) (-1.4) (1.1)
$R^2 = 0.4$, DW = 2.22, n = 89 (14a)

$$u_{2B,t+1} = 2448 + 0.34\ u_{2B,t} - 445\ k_{2,t} - 251\ k_{1,t} + 112 k_{1,t}\ k_{2,t}$$
(3.1) (3.0) (-1.0) (-1.4) (0.7)
$R^2 = 0.3$, DW = 1.9, n = 89 (14b)

$$u_{t+1} = 1879 + 0.42\ u_t - 194\ k_t - 94\ k_{opp} + 22\ k_t\ k_{opp,t}$$
(5.3) (5.8) (-2.5) (-1.3) (0.4)
$R^2 = 0.4$, DW = 1.93, n = 178 (14c)

As was the case with the corresponding static model, the null hypothesis of zero interaction term coefficient cannot be rejected at the 5 percent level of significance. Hence the other dynamic model, without the interaction term (Eq. 12c), is retained for further investigation. The dynamic speed-concentration model can then be rewritten in a form similar to Equation 6 as follows:

$$u_{t+1} = u_t - \Delta t/32\ [(u_t - (1828 - 291\ k_t - 118\ k_{opp,t})] \quad (15)$$

We can test if the differences between the "equilibrium" speed-concentration model parameters are different from those of the previously calibrated "static" model (Eq. 10). To do so, the static model parameters were restricted to those of the equilibrium one (Eq. 13), and two regression models were considered: with and without the restriction. The sum of squared errors for the restricted model, Q^R, and the unrestricted one (corrected for serial correlation), Q^U, were calculated as follows, respectively:

$$Q^R = \sum_{i=1}^{n}\left[u_i - \left(1828 - 291k_i - 118k_{opp,i}\right)\right]^2 \quad (16a)$$

$$Q^U = \sum_{i=1}^{n}\left[u_i - \left(1907 - 336k_i - 128k_{opp,i}\right)\right]^2 \quad (16b)$$

Thus, the calculated F* statistic is equal to 1.1, while the theoretical value of $F_{3,175}$ at the 5 percent level of significance is 2.68; thus, we can conclude that the restrictions are valid, i.e., the calibrated static speed-concentration model parameters (Eq. 10) are not significantly different from those of the equilibrium model.

5 Conclusions

In this paper, two types of speed-concentration model specifications were investigated for pedestrian movement in two opposite directions: static and dynamic. The static formulation assumes that the average speed of a stream of pedestrians is determined solely by the prevailing concentration of those moving in the same direction as well as those moving in the opposite one. On the other hand, the dynamic formulation assumes that this average speed at a given time further depends also on the speed at the previous time step.

Both types of models were estimated, using data collected for this purpose. The estimated models revealed some interesting features of the process. The speed of a group of pedestrians moving in one direction, in a bi-directional movement setting, was found, as expected, to depend on the concentration levels of each of the two opposing pedestrian streams present. Furthermore, these models showed that the impedance, on users' speed, of people moving in the same direction is more than twice that of those moving in the opposite direction. Another important aspect of these calibrated models is that the marginal impedance of each of the two types of concentrations involved, on the speed of movement, is independent of the level of the other stream concentration. There is no interaction term between the two types of concentration in the estimated speed-concentration model.

The estimation results of the dynamic model, suggested that it takes 32 seconds, on average, for a stream of pedestrians to adjust its average speed of movement to the equilibrium one. This value was inferred from the calibrated results, though not directly identified in the calibration. However, this value may be somewhat higher than one would expect given the flexibility of human beings and their ability to change speed quickly in response to stimulus. Thus, more investigation of the dynamic formulation, which incorporates all three processes proposed in connection with Eq. 5, is needed, before a more definitive conclusion is reached.

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