KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

Second Semester (1430/1431)

Solution Final Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 50 Time: Three hours

Marks:

Multiple Choice (1-20)	
Question # 21	
Question # 22	
Question # 23	
Question # 24	
Question # 25	
Question # 26	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$\{a,b,c,d\}$	b	a	d	b	a	c	d	c‘	a	a	b	a	a	a	b	c	a	d	b	a

Q. No: 1 If $\sum_{k=1}^{6}(k^2+3k+2\alpha)=130$, then the value of α is equal to:

(a) 2 $\quad$ (b) -2 $\quad$ (c) 1 $\quad$ (d) 3

Q. No: 2 The average value of the function $f(x)=\dfrac{1}{\sqrt{x^2+1}}$ on $[0,1]$ is equal to:

(a) $\sinh^{-1}(1)$ $\quad$ (b) $\cosh^{-1}(1)$ $\quad$ (c) $\sin^{-1}(1)$ $\quad$ (d) $\tan^{-1}(1)$

Q. No: 3 The number z that satisfies the conclusion of the Mean value Theorem for $f(x)=x^2$ on $[0,2]$ is:

(a) $\sqrt{\dfrac{8}{3}}$ $\quad$ (b) $\dfrac{8}{\sqrt{3}}$ $\quad$ (c) $\sqrt{\dfrac{2}{3}}$ $\quad$ (d) $\dfrac{2}{\sqrt{3}}$

Q. No: 4 If $F(x)=\displaystyle\int_{x-1}^{x+1}\tan(t^2)\,dt$, then $F'(x)$ is equal to:

(a) $\tan(x^2+2x+1)+\tan(x^2-2x+1)$ $\quad$ (b) $\tan(x^2+2x+1)-\tan(x^2-2x+1)$

(c) $\tan(x^2+1)-\tan(x^2-1)$ $\quad$ (d) 0

Q. No: 5 The value of the integral $\displaystyle\int_0^1 3^x dx$ is equal to:

(a) $\dfrac{2}{\ln 3}$ $\quad$ (b) $\dfrac{3}{\ln 3}$ $\quad$ (c) 3 $\quad$ (d) 2

Q. No: 6 If $f(x)=x^x$, then $f'(1)$ is equal to:

(a) 0 $\quad$ (b) e $\quad$ (c) 1 $\quad$ (d) $\dfrac{1}{e}$

Q. No: 7 $\displaystyle\lim_{x\to\infty}\left(\frac{x}{\sqrt{x}-1}-\frac{x}{\sqrt{x}+1}\right)$ is equal to:

(a) ∞ $\quad$ (b) 1 $\quad$ (c) 0 $\quad$ (d) 2

Q. No: 8 If $F(x)=\tan^{-1}x+\tan^{-1}\dfrac{1}{x}\ (x\neq 0)$, then $F'(x)$ is equal to:

(a) $\dfrac{2}{1+x^2}$ $\quad$ (b) $\dfrac{-1}{1+x^2}$ $\quad$ (c) 0 $\quad$ (d) $\dfrac{1}{1+\left(\frac{1}{x^2}\right)}$

Q. No: 9 The integral $\int \frac{x}{\cos^2(x^2)} dx$ is equal to:

(a) $\frac{1}{2}\tan(x^2)+c$ (b) $\tan(x^2)+c$ (c) $\frac{1}{2}\tan(x)+c$ (d) $-\frac{1}{\cos(x^2)}$

Q. No: 10 To evaluate the integral $\int \frac{3}{x^2\sqrt{9x^2-4}} dx$, we use the substitution:

(a) $x=\frac{2}{3}\sec\theta$ (b) $x=\frac{2}{3}\tan\theta$ (c) $x=\frac{3}{2}\sec\theta$ (d) $x=\frac{4}{9}\tan\theta$

Q. No: 11 The partial fraction decomposition of $\frac{x^2-2}{1-x^3}$ takes the form:

(a) $\frac{A}{1-x}+\frac{Bx+C}{1-x+x^2}$ (b) $\frac{A}{1-x}+\frac{Bx+C}{1+x+x^2}$ (c) $\frac{A}{1-x}+\frac{Bx+C}{1+x^2}$ (d) $\frac{A}{1-x}+\frac{C}{1+x+x^2}$

Q. No: 12 The substitution $u=\tan\left(\frac{x}{2}\right)$ transforms the integral $\int \frac{1}{1+\cos x} dx$ into:

(a) $\int du$ (b) $\int \frac{1}{1+u} du$ (c) $\int \frac{2}{1-u} du$ (d) $\int \frac{2}{1+u} du$

Q. No: 13 The area of the region bounded by the graphs of $y=\cos x$, $y=\sin x$, $x=0$ and $x=\frac{\pi}{4}$ is equal to:

(a) $\sqrt{2}-1$ (b) 0 (c) $\sqrt{2}+1$ (d) $1-\sqrt{2}$

Q. No: 14 The slope of the tangent line at the point corresponding to $t=\frac{\pi}{2}$ on the curve given parametrically by the equations $x=\sin^2 t$, $y=\cos t$; $\frac{\pi}{2}\leq t\leq 2\pi$ is:

(a) ∞ (b) -1 (c) 0 (d) 1

Q. No: 15 If a graph has polar equation $r=\frac{1}{2\sin\theta+\cos\theta}$, then its equation in $xy-$system is:

(a) $x+2y+1=0$ (b) $x+2y-1=0$ (c) $2x+y+1=0$ (d) $2x+y-1=0$

Q. No: 16 The length of the curve $C: x=2\cos t,\ y=2\sin t;\ 0\leq t\leq 1$ is equal to:

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) 4

Q. No: 17 The surface area resulting by revolving the graph of the equation $x=y$, $0\leq y\leq 4$ around the $y-$axis is equal to:

(a) $16\sqrt{2}\pi$ (b) $\sqrt{2}\pi$ (c) $16\sqrt{2}$ (d) $8\sqrt{2}\pi$

Q. No: 18 If a point has $xy-$coordinates $(x, y) = (1, 1)$ then one of its (r, θ) $-$coordinates is:

(a) $\left(1, \frac{\pi}{2}\right)$ (b) $\left(1, \frac{\pi}{4}\right)$ (c) $\left(2, \frac{\pi}{4}\right)$ (d) $\left(\sqrt{2}, \frac{\pi}{4}\right)$

Q. No: 19 The slope of the tangent line to the graph of the equation $r = 2$ at $\theta = \frac{\pi}{4}$ is:

(a) 1 (b) -1 (c) 0 (d) ∞

Q. No: 20 The polar equation that has the same graph as the equation:
$x^4 + 2x^2y^2 + y^4 = 2xy$, is:

(a) $r^2 = \sin 2\theta$ (b) $r^2 = \dfrac{\sin 2\theta}{1 + \frac{1}{2}\sin 2\theta}$ (c) $r^2 = \cos 2\theta$ (d) $r^2 = \sin\theta\cos\theta$

Full Questions

Question No: 21 Approximate the integral $\int_0^2 \frac{1}{x^5+1} dx$ using the **Trapezoidal rule** with $n = 4$. [**4**]

Solution:

Let $f(x) = \dfrac{1}{x^5+1}$. $\Delta x = \frac{1}{2}$

So $x_0 = 0$, $x_1 = \frac{1}{2}$, $x_2 = 1$, $x_3 = \frac{3}{2}$ and $x_4 = 2$.

$$\begin{aligned}\int_0^2 \frac{1}{x^5+1} dx &\approx \tfrac{2-0}{2\times 4}\left\{f(0) + 2f\left(\tfrac{1}{2}\right) + 2f(1) + 2f\left(\tfrac{3}{2}\right) + f(2)\right\} \\ &\approx \tfrac{1}{4}\left\{1 + (2 \times 0.9697) + \left(2 \times \tfrac{1}{2}\right) + (2 \times 0.11636) + 0.0303\right\} \\ &\approx \tfrac{1}{4}\{1 + 1.9394 + 1 + 0.23272 + 0.0303\} \\ &\approx \tfrac{1}{4}\{4.2024\} \\ &\approx 1.0506.\end{aligned}$$

Question No: 22 **Evaluate** $\int \frac{x^2}{x^2-1} dx$. [**4**]

Solution:

$$\frac{x^2}{x^2-1} = 1 + \tfrac{1}{2(x-1)} - \tfrac{1}{2(x+1)}$$

So

$$\begin{aligned}\int \frac{x^2}{x^2-1} dx &= \int 1dx + \int \tfrac{1}{2(x-1)} dx - \int \tfrac{1}{2(x+1)} dx \\ &= x + \tfrac{1}{2}\ln|x-1| - \tfrac{1}{2}\ln|x+1| + c.\end{aligned}$$

Question No: 23 Determine whether the improper integral $\int_0^2 \frac{1}{(x-1)^{\frac{2}{3}}}dx$ converges or diverges and if it converges find its value. [**5**]

Solution:

$$\begin{aligned}
\int_0^2 \frac{1}{(x-1)^{\frac{2}{3}}}dx &= \int_0^1 \frac{1}{(x-1)^{\frac{2}{3}}}dx + \int_1^2 \frac{1}{(x-1)^{\frac{2}{3}}}dx \\
&= \lim_{t\to 1^-}\int_0^t (x-1)^{-\frac{2}{3}}\,dx + \lim_{t\to 1^+}\int_t^2 (x-1)^{-\frac{2}{3}}\,dx \\
&= \lim_{t\to 1^-}\left[\frac{(x-1)^{\frac{1}{3}}}{\frac{1}{3}}\right]_0^t + \lim_{t\to 1^+}\left[\frac{(x-1)^{\frac{1}{3}}}{\frac{1}{3}}\right]_t^2 \\
&= \lim_{t\to 1^-} 3\left[(t-1)^{\frac{1}{3}} - (-1)^{\frac{1}{3}}\right] + \lim_{t\to 1^+} 3\left[(1)^{\frac{1}{3}} - (t-1)^{\frac{1}{3}}\right]_t^2 \\
&= 3+3 \\
&= 6
\end{aligned}$$

Hence $\int_0^2 \frac{1}{(x-1)^{\frac{2}{3}}}dx$ converges and $\int_0^2 \frac{1}{(x-1)^{\frac{2}{3}}}dx = 6.$

Question No: 24 Find the length of the arc of the graph of the function

$f(x) = \int_0^x \sqrt{\cos t}\ dt;\ 0 \le x \le \frac{\pi}{2}.$ [**5**]

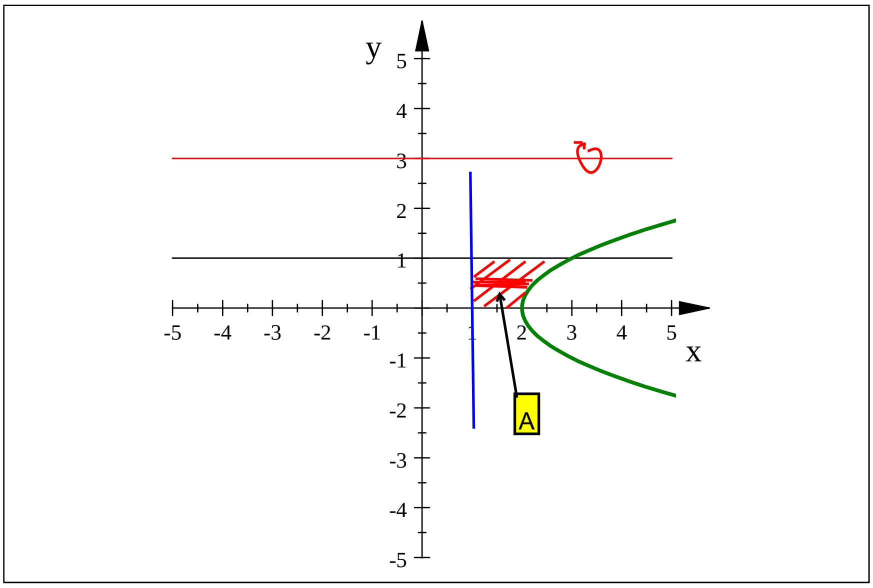
Solution:

$$\begin{aligned}
L_0^{\frac{\pi}{2}} &= \int_0^{\frac{\pi}{2}} \sqrt{1+(f'(x))^2}dx \\
&= \int_0^{\frac{\pi}{2}} \sqrt{1+(\sqrt{\cos x})^2}dx \\
&= \int_0^{\frac{\pi}{2}} \sqrt{1+\cos x}dx \\
&= \int_0^{\frac{\pi}{2}} \sqrt{2\cos^2\left(\frac{x}{2}\right)}dx \\
&= \sqrt{2}\int_0^{\frac{\pi}{2}} \left|\cos\left(\frac{x}{2}\right)\right| dx = \sqrt{2}\int_0^{\frac{\pi}{2}} \cos\left(\frac{x}{2}\right) dx \\
&= \sqrt{2}\left[2\sin\left(\frac{x}{2}\right)\right]_0^{\frac{\pi}{2}} = 2
\end{aligned}$$

Question No: 25 Let R be the region bounded by the graph of the equations $x = y^2 + 2$, $x = 1$ and the horizontal lines $y = 0$, $y = 1$.

Sketch the region R and **set up** (Do not evaluate) an integral that can be used to find the **volume** of the solid generated by revolving the region R around the **line** $y = 3$. (**Use Cylindrical Shell**) [**6**]

Solution:



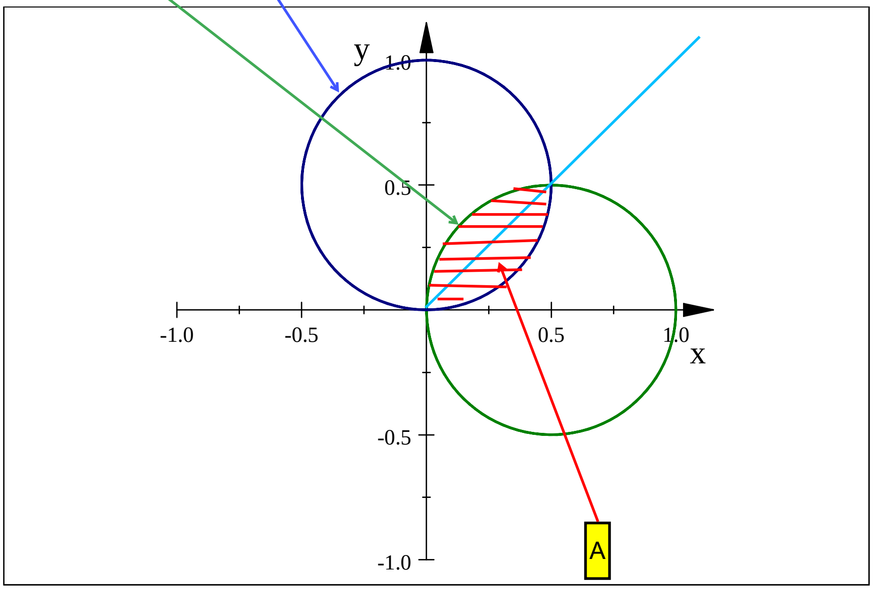
$$A = 2\pi \int_0^1 (3 - y)\,(y^2 + 1)\,dy$$

Question No: 26 Let R be the region that lies inside the graphs of **both** the equations:

$r = \cos\theta$ and $r = \sin\theta$.

Sketch the region R and **set up** (Do not evaluate) an integral that can be used to find its **area**. [**6**]

Solution:



$$A = \frac{1}{2}\int_0^{\frac{\pi}{4}} \sin^2\theta d\theta + \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2\theta d\theta$$