KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

Second Semester (1431/1432)

Solution Final Exam

Programmable Calculators are Not Authorized

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 50 Time: Three hours

Marks:

Multiple Choice (1-20)	
Question # 21	
Question # 22	
Question # 23	
Question # 24	
Question # 25	
Question # 26	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$\{a,b,c,d\}$	d	a	b	c	d	b	c	d	b	a	a	c	a	a	b	c	c	d	a	b

Q. No: 1 $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(\dfrac{k}{n^2}\right)$ is equal to:

(a) 0 (b) ∞ (c) 2 (d) $\frac{1}{2}$

Q. No: 2 The average value of the function $f(x) = e^x$ on $[0, \ln 2]$ is equal to:

(a) $\frac{1}{\ln 2}$ (b) $\frac{2}{\ln 2}$ (c) 1 (d) 2

Q. No: 3 The value of the integral $\displaystyle\int_0^2 |x-1|\,dx$ is equal to:

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2

Q. No: 4 If $F(x) = x\displaystyle\int_{\sqrt{\pi}}^{x} \cos(t^2)\,dt$, then $F'(\sqrt{\pi})$ is equal to:

(a) 0 (b) $\sqrt{\pi}$ (c) $-\sqrt{\pi}$ (d) 1

Q. No: 5 The integral $\displaystyle\int \frac{\tan^2 x}{\sec x}dx$ is equal to:

(a) $\ln|\sec x + \tan x| + \sin x + c$ (b) $\ln|\sec x| - \sin x + c$

(c) $\ln|\sec x + \tan x| - \cos x + c$ (d) $\ln|\sec x + \tan x| - \sin x + c$

Q. No: 6 If $f(x) = \ln(\ln x)$ then $f'(x)$ is equal to:

(a) $\frac{1}{\ln x}$ (b) $\frac{1}{x\ln x}$ (c) $\frac{-1}{(\ln x)^2}$ (d) $\frac{-1}{x\ln x}$

Q. No: 7 $\lim\limits_{x\to\infty}\left(\frac{x^2}{x-1} - \frac{x^2}{x+1}\right)$ is equal to:

(a) ∞ (b) 1 (c) 2 (d) 0

Q. No: 8 To evaluate the integral $\displaystyle\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}dx$, we use the substitution:

(a) $u = \sqrt{x}$ (b) $u = \sqrt[4]{x}$ (c) $x = \sqrt[6]{u}$ (d) $u = \sqrt[6]{x}$

Q. No: 9 The integral $\displaystyle\int \frac{\cosh x}{1+\sinh^2(x)}dx$ is equal to:

(a) $-\tan^{-1}(\sinh x) + c$ (b) $\tan^{-1}(\sinh x) + c$ (c) $\frac{1}{1+\cosh x} + c$ (d) $\tanh^{-1}(\sinh x) + c$

Q. No: 10 To evaluate the integral $\int \frac{\sqrt{x^2-25}}{x}dx$, we use the substitution:

$(a)\ \ x = 5\sec\theta \qquad (b)\ \ x = 25\sec\theta \qquad (c)\ \ x = 5\tan\theta \qquad (d)\ \ x = 25\tan\theta$

Q. No: 11 The partial fraction decomposition of $\frac{x^3+2}{(x^2-1)(x^2+2)}$ takes the form:

$(a)\ \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+2} \quad (b)\ \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x^2+2} \quad (c)\ \frac{A}{x^2-1} + \frac{B}{x^2+2} \quad (d)\ \frac{Ax+B}{x^2-1} + \frac{Cx+D}{x^2+2}$

Q. No: 12 The improper integral $\int_0^1 x^{\alpha}dx$ converges **if and only if**

$(a)\ \ \alpha > 1 \qquad (b)\ \ -2 < \alpha < -1 \qquad (c)\ \alpha > -1 \qquad (d)\ \ \alpha < -2$

Q. No: 13 The area of the region bounded by the graphs of the functions $x = -y^2$ and $x = -1$ is equal to:

$(a)\ \ \frac{4}{3} \qquad (b)\ \ \frac{2}{3} \qquad (c)\ \ \frac{8}{3} \qquad (d)\ \ \frac{3}{4}$

Q. No: 14 Let C be the curve given parametrically by: $x = t^2 + t\ ,\ y = t^2 + 3\ \ t \in \mathbb{R}$.

The point on C at which the slope of the tangent line equal to 2 is given by:

$(a)\ \ (0,4) \qquad (b)\ \ (2,4) \qquad (c)\ \ \left(\frac{3}{4}, \frac{13}{4}\right) \qquad (d)\ \ (4,4)$

Q. No: 15 If a graph has polar equation $r = \csc\theta$, then its equation in $xy-$system is:

$(a)\ \ x = 1 \qquad (b)\ \ y = 1 \qquad (c)\ \ x + 1 = 0 \qquad (d)\ \ y + 1 = 0$

Q. No: 16 The length of the curve $C : x = \cos(4t)\ ,\ y = \sin(4t)\ ,\ \ 0 \le t \le \frac{\pi}{4}$ is equal to:

$(a)\ \ \frac{\pi}{2} \qquad (b)\ \ 2\pi \qquad (c)\ \ \pi \qquad (d)\ \ 4\pi$

Q. No: 17 The integral for finding the surface of revolution obtained by revolving the curve $y = \sqrt{4-x^2}$, $x \in [-2,2]$ about the $x-$axis is:

$(a)\ \int_{-2}^{2} 2\pi\left(\sqrt{4-x^2}\right)^2 dx \quad (b)\ \int_{-2}^{2} 2\pi\sqrt{4-x^2}dx \quad (c)\ \int_{-2}^{2} 4\pi dx \quad (d)\ \int_{-2}^{2} \pi\sqrt{4-x^2}dx$

Q. No: 18 If a point has $xy-$coordinates $(x,y) = (1,1)$ then one of its (r,θ)-coordinates is:

$(a)\ \ \left(1, \frac{\pi}{4}\right) \qquad (b)\ \ \left(2, \frac{\pi}{4}\right) \qquad (c)\ \ \left(\sqrt{2}, \frac{-\pi}{4}\right) \qquad (d)\ \ \left(-\sqrt{2}, \frac{5\pi}{4}\right)$

Q. No: 19 The equation in polar coordinates for the line $y = x - 1$ is:

$(a)\ r = \frac{1}{\cos\theta - \sin\theta} \quad (b)\ r = \frac{1}{\cos\theta + \sin\theta} \quad (c)\ r = \frac{1}{\cos\theta} + \frac{1}{\sin\theta} \quad (d)\ \ r = \cos\theta + \sin\theta$

Q. No: 20 The parametric equations of the circle centered at the origin and with radius 5 is given by

$(a)\ \begin{cases} x = \cos(5\theta) \\ y = \sin(5\theta) \end{cases} \quad (b)\ \begin{cases} x = 5\cos(\theta) \\ y = 5\sin(\theta) \end{cases} \quad (c)\ \begin{cases} 5x = \cos(\theta) \\ 5y = \sin(\theta) \end{cases} \quad (d)\ \begin{cases} x = \cos(\theta) \\ y = \sin(\theta) \end{cases}$

Full Questions

Question No: 21 Approximate the integral $\int_0^2 \frac{x}{\sqrt{x+1}}dx$ using the **Simpson's rule** with $n = 4$. [**5**]

Solution:

Let $f(x) = \frac{x}{\sqrt{x+1}}$.

$\Delta x = \frac{2}{4} = 0.5$

$x_0 = 0,\ \ x_1 = 0.5,\ \ x_2 = 1,\ \ x_3 = 1.5\ \text{ and }\ x_4 = 2.$ (1)

$$\begin{aligned}\int_0^2 \frac{x}{\sqrt{x+1}}dx &\approx \tfrac{2-0}{3\times 4}\{f(0) + 4f(0.5) + 2f(1) + 4f(1.5) + f(2)\} && (1)\\ &= \tfrac{1}{6}\{0 + 4\,(0.408\,25) + 2(0.707\,11) + 4(0.948\,68) + 1.\,154\,7\} && (1)\\ &= \tfrac{1}{6}\{0 + 1.\,633 + 1.\,414\,2 + 3.\,794\,7 + 1.\,154\,7\}\\ &= \tfrac{1}{6}\{7.\,996\,6\} \approx 1.\,332\,8 && (2)\end{aligned}$$

Question No: 22 **Evaluate** $\int \frac{x}{\sqrt{3-2x-x^2}}dx$ [**5**]

Solution:

Let $u = x + 1 \Rightarrow du = dx$ (0.5)

$$\begin{aligned}\int \frac{x}{\sqrt{3-2x-x^2}}dx &= \int \frac{x}{\sqrt{4-(x+1)^2}}dx && (0.5)\\ &= \int \frac{u-1}{\sqrt{4-u^2}}du && (1)\\ &= -\sqrt{4-u^2} - \sin^{-1}\left(\tfrac{u}{2}\right) + c && (1+1)\\ &= -\sqrt{4-(x+1)^2} - \sin^{-1}\left(\tfrac{x+1}{2}\right) + c && (1)\end{aligned}$$

Question No: 23 **Evaluate** $\int \frac{x^2+1}{x\,(x-2)}dx$ [**5**]

Solution:

$$\frac{x^2+1}{x\,(x-2)} = 1 - \tfrac{1}{2x} + \tfrac{5}{2(x-2)} \qquad (0.5+1+1)$$

So

$$\int \frac{x^2+1}{x\,(x-2)}dx = x - \tfrac{1}{2}\ln|x| + \tfrac{5}{2}\ln|x-2| + c \qquad (0.5+1+1)$$

Question No: 24 Determine whether the improper integral $\int_{-\infty}^{0} xe^{2x}dx$ converges or diverges and if it converges find its value [**5**]

Solution:

$$\begin{aligned}\int_{-\infty}^{0} xe^{2x}dx &= \lim_{t\to-\infty}\int_{t}^{0} xe^{2x}dx \\ &= \lim_{t\to-\infty}\left(\left[\tfrac{1}{2}xe^{2x}\right]_t^0 - \tfrac{1}{2}\int_t^0 e^{2x}dx\right) \quad (2) \\ &= \lim_{t\to-\infty}\left[-\tfrac{1}{2}te^{2t} - \tfrac{1}{4} + \tfrac{1}{4}e^{2t}\right] \quad (1) \\ &= -\tfrac{1}{4} \quad (1)\end{aligned}$$

So improper integral $\int_{-\infty}^{0} xe^{2x}dx$ converges and $\int_{-\infty}^{0} xe^{2x}dx = -\frac{1}{4}$. (1)

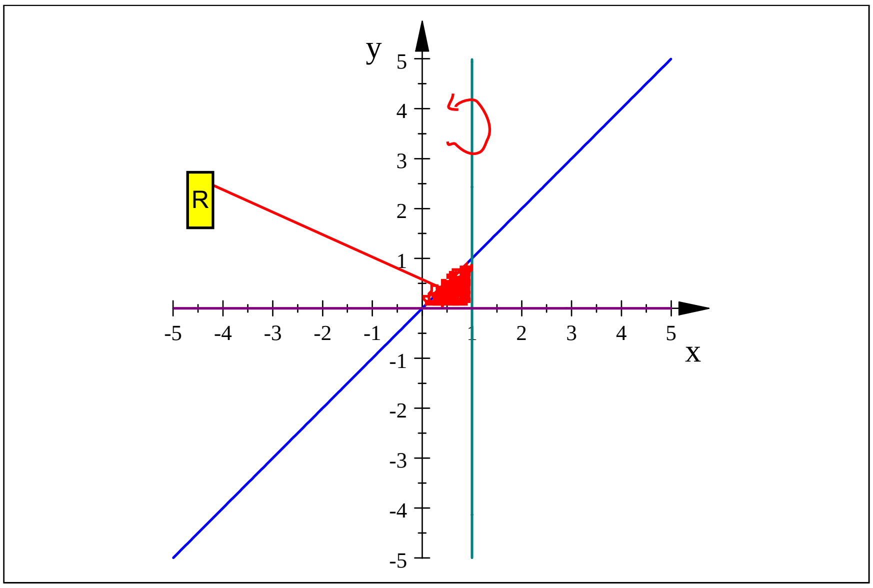
Question No: 25 *a*) **Sketch** the region R bounded by the graph of the equations:

$y = x$, $y = 0$ and $x = 1$. [**2**]

b) **Find** the **volume** of the solid generated by revolving the region R around the line $x = 1$. (Use any method) [**3**]

Solution:

a)



$y = x$ and $x = 1$ (2)

$b)$

By cylindricall shell Method OR By Disk Method

$$V = 2\pi \int_0^1 (1-x)\, x dx \quad (2) \qquad\qquad V = \pi \int_0^1 (1-x)^2\, dx \quad (2)$$

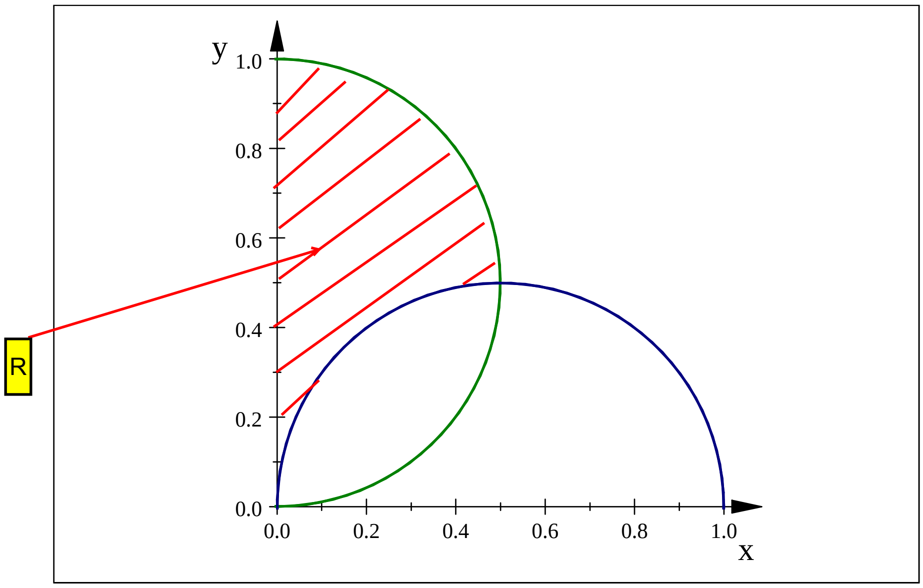
$$= \frac{\pi}{3} \quad (1) \qquad\qquad = \frac{\pi}{3} \quad (1)$$

Question No: 26 $a)$ **Sketch** the region R that is **inside** $r = \sin\theta$ and **outside** $r = \cos\theta$, for $\theta \in \left[0, \frac{\pi}{2}\right]$. [**3**]

$b)$ **Set up** (Do not evaluate) an integral that can be used to find the **area** of R. [**2**]

Solution:

$a)$



$r = \sin\theta$ and $r = \cos\theta$ (3)

$b)$ Area (R)= $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\sin^2\theta - \cos^2\theta\right) d\theta$ (2)